

Biometrics

Moderators:

JIM M. GULDIN

USDA Forest Service

and

RALPH AMATEIS

Virginia Polytechnic Institute

EMPIRICALLY DERIVED OPTIMAL GROWTH EQUATIONS FOR HARDWOODS AND SOFTWOODS IN ARKANSAS

Don C. Bragg¹

Abstract—Accurate growth projections are critical to reliable forest models, and ecologically based simulators can improve silvicultural predictions because of their sensitivity to change and their capacity to produce long-term forecasts. Potential relative increment (PRI) optimal diameter growth equations for loblolly pine, shortleaf pine, sweetgum, and white oak were fit to data from the Arkansas portion of the Eastwide Forest Inventory Data Base (EFIDB). Large sample sizes are necessary for successful application of the PRI methodology, and in aggregate almost 29,000 trees were used to develop these models. In the final model versions, only a handful (< 30 per species) of the fastest growing trees given their species, size, and growing conditions were retained from the Arkansas EFIDB. Shortleaf pine, sweetgum, and white oak all generated skewed model curves, while loblolly pine produced a monotonically declining curve. Comparison of these optimal increment models across tree size indicated that loblolly pine had higher potential than the other species until ~ 10 cm in diameter at breast height (d.b.h.), after which sweetgum and white oak overtook it at intermediate sizes. However, loblolly pine optimal performance decreased at a lesser rate than any of the other species, so that by 60 cm d.b.h. it once again had the greatest potential. The other taxa outperformed shortleaf pine throughout most of the diameter range considered, while sweetgum proved intermediate between shortleaf and white oak. These optimal diameter functions are a valuable first step in the development of forest simulators.

INTRODUCTION

Foresters have increasingly used models to predict long-term stand dynamics. Empirically based growth and yield models, e.g., Lynch and others (1999), Wykoff and others (1982), are popular because they are relatively easy to parameterize. However, the rigid nature of these designs, their finite analysis options, and their lack of ecological mechanism have limited their applicability beyond short-term growth-and-yield prediction. Ecological process models are becoming more widespread, e.g., Botkin and others (1972), Bragg (1999), Pacala and others (1993), in part because of their greater complexity and flexibility. However, these models often lack an empirical foundation and sometimes rely upon questionable assumptions. Blending the positive features of empirical and ecological models should improve the reliability of long-term forecasts of forest dynamics.

Most forest simulators include some kind of individual tree growth model. A fundamental goal of this increment model is to predict realized growth accurately, and there are at least two different ways to approach this problem. Most empirical models use a fitted statistical response where increment is either added or subtracted from a standard level, depending on how favorable conditions are for growth, e.g., Wykoff and others (1982). While commonly applied, this design limits the growth function to a specified set of modifiers, thus restricting its adaptability. The other primary approach employs a potential increment function that is rescaled downward based on departures from

optimal growth conditions, e.g., Botkin and others (1972), Bragg (2001). Thus, one predicts realized growth from its departure from optimal growth using appropriate modifier function(s). In principle, this strategy has greater flexibility for ecological modeling because environmental response functions can be more sophisticated and mechanistic. However, one of the biggest challenges to optimal growth modeling lies in the development of an acceptable response curve.

Researchers have developed and evaluated numerous designs of potential growth equations (Botkin and others 1972, Moore 1989, Pacala and others 1993, Zeide 1993). Most recently, Bragg (2001) developed the Potential Relative Increment (PRI) methodology to fit inventory data to an ecologically robust function, thus linking desirable theoretical and statistical properties. This paper presents optimal PRI increment models for loblolly pine (*Pinus taeda* L.), shortleaf pine (*P. echinata* Mill.), sweetgum (*Liquidambar styraciflua* L.), and white oak (*Quercus alba* L.) in Arkansas using data from the Eastwide Forest Inventory Data Base (EFIDB) (Hansen and others 1992).

METHODS

The details of the PRI method are beyond the scope of this paper (see Bragg 2001). Briefly, all records of the species of interest with positive growth were selected for processing. After identifying this initial group, those individuals growing at the greatest rate for each 2-cm diameter at breast height (d.b.h.) class (one tree per size class) were segregated into a maximal actual increment

¹Research Forester, USDA Forest Service Southern Research Station, P.O. Box 3516 UAM, Monticello, AR 71656 USA

Table I-Statistics on the four species extracted from the Arkansas portion of the Eastwide Forest Inventory Data Base

Species	Original sample size	Minimum d.b.h.	Maximum d.b.h.	Standard deviation
..... Centimeters				
Loblolly pine	11,340	2.8	88.6	13.52
Shortleaf pine	7,587	2.8	70.1	10.57
Sweetgum	3,906	2.8	111.3	14.13
White oak	6,089	2.8	100.8	13.70

Table P-Final model regression coefficients (b₁, b₂, and b₃), goodness-of-fit, and final sample sizes

Species	b ₁	b ₂	b ₃	R ²	Loss ^a	Final n ^b
Loblolly pine	2.708480	-1.033813	0.993497	0.9975	0.00284	29
Shortleaf pine	1.171747	-.623995	.962244	.9953	.00278	25
Sweetgum	.439226	-.053773	.944559	.9942	.00109	16
White oak	.273683	.086414	.955673	.9960	.00028	12

^a Loss = $\sum(\text{observed} - \text{predicted})^2$.

^b Final number of points used to fit the optimal potential relative increment curves and generate the R² values in this table.

pool. From these, a best subset was identified to fit the PRI growth function:

$$PRI = b_1 D_{MAX}^{b_2} b_3^{D_{MA}} \quad (1)$$

where

D_{MAX} = the d.b.h. of the maximally performing individual (by size class) and

b₁ to b₃ = species-specific nonlinear ordinary least squares regression coefficients.

Optimal increment is the product of PRI and current d.b.h., while realized increment can be estimated by multiplying optimal increment with limiting environmental scalar(s) (Bragg 2001). In this final step, factors such as competition and site quality come into play.

The spatially extensive sample found in the EFIDB covers most of the possible variation in the environment. However, the odds of finding a Forest Inventory and Analysis plot with the perfect combination of site quality, stand density, and genetics to produce a truly optimal growth environment are negligible. Therefore, the PRI methodology is a conservative representation of potential diameter growth (Bragg 2001). The inference that optimal conditions can be approximated from inventory data requires a large sample of trees of the desired species from an extensive area. To

ensure adequate representation, almost 29,000 individuals from the taxa of interest were selected from the Arkansas portion of the EFIDB (11,340 loblolly pines, 7,587 shortleaf pines, 3,906 sweetgums, and 6,089 white oaks) (table 1).

Only a small fraction (< 30 per species) of the records were retained for the final models (table 1). Loblolly pine provides an example of the iterative fitting process.

Originally, over 11,000 records were considered usable, covering most of the range of possible size and increment with little apparent measurement error (fig. 1A). The exception is an outlier identified by the arrow in figures 1A and 1 B. This tree apparently grew from 61.0 cm d.b.h. to 90.2 cm d.b.h. in 7.2 years (an average of 4.2 cm annually), a highly dubious rate given the size of the tree. Of the initial multitude of records, 42 loblolly were chosen, one for each respective diameter class (fig. 1B). Since the objective of the methodology was to identify an optimal growth curve, individuals within the d.b.h. class structure that did not maximize this function were removed (including the outlier). Thus, a final subset of 29 loblolly pines was retained for curve fitting (fig. 1C). This process was repeated for the other species until a suite of models was developed.

RESULTS AND DISCUSSION

Figure 2A illustrates that optimal increment performance is a distinct function of species and size. Translated into the more interpretable measure of potential annual d.b.h.

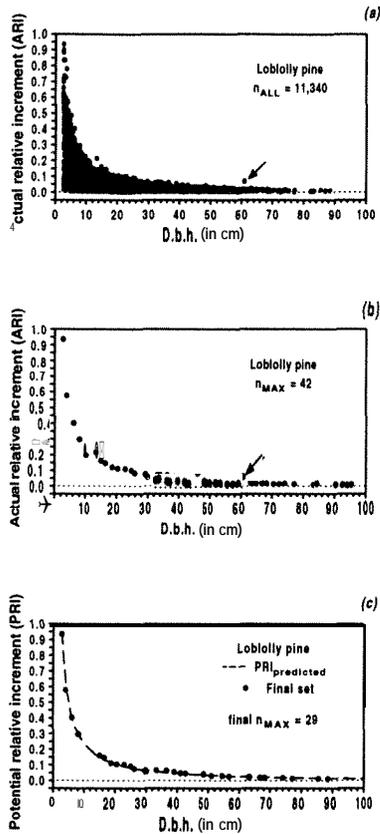


Figure 1-Graphical demonstration of the potential relative increment (PRI) methodology for loblolly pine in Arkansas. After the initial sample (n_{ALL}) of 11,340 loblolly pines was chosen (A), only the points (n_{MAX}) representing the highest growth performance within each 2-cm d.b.h. class remained after an initial filtering (B). Further discrimination resulted in a subset (final $n_{MAX} = 29$) used to develop the PRI model (C). Note the arrows in (A) and (B) that identify the outlier removed before final model fitting.

growth (fig. 2B), differences in performance become even more marked. Shortleaf pine, sweetgum, and white oak all produced skewed model curves with different local maxima and trajectories, while loblolly pine yielded a monotonically declining curve with a maximum at the smallest d.b.h. class. Thus, for the smaller diameters (< 12 cm), loblolly pine had the potential to outgrow any of the other species in this sample, especially **shortleaf** pine and **sweetgum**. However, between 12 and 30 cm d.b.h., both **sweetgum** and white oak were predicted to have higher potential performance than loblolly pine, with white oak continuing this trend to 60 cm. From 60 centimeters on, loblolly regained its dominance over the other species.

Shortleaf pine failed to approach the maximal performance of **sweetgum** and white oak until very large diameters, and never matched **loblolly's** potential. **Sweetgum** performed at an intermediate level until larger diameters were reached, upon which its optimal performance decreased noticeably. Note that these results are for predictions of potential increment, not those realized in the field: actual diameter growth will be a function of factors such as

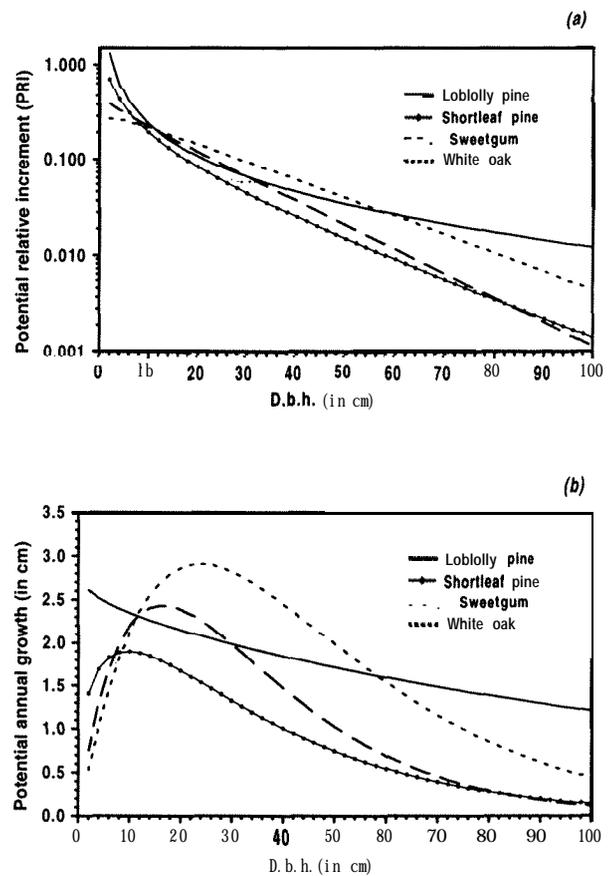


Figure 2-Annual potential relative increment (PRI) (A) and potential annual growth (B) for loblolly pine, shortleaf pine, sweetgum, and white oak. The differences in the PRI curves yield dramatic differences in optimal growth performance between species.

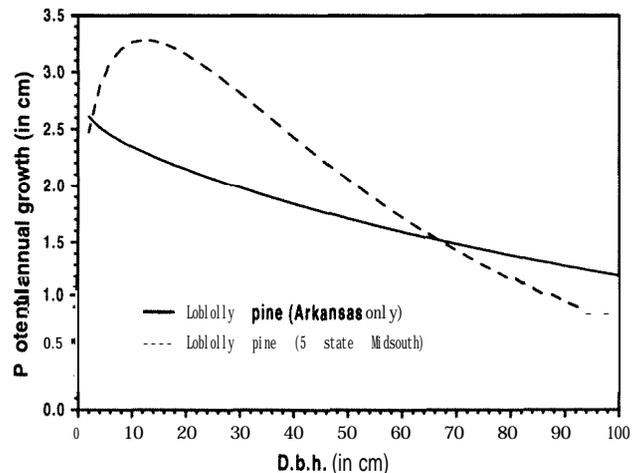


Figure S-Comparison of the Arkansas potential relative increment model of loblolly pine (solid line) and the **Midsouth** model (Unpublished manuscript, DC. Bragg, Research Forester, USFS Southern Research Station, P.O. Box 3516 UAM, Monticello, AR 71656) that included Arkansas (dashed line). The **Midsouth** model benefited from supplemented and extended size classes, thus resulting in a noticeably different prediction of potential tree performance.

localized edaphic, climatic, and competitive conditions, photosynthetic surface area, tree moisture status, genetic predisposition, or the presence of pathogens.

From these data, it appears loblolly pine has the potential to add the greatest diameter increment (2.6 cm annually) at the smallest size, while shortleaf pine peaks (~ 1.9 cm annually) at approximately 8 to 10 cm d.b.h., sweetgum reaches a maximum (-2.4 cm annually) at 15 to 18 cm d.b.h., and white oak crests (-2.9 cm annually) at approximately 25 cm d.b.h.. These results differ from a more extensive set of PRI curves fit to an inventory pool for the Midsouth (Arkansas, Louisiana, Missouri, Oklahoma, and Texas).^{*} Using loblolly pine as an example, noticeable differences in potential increment are apparent at both small and large diameters (fig. 3).

Under the Midsouth model, a skewed model form replaces the monotonically declining model of the Arkansas-only data, with a new, higher maximal annual growth peak of > 3.2 cm now found at ~ 15 cm d.b.h. Optimal growth potential remains higher until loblolly pine reaches > 65 cm d.b.h., after which it drops below the Arkansas model. The data used for the Midsouth model changed the curve shape dramatically by adding points at small diameters that produced more optimistic optimal performance while simultaneously contributing new observations in the larger diameter classes. Pooling can increase confidence in results by supplementing and/or extending the range of sample data. In some cases, though, pooling may overestimate local growth potential if environmental and genetic conditions are significantly different from the more limited study area. Because of similarities in the environmental and loblolly genetic conditions in the Midsouth, the increase in potential optimal performance noted in figure 3 should not cause major problems when applied in Arkansas.

CONCLUSIONS

Optimal tree diameter growth performance is a function of both species and size. In this Arkansas sample, loblolly pine and white oak outperformed sweetgum and shortleaf pine. However, all species considered in this paper can potentially add 2 to 3 cm of diameter annually. The ability to differentiate species performance based on standardized growth functions should help the forest research community, especially if the inventory information is widely available.

Large public databases like the EFIDB can assist the development of silvicultural and mensurational applications. Their considerable spatial extent, rigorous sampling design, and broad range of species and size classes also favor their use in other fields, especially ecological modeling. The development of empirically derived optimal growth models provides the basis for forest simulators grounded in both theory and reality.

ACKNOWLEDGMENTS

I thank the following people and institutions for their support of this work: Dave Roberts (Utah State University), Tom Crow (USDA Forest Service, North Central Research Station), and Jim Guldin (USDA Forest Service, Southern Research Station). Reviews from the following improved the quality of this manuscript: Jim Guldin, Boris Zeide (University of Arkansas at Monticello), Bernie Parresol (USDA Forest Service, Southern Research Station), and David Larsen (University of Missouri at Columbia).

LITERATURE CITED

- Botkin, D.B.; Janak, J.F.; Wallis, J.R. 1972. Some ecological consequences of a computer model of forest growth. *Journal of Ecology*. 60: 849-872.
- Bragg, D.C. 1999. Multi-scalar spatial modeling of northern forest dynamics: foundations, theories, and applications. Logan, UT: Utah State University. 532 p. Ph.D. dissertation.
- Bragg, D.C. 2001. Potential relative increment (PRI): a new method to empirically derive optimal tree diameter growth. *Ecological Modelling*. 137: 77-92.
- Hansen, M.H.; Frieswyk, T.; Glover, J.F.; Kelly, J.F. 1992. The Eastwide Forest Inventory Data Base: users manual. Gen. Tech. Rep. NC-151. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 48 p.
- Lynch, T.B.; Hitch, K.L.; Huebschmann, M.M.; Murphy, P.A. 1999. An individual-tree growth and yield prediction system for even-aged natural shortleaf pine forests. *Southern Journal of Applied Forestry*. 23: 203-211.
- Moore, A.D. 1989. On the maximum growth equation used in forest gap simulation models. *Ecological Modelling*. 45: 63-67.
- Pacala, S.W.; Canham, C.D.; Silander, J.A. 1993. Forest models defined by field measurements: I. The design of a northeastern forest simulator. *Canadian Journal of Forest Research*. 23: 1980-1988.
- Wykoff, W.R.; Crookston, N.L.; Stage, A.R. 1982. User's guide to the stand prognosis model. Gen. Tech. Rep. INT-133. Ogden, UT: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station. 112 p.
- Zeide, B. 1993. Analysis of growth equations. *Forest Science*. 39: 594-616.

² Bragg, D.C. Unpublished manuscript.

A METHOD FOR ASSESSING ECONOMIC THRESHOLDS OF HARDWOOD COMPETITION

Steven A. Knowe¹

Abstract—A procedure was developed for computing economic thresholds for hardwood competition in pine plantations. The economic threshold represents the break-even level of competition above which hardwood control is a financially attractive treatment. Sensitivity analyses were conducted to examine the relative importance of biological and economic factors in determining economic thresholds. Growth models were used to determine the level of hardwood basal area (HBA) at which the cost of hardwood control equals the reduction in net present value of the stand due to competition. A basal area prediction model was fit with absolute HBA, rather than percent HBA, and then used to simulate the effects of hardwood competition in loblolly pine plantations. Generalized yield response models at age 25 were developed by site index and HBA, and used to compute HBA when the net present value of the pine response was zero. A hardwood basal area growth model was developed for projecting hardwood basal area to age 3, which is when release treatments would be applied. Sensitivity analyses examined the relative importance of site index, interest rate, pine stumpage value, and treatment cost in determining economic thresholds. The most important biological factor was site index, and interest rate was the most important economic factor. Pine stumpage value and cost of hardwood control treatment were relatively unimportant in determining economic thresholds.

INTRODUCTION

Control of competing vegetation has become a common silvicultural practice for managing pine plantations in the Southeast. Budgetary and environmental considerations require that vegetation management treatment be prescribed on the basis of site-specific analysis of costs and benefits. To be most effective, vegetation treatments must be applied at young ages. However, information on the long-term benefits of vegetation management is inadequate, and response to different treatments cannot be reliably extrapolated to rotation age as required for economic analyses.

Research over the last 25 years has shown substantial increases in pine growth following hardwood control (Clason 1978, Cain and Mann 1980, Glover and Dickens 1985, Glover and Zutter 1993, Miller and others 1995, Quicke and others 1996). Despite these efforts, forest managers have few quantitative tools to assess "how much is too much" for specific site and stand conditions. According to Wagner (1993), developing objective and quantitative systems to evaluate the response to proposed treatments is one of the highest priorities for vegetation management research. Such decision support tools are needed to ensure that treatments are prescribed only when the long-term changes in stand development can be economically justified and balanced with ecological considerations (Wagner 1994).

The economic threshold—the hardwood density at which the discounted value of the gain in timber volume at rotation age following a competition control treatment equals the

discounted cost of the competition control treatment (Cousens 1987)—serves as a basis for justifying vegetation treatments. The economic threshold approach involves computing net present value (NPV) for competition control treatments and determining the level of hardwood competition that produces an NPV of \$0/ac in the treated stand.

$$NPV = \frac{\text{Volume Gained} \times \text{Stumpage Value}}{(1+i)^t} - \frac{\text{Treatment Cost}}{(1+i)^t} = 0 \quad [1]$$

where i = interest rate (percent), r = rotation age (years), and t = age of hardwood control treatment (years). Estimating the volume gained following competition control is essential to computing the economic thresholds.

The method of determining the economic threshold level of hardwood competition consists of 3 steps, and is demonstrated for loblolly pine plantations. Yield is simulated for various levels of site index, planting density, and HBA. The second step is to use predicted yield, pine stumpage value, hardwood treatment cost, and interest rate to compute the economic threshold level of hardwood competition at rotation age. The final step is to project the economic threshold level of hardwood competition at rotation age to the age when a release treatment would be applied. Sensitivity of biological factors (site index and planting density) and economic factors (interest rate, pine stumpage value, and treatment cost) on the economic threshold level of hardwood competition also is examined.

¹Assistant Professor, Department of Forestry, Wildlife and Fisheries, University of Tennessee, Knoxville, TN 37901-1071.

METHODS

Predicted Yield

The first step in determining economic thresholds is to predict the yield of loblolly pine plantations with varying amounts of hardwood competition. Three computer models have been developed (Burkhart and Sprinz 1984, Smith and Hafley 1986, and Knowe 1992) to simulate the effects of hardwood competition in loblolly pine plantations. Knowe (1992) compared the assumptions and methodology of these yield systems. Data used to develop the models were obtained primarily from Piedmont and Upper Coastal Plain sites. When hardwood competition is present, all three existing models produce negatively skewed diameter distributions, with predominately small-diameter trees and few large-diameter trees. These models use a negative exponential relationship between pine basal area and percent HBA: greater pine reductions occur at low levels of hardwood competition than at high levels.

A major limitation of all three existing models is that percent HBA is used as a predictor variable. Pine basal area can be obtained from total basal area and percent HBA. Furthermore, total basal area must be known in order to compute percent HBA. Therefore, using percent HBA implies that the basal area of both the pine and hardwood components is known. In addition, the long-term dynamics of percent HBA are not well documented, with one notable exception (Glover and Zutter 1993), and it cannot be reliably predicted (Harrison and Borders 1996).

A major difference in the loblolly pine plantation yield prediction systems is the amount of pine basal area displaced by hardwood competition. The Burkhart and Sprinz (1984) model implies that 1 ft²/acre of HBA replaces 1.26 ft²/acre of pine basal area at 10 percent HBA and 2.11 ft²/acre of pine basal area at 30 percent HBA. The model developed by Smith and Hafley (1986) implies replacement ratios of 0.88 ft² and 0.93 ft² of pine basal area per ft² of HBA at 10 and 30 percent HBA, respectively. The Knowe (1992)

model implies a replacement ratio of 0.97:1 (ft² pine basal area/ft² hardwood basal area) at 10 percent HBA and 0.99:1 at 30 percent HBA.

The pine basal area and diameter distribution models developed by Knowe (1992) were chosen for demonstrating the method of computing economic threshold level of hardwood competition. The pine basal area model was refit by using absolute HBA rather than percent HBA. The resulting equation accounted for only 1.5 percent less of the variation in observed pine basal area than the model with percent HBA. Dominant height, survival, individual tree height, and volume were predicted by using the functions developed by Borders and others (1990).

Loblolly pine yield at age 25 years was simulated using 0, 5, 10, 15, 20, and 25 ft²/acre of HBA in stands with site index (base age 25) values of 50 to 80 feet, in 5-foot increments, and planting densities of 500-900 trees/acre, in increments of 100 trees/acre. The relationship between loblolly pine yield and hardwood basal area was linear for all combinations of site index and planting density, so simple linear regression models were developed for each level of site index and planting density:

$$Y = b_0 - b_1 \text{HBA} \quad [2]$$

where Y = loblolly pine yield (tons/acre) at age 25 and HBA = hardwood basal area (ft²/acre). Inspection of the intercepts (b₀) and slopes (b₁) for all 30 combinations of site index and planting density indicated a linear relationship with site index but no relationship with planting density.

The final step is to project the economic threshold level of hardwood competition at rotation age (25 years) to an age when a release treatment would be applied. In this example, release treatments were applied at age 3 years. As previously mentioned, long-term data on hardwood basal area growth in loblolly pine plantations is very limited. The one notable exception involves a well-documented site preparation study in the upper Coastal Plain of Alabama

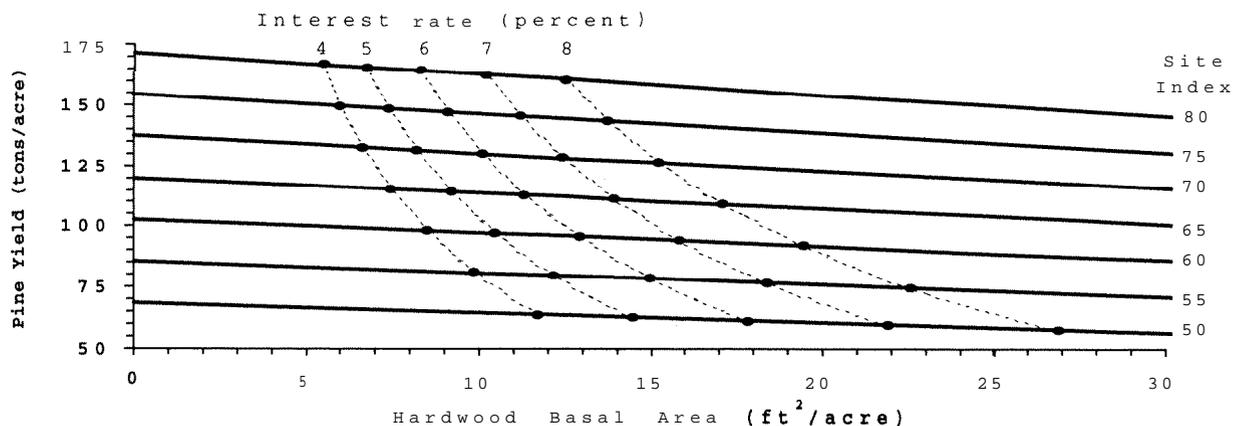


Figure 1-Relationship between loblolly pine yield and hardwood basal area at age 25 for site index between 50 and 80. The dashed lines represent the economic threshold level of hardwood basal area for interest rates between 4 and 8 percent. Additional inputs: pine stumpage value = \$30/ton and hardwood control treatment cost = \$60/acre.

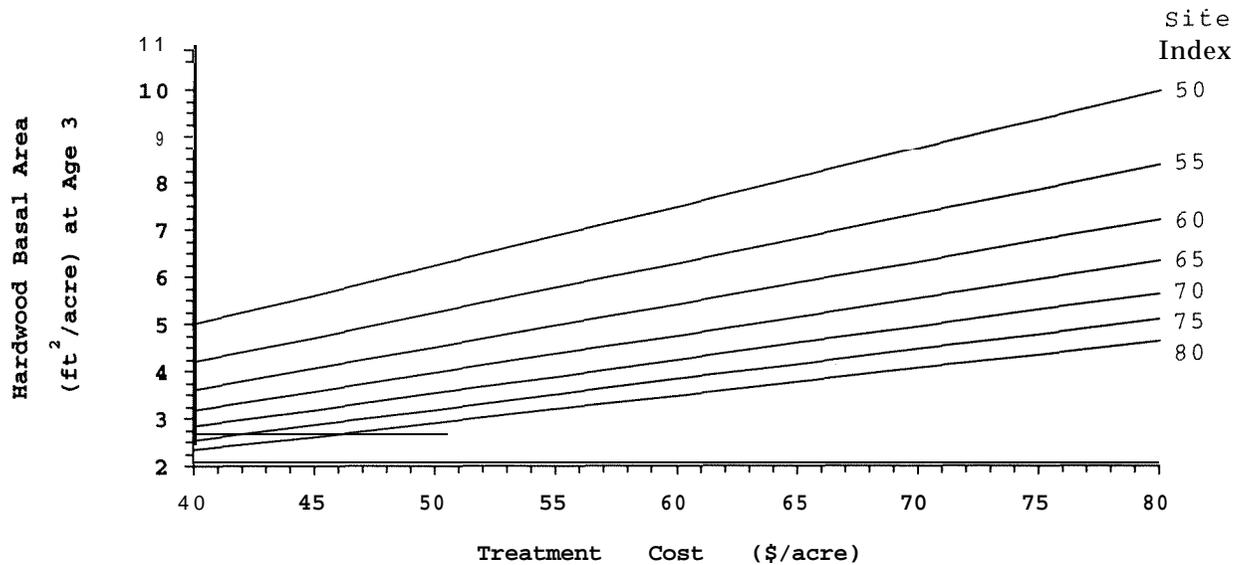


Figure Z-Economic threshold level of hardwood basal area at hardwood control treatment age 3 years for interest rates between 4 and 8 percent. Additional inputs: pine stumpage value = \$30/ton and hardwood control treatment cost = \$60/acre.

(Glover and Zutter 1993). This study included 5 replications of an untreated check plus chemical (injection and two methods of cut-surface treatment), mechanical (bulldozer scarification), and manual (girdling) treatments. Surviving hardwoods and resprouts developed along with the planted loblolly pine for 27 years after treatment.

Fifty observations of average HBA for each of the six site preparation treatments at ages 1-4, 6, 11, 13, 22, and 24 years were used in the analyses. Data for age 27 were excluded from the regressions because hardwood basal area growth was negative between ages 24 and 27 years for several treatments, and a more complex equation would be required to describe this downward trend. In addition, hardwood data were not available for one of the cut-surface treatments at plantation ages 11 and 13 years. Observed HBA-age pairs for each treatment were arranged into 45 non-overlapping growth intervals (e.g., ages 1-2, 2-3, 3-4, 4-6, 6-11, etc.). Graphs of these data suggested several potential equations for describing the observed patterns of HBA growth. Tests for differences in the growth rates among site preparation treatments were also conducted by incorporating indicator variables into the equation that best fit the observed data.

Statistical differences in hardwood growth rates were detected among the site preparation treatments. Average growth rate for the herbicide treatments (injection only, girdle+herbicide, and chain frill+herbicide) was slower than for the non-herbicide treatments (no treatment, girdle only, and bulldozer scarification). However, this difference was not of practical importance because the equation with treatment-specific growth rates accounted for only 0.3% more of the variation in projected HBA than the reduced model. A single equation can be used to predict hardwood basal area (HBA_t) at any plantation age (X) using current hardwood basal area (HBA) and current age (Age):

$$\text{HBA}_t = \text{HBA} \exp\{0.0395 \cdot (X - \text{Age})\} \quad [3]$$

This equation accounted for 98% of the variation in projected HBA. In this example, the economic threshold level of hardwood basal at a rotation age of 25 years is projected to a hardwood-control treatment age of 3 years by multiplying HBA at rotation age by 0.4194. This implies that about 42 percent of the HBA at 25-year-old stands is present in 3-year-old stands, when release treatments are applied.

Sensitivity Analyses

The sensitivity analysis was conducted for two reasons. The first is to examine predictions at extreme values of input variables to determine whether the model and assumptions are reasonable. The second reason is to assess the relative importance of biological and economic factors used in determining economic thresholds. Economic factors included in the sensitivity analyses were interest rates of 4 to 8 percent; pine stumpage values of \$25/ton to \$35/ton; and hardwood treatment costs of \$50/acre to \$90/acre. The relative importance of the biological and economic factors was examined by varying one factor while holding the remaining factors constant. The more influential factors result in greater variations in the economic threshold level of HBA than the less important factors.

RESULTS

Linear relationships were observed between the intercepts (b₀) and slopes (b₁) of the yield equation in [2] and site index for all combinations of site index and planting density. Therefore, loblolly pine yield at age 25 in [2] can be generalized as:

$$Y = [-103.0979 + (3.4325 \text{ SI})] - [0.3709 - (0.0155 \text{ SI})] \times \text{HBA} \quad [4]$$

where SI = site index (base age 25) and other terms as previously defined. Volume gained following competition

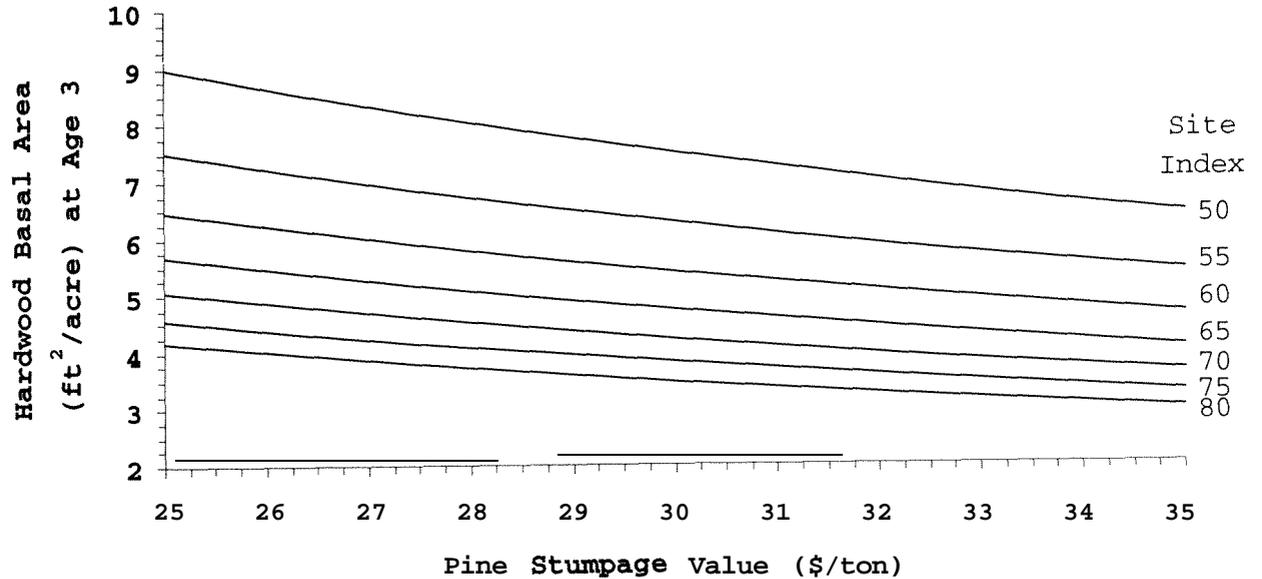


Figure 3-Economic threshold level of hardwood basal area at hardwood control treatment age 3 years for pine stumpage values between \$25/ton and \$35/ton. Additional inputs: interest rate=6 percent and hardwood control treatment cost=\$60/acre.

control (VG) is the difference in yield for stands without hardwoods (HBA = 0) and stands with hardwoods. Combining [2] and [4], VG is:

$$\begin{aligned}
 VG &= [b_0 - b_1(0)] - [b_0 - b_1(\text{HBA})] & [5] \\
 &= [b_0 - b_0] + b_1(\text{HBA}) \\
 &= b_1(\text{HBA}) \\
 &= [-0.3709 + (0.0155 \times \text{SI})] \times \text{HBA}
 \end{aligned}$$

Note that the sign of b_1 changes from negative in [4] to positive in [5], which changes the sign of the component coefficients. When response to hardwood control is expressed as $b_1 \text{HBA}$, the economic threshold level of hardwood basal area (HBA_e) for a 25-year rotation ($r = 25$) and hardwood control treatment at age 3 ($t = 3$) can be computed by solving [1] for HBA as follows:

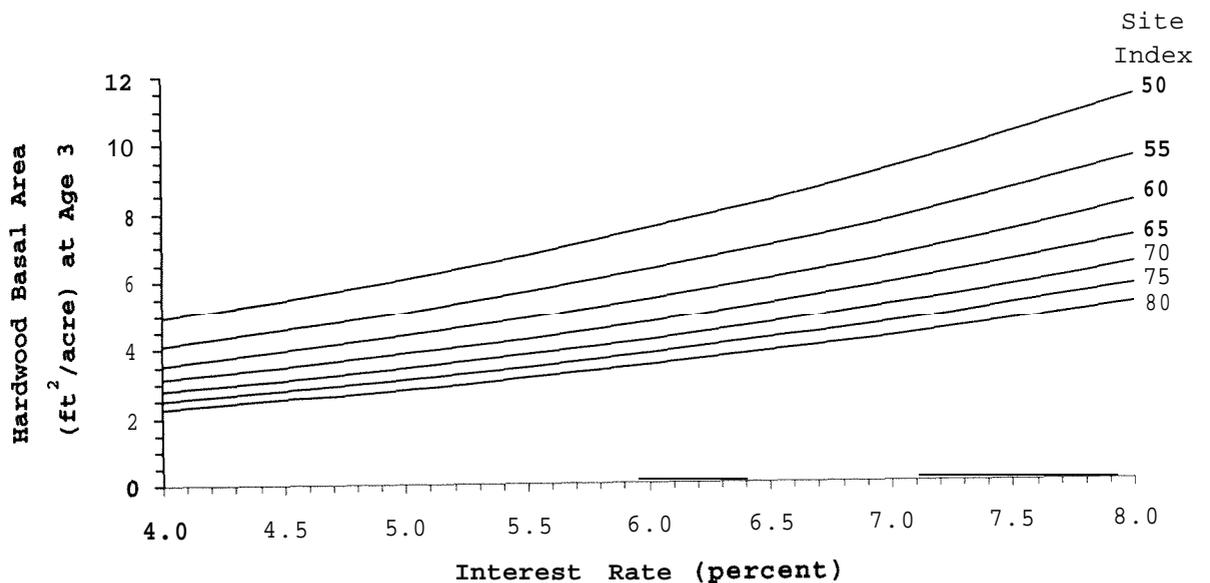


Figure 4-Economic threshold level of hardwood basal area at hardwood control treatment age 3 years for hardwood treatment costs between \$40/acre and \$80/acre. Additional inputs: interest rate=6 percent and pine stumpage value=\$30/ton.

$$\text{HBA}_{r,t} = \frac{\text{TC} \times (1+i)^{22}}{\text{SV} \times [-0.3709 + (0.0155 \times \text{SI})]} \quad [6]$$

where i = interest rate, SV = pine stumpage value (\$/ton), TC = hardwood treatment cost (\$/acre), and $r-t = 22$ years. The economic threshold level of hardwood basal area at rotation age is projected to a hardwood-control treatment age of 3 years by multiplying HBA at rotation age by 0.4194. The effect of varying interest rate on economic threshold level of hardwood basal area at age 25 is shown in figure 1 for fixed pine stumpage value and hardwood treatment cost. As expected, yield increases with increasing site index and decreases with increasing HBA. When interest rate = 6 percent, site index = 65, pine stumpage value = \$30/ton, and treatment cost = \$60/acre, for example, the economic threshold hardwood basal area is about 11.5 ft²/acre and expected yield is about 112 tons/acre. The difference in yield between interest rates is not equal, and is larger at lower site index than higher. This implies an interaction between interest rate and site index.

Multiplying HBA at rotation age by 0.4194 provides an estimate of HBA at age 3, which is when hardwood control treatments would be prescribed (figure 2). Using the previous example, the economic threshold hardwood basal area is about 4.7 ft²/acre at age 3. This is interpreted as the minimum amount of hardwood competition that must be present for a \$60/acre release treatment to be financially attractive under those circumstances.

The effect of varying pine stumpage value on economic threshold level of hardwood basal area at age 3 is shown in figure 3 for fixed interest rate and hardwood treatment cost. In this case, the economic threshold level of hardwood basal area decreases with increasing pine stumpage value and site index. For example, when pine stumpage value = \$30/ton, site index = 65, interest = 6 percent, and treatment cost = \$60/acre, the economic threshold hardwood basal area is about 4.7 ft²/acre. The difference in economic thresholds across pine stumpage values is nearly linear, and the difference is larger at lower site index than at higher site index.

The effect of varying hardwood treatment cost on economic threshold level of hardwood basal area at age 3 is shown in figure 4 for fixed interest rate and pine stumpage value. As with interest rates, the economic threshold level of hardwood basal area increases with increasing treatment cost and decreasing site index. When treatment cost = \$60/acre, site index = 65, interest = 6 percent, and pine stumpage value = \$30/ton, the economic threshold hardwood basal area is about 4.7 ft²/acre. The difference in economic thresholds across hardwood treatment costs is nearly linear, and the difference is larger at lower site index than at higher site index.

DISCUSSION AND SUMMARY

The concept of economic thresholds was applied to hardwood competition in loblolly pine plantations, and procedures were developed for estimating threshold levels of hardwood basal area. The sensitivity analysis of biological

and economic factors affecting the threshold level of hardwood basal area indicated that both interest rate and site index were more influential factors than stumpage value and treatment cost.

Interest rate has the greatest influence on economic thresholds, especially on poor sites. A 1 percent increase in interest rate increases threshold by 1-2 ft²/acre on good sites and by 5 ft²/acre on poor sites. A \$5/acre increase in treatment cost increases economic threshold level of hardwood basal area by 0.50 ft²/acre on good sites and by 0.75 ft²/acre on poor sites. Increasing loblolly pine stumpage value decreases threshold by 0.5 ft²/acre on good sites and by 1.0 ft²/acre on poor sites. Growth models used to simulate hardwood competition may have profound effects on the biological and economic interpretations. The pattern of negative exponential response of pines to hardwood competition implies that low levels of hardwood basal area would produce a greater proportional reduction in pine yield than at higher levels of hardwood competition. Thus, the Burkhart and Sprinz (1984) model may be more appropriate at low levels of hardwood basal area while the Knowe (1992) model may be more appropriate at the higher levels of hardwood competition. Additional considerations are the pine:hardwood replacement ratio and hardwood dynamics. A more comprehensive pine release dataset, with hardwood information, is needed to refine the economic threshold method presented in this study.

ACKNOWLEDGMENTS

Tom Fox, Tim Harrington, Steve Radosevich, Barry Shiver, and Bob Wagner helped to develop the economic threshold concept.

REFERENCES

- Borders, B.E.; W.M. Harrison; D.E. Adams; R.L. Bailey; L.V. Pienaar. 1990. Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Florida, and Alabama. Plantation Manage. Res. Coop. Tech. Report 1990-2, Athens, GA: School of Forestry Resources, Univ. of Georgia. 65 p.
- Burkhart, H.E.; P.T. Sprinz. 1984. A model for assessing hardwood competition effects on yields of loblolly pine plantations. Publ. No. FWS-3-84, Blacksburg, VA: School of Forestry and Wildlife Resources, VPI&SU. 55 p.
- Cain, M.D.; W.F. Mann, Jr. 1980. Annual brush control increases early growth of loblolly pine. Southern Journal of Applied Forestry. 4: 67-70.
- Clason, T.R. 1978. Removal of hardwood vegetation increases growth and yield of a young loblolly pine stand. Southern Journal of Applied Forestry. 2: 96-97.
- Cousens, R. 1987. Theory and reality of weed control thresholds. Plant Protection Quarterly. 2(1): 13-20.
- Glover, G.R.; D.F. Dickens. 1985. Impact of competing vegetation on yield of southern pines. Ga. For. Res. Pap. 59. 14 p.
- Glover, G.R.; B.R. Zutter. 1993. Loblolly pine and mixed hardwood dynamics for 27 years following chemical, mechanical, and manual site preparation. Canadian Journal of Forest Research. 23: 2126-2132.

- Harrison, W.M.; B.E. Borders.** 1996. Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Alabama, and Florida. *Plantation Manage. Res. Coop. Tech. Report 1996-1*, Athens, GA: School of Forestry Resources, University of Georgia. 65 p.
- Knowe, **S.A.** 1992. Basal area and diameter distribution models for loblolly pine plantations with hardwood competition in the Piedmont and upper Coastal Plain. *Southern Journal of Applied Forestry*. 16: 93-98.
- Miller, J.H.; B.R. Zutter; S.M. Zedaker; M.B. Edwards; R.A. Newbold.** 1995. A regional framework of early growth response for loblolly pine relative to herbaceous, woody, and complete competition control: The COMProject. Gen. Tech. Rep. SO-117, New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 48 p.
- Quicke, H.E.; G.R. Glover; D.K. Lauer.** 1996. Herbicide release of 3-year-old loblolly pine from competing hardwoods in Arkansas. *Southern Journal of Applied Forestry*. 20: 121-126.
- Smith, W.D.; W.L. Hafley.** 1986. Simulating the effect of hardwood encroachment on loblolly pine plantations. In: Phillips, DR., comp. *Proceedings of the Fourth biennial southern silvicultural research conference*; 1986 November 4-6, Atlanta, GA. Gen. Tech. Rep. SE-42. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station: 180-186.
- Wagner, R.G.** 1993. Research directions to advance forest vegetation management in North America. *Canadian Journal of Forest Resources*. 23: 2317-2327.
- Wagner, R.G.** 1994. Toward integrated forest vegetation management. *Journal of Forestry*. 92: 26-30.

EMPIRICAL ALLOMETRIC MODELS TO ESTIMATE TOTAL NEEDLE BIOMASS FOR LOBLOLLY PINE

Hector M. De los Santos-Posadas and Bruce E. Borders¹

Abstract-Empirical geometric models based on the cone surface formula were adapted and used to estimate total dry needle biomass (TNB) and live branch basal area (LBBA). The results suggest that the empirical geometric equations produced good fit and stable parameters while estimating TNB and LBBA. The data used include trees from a spacing study of 12 years old and a set of fully measured trees on the coastal plain of North and South Carolina of ages 10 to 25.

INTRODUCTION

One of the most important factors contributing to the development of a stand is the amount of leaf biomass. Although its contribution to total tree biomass is only 4 to 6 percent, leaf biomass is responsible for most of the transpiration-respiration processes and total carbon uptake in the tree (Zhang, 1997). Leaf biomass has proven to be very sensitive to climatic patterns and silvicultural treatments, thus quantification of leaf biomass may be important for explaining productivity of forest stands.

Accurate estimation of leaf biomass may not only improve estimation of the potential growth rate but can be used to characterize other stand conditions. For example leaf area is useful as an index of productivity and vigor that explains a potential source of variability in stand response to silvicultural treatments. O'Hara (1989) states that thinned stands have higher transpiration/respiration rates and require greater sapwood area to supply a given amount of leaf biomass. In this case, the reduction in the number of trees makes more water and nutrients available producing more conductive tissue that remains healthy for more time. This is also true for stands growing on good quality sites.

Leaf biomass and leaf area may become an important input to a new generation of growth and yield models that are more site specific than today's models. It is anticipated that since leaf biomass is sensitive to environmental and silvicultural factors, models that use it to project growth will also be more sensitive to these factors. The most wide spread approach for estimating leaf area and needle biomass is based on allometric relationships. The basic allometric relationship between leaf biomass and stem size (diameter or stem area) is based on the pipe model theory proposed by Shinozaki and others (1964). Based on this work, Waring and others (1981) suggested that the amount of foliage is proportional to the amount of conductive tissue present on the stem, which for conifers is the sapwood. In geometric terms leaf biomass should be related not only to the transversal area but the geometry of the crown.

Several of these studies show that, in general, these allometric relationships are not completely linear or are only linear for a given age class. Most of the equations developed to estimate biomass are linear in logarithmic units or intrinsically non-linear. Baldwin (1989) presented equations to compare the fit of leaf biomass from DBH and the sapwood area (cm²) at breast height and live crown height, finding that DBH was the best independent variable to estimate needle and branch biomass for loblolly pine. Long and Smith (1988) and Long and Smith (1989) developed non-linear models for *Pinus contorta* and *Abies lasiocarpa* that include crown size observations, making the equations more tree specific and sensitive to stand density. McCrady and Jokela (1998) used the pipe model theory and assume that leaf biomass is proportional to total tree volume suggesting that the amount of leaf area/leaf biomass is strongly related with the geometry of the tree biomass.

The main objective of this study is to develop site/tree specific allometric needle biomass prediction equations such that the total tree dry needle biomass (TNB) prediction equations are sensitive to stand density and stand structure. The new models should improve leaf area estimation and provide a means of differentiating total stand biomass growth for stands that have similar size stem dimensions but different amounts of leaf biomass.

MATERIALS AND METHODS

The research was conducted in a loblolly pine spacing study established at the B.F Grant Memorial Forest near Eatonton, Georgia (Pienaar and others, 1997). The study was planted with genetically improved seedlings in March 1983 at a 6 by 6 ft spacing (1.81 by 1.81 m). In July 1983, 24 one fifth-acre treatment plots each with a one-tenth acre interior measurement plot (0.08 ha) were installed with planting densities of 100, 200, 400, 600, 800 and 1000 trees per acre (247, 494, 988, 1483, 1977 and 2471 trees per hectare, respectively). The experimental plots were completely randomized with four replications of each density. The study is located on an old agricultural field

¹Graduate Student, Professor, D.B. Warnell School of Forest Resources, University of Georgia, Athens, GA 30602-2152, respectively.

Calculated Total Needle Biomass and Live Branch Basal Area for the B.F. Grant Spacing study

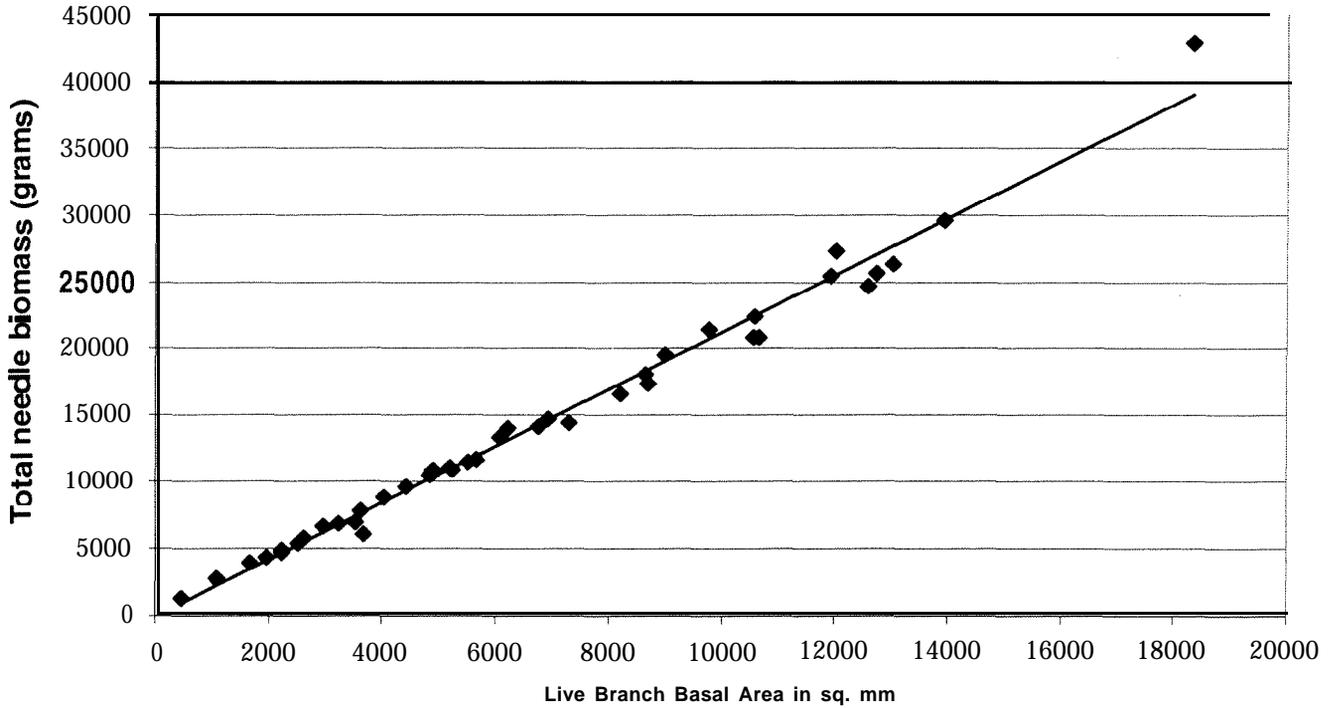


Figure 1— Total Dry Needle Biomass (TNB) vs Live Branch Basal Area (LBBA) for the B. F. Grant spacing study.

Observed Total Needle Biomass and Live Branch Basal Area for 28 tree form the Coastal plain of South and North Carolina (Brister database)

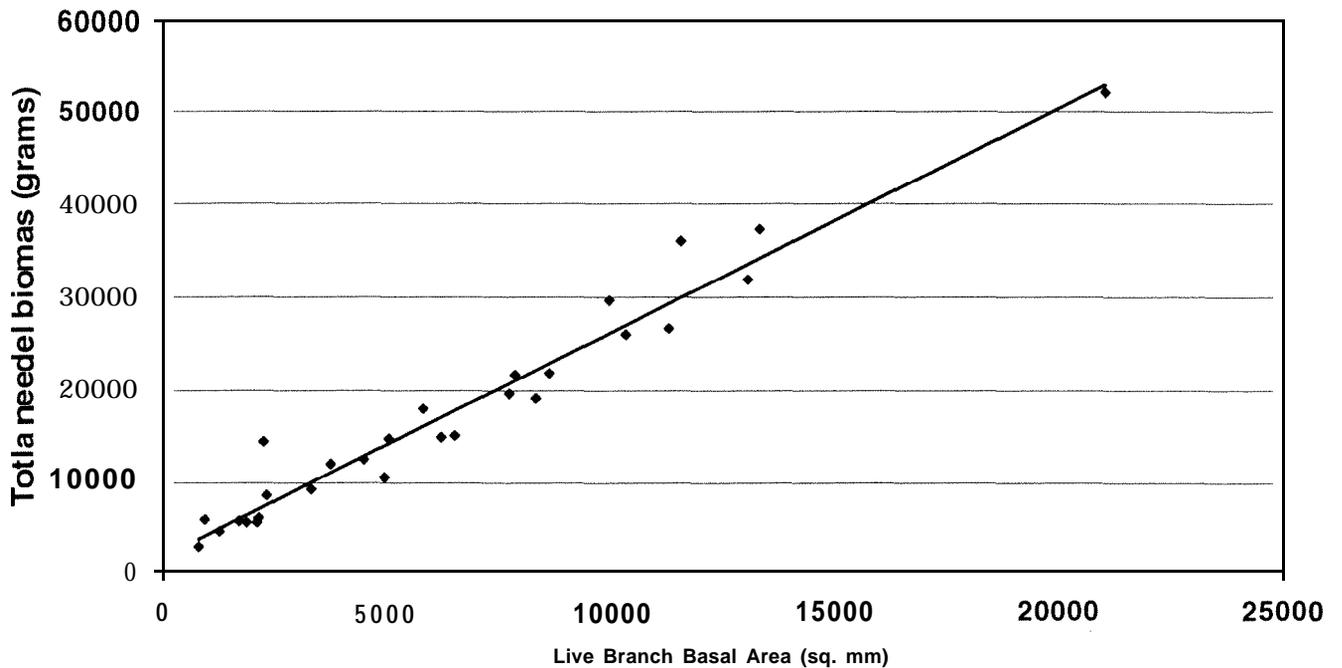


Figure 2— Total Dry Needle Biomass (TNB) vs Live Branch Basal Area (LBBA) for the Brister database on the Coastal plain of North and South Carolina.

which had been previously planted with soybeans. Herbicide was first applied in May 1984 and in 1985, wildlings of both loblolly pine and sweetgum (*Liquidambar styraciflua*) were mechanically removed in 1985. As a result all the plots have been growing essentially free of competing vegetation throughout the life of the study.

In July 1997 felled trees from the first thinning of the B. F. Grant spacing study were sampled to produce needle biomass estimators. Trees from all plots, except the 247 trees/ha plots, were cut to reach a residual target number of trees. The thinning was primarily from below, however good spatial distribution of residual stems was also a criterion for tree selection. Forty-three trees were selected and sampled using the methodology described by Xu (1997) to develop a two-level allometric estimation procedure: equations based on branch measurements were used to estimate dry needle branch biomass and in turn other equations were used to estimate total tree dry needle biomass. Sample trees came from four crown classes: dominant (12), co-dominant (21), intermediate (8) and suppressed (2). The selected trees were free of deformities and fusiform rust.

The variables measured for each tree include diameter at breast height (D) in cm, total height from the stump in m, stump height in m, and live crown height in m. The crown also was carefully measured and divided in five equal length sections along the stem. One branch was randomly taken in each section and the parts of the branch with foliage were collected, identified for tree and section number and bagged for further laboratory analysis. For each sample, branch length (m) and basal branch diameter (mm), at approximately 20 mm from the base of the branch, were obtained with a measuring tape and a digital caliper, respectively. The branch height (BH) and basal branch diameter (BBD) for every live branch was also recorded.

In the laboratory, the foliage of each sample branch was separated from the branch fragment, identified, and dried at 70° C until reaching a constant weight. The needle weight was then measured with a balance to the nearest gram. An additional database from a similar study on loblolly pine was provided by Professor Graham H. Brister and used to test the models by estimate LBBA and TNB per tree.

RELATIONSHIP BETWEEN TOTAL NEEDLE BIOMASS AND LIVE BRANCH BASAL AREA

In an attempt to increase the precision of estimation for total dry needle biomass for the entire tree the relationship between the total dry needle biomass (TNB) and the total live branch basal area in mm² (LBBA) was analyzed. Graphical analysis shows (figure 1) that this relationship is very strongly linear and stable for the trees in this study. This behavior is logical since branch needle biomass was estimated with a linear equation that uses the branch diameter squared (De los Santos, 1998). To verify this relationship a database generated by Brister in 1977 for 28 loblolly pine trees was used (figure 2). All trees in this database were located in the coastal plain of North and South Carolina and total needle biomass was obtained by removing and weighing all needles in each branch. Clearly the relationship between LBBA and the observed TNB is very similar to the relationship observed for the 43 trees in

the B. F. Grant spacing study. A similar relationship was found by Whitehead (1990) in *Pinus radiata* for branch basal area and leaf area clustered on "branch complexes" but was never aggregated to estimate the total leaf area per tree. This relationship seems less appropriate for shade intolerant trees growing at wide spacing and/or old stands that have changed the excurrent growing pattern from the young-middle ages to a more sympodic pattern. In these old age trees the crown expands more longitudinally than vertically, the branches become more massive and ultimately accumulate heartwood.

Regression analysis (table 1) with both databases shows a strong and stable correlation between these two characteristics. The slope of the regression line can be interpreted as the amount of needle biomass sustained by each unit of conductive tissue surface area attached to the stem. The differences in the slope can be attributed in part to the process of allometric estimation versus measured biomass and by differences in site quality and management at each site. Trees in the B. F. Grant spacing study sustain more needle biomass than the sites in the coastal plain which is most likely due not only to differences in nutrient availability but to the amount and types of competing vegetation.

GEOMETRIC MODELS

Since LBBA and TNB are linearly and highly correlated it may be useful to focus on prediction of LBBA using geometric based models. Thus the hypothesis is that LBBA should be proportional to the stem surface area occupied by live crown. The main assumption is that the estimation based on tree characteristics will be more precise for LBBA than for the TNB.

The basic form for the cone surface was modified to be used as empirical models. Since the diameter at the base of the live crown was not obtained, diameter at breast height (D) and crown length (L) are used in these model forms. The constant p was replaced by a scale parameter in the formulations. These structures were motivated by the description that Steill (1964) cited by Seymour and Smith, (1987) used for crown volume for *Pinus resinosa*. He found a very good correlation between foliage weight and crown volume estimated with a paraboloid formula. The models derived are:

Cone formulation 1

$$(1) \quad B = a \left(\frac{D}{2} \right) * \sqrt{\left(\frac{D}{4} \right)^b + L^c}$$

Cone formulation 2

$$(2) \quad B = a \left(\frac{D}{2} \right) * \left[\left(\frac{D}{4} \right)^b + L^c \right]^d$$

Where:

a, b, c and d are the parameters to be estimated, B = LBBA or some other biomass crown component as needle biomass, all else is as defined above.

Table I-Analysis of Variance for Total dry needle biomass (TNB) vs Live Branch Basal Area (LBBA) vs. for B.F. Grant spacing study data and North and South Carolina coastal plain (Brister Data) $TNB = \alpha + \beta(LBBA) + e_i$

B.F. Grant Database

Source	df	Analysis of Variance			
		SSE	MSE	F Value	Prob > F
Regression	1	691369020.8	691369020.8	3049.81	.0000001
Residual	40	9067698.453	226692.4613		
Total	41	700436719.3			

R-square = 0.98705

	Coefficients	Standard Error	t stat	P-value
Intercept	120.8546	139.7541	0.8647	0.392324
Slope	0.4638	0.0084	55.2251	0.00001

Brister Database

Source	df	Analysis of Variance			
		SSE	MSE	F Value	Prob > F
Regression	1	544452712	544452712	497.139	.0000001
Residual	24	26284127.89	1095171.995		
Total	25	570736839.8			

R-square = 0.95394

	Coefficients	Standard Error	t stat	P-value
Intercept	-436.412	382.2230	-1.14177	0.264812
Slope	0.391966	0.01758	22.2966	.0000001

Table 2— Fit statistics and parameter estimates for cone formulation 1 on the B.F. Grant spacing study data

	DF Model	DF Error	SSE	MSE	Root MSE	R-square
TNB	3	40	8348	208.6878	14.44603	0.9278
LBBA	3	40	35465	886.6308	29.77635	0.9304
	Parameter	Estimate	Standard Error	Aprox T ratio	Aprox Prob > T	
TNB	<i>a</i>	15.73783	4.52413	3.48	0.0012	
	<i>b</i>	3.456465	0.32257	10.72	0.0001	
	<i>c</i>	4.219977	0.3263	12.93	0.0001	
LBBA	<i>a</i>	30.31826	8.39138	3.61	0.0008	
	<i>b</i>	3.61632	0.29436	12.29	0.0001	
	<i>c</i>	4.230212	0.33276	12.71	0.0001	

Table 3— Fit statistics and parameter estimates for cone formulation 2 on the B.F. Grant spacing study data

	DF Model	DF Error	SSE	MSE	Root MSE	R-square
TNB	3	40	8441	211.0289	14.52683	0.927
LBBA	3	40	35908	897.6995	29.96163	0.9295
	Parameter	Estimate	Standard Error	Aprox T ratio	A prox Prob > T	
TNB	<i>a</i>	12.82514	4.35711	2.94	0.0054	
	<i>b</i>	2.450571	0.09015	9.47	0.0001	
	<i>c</i>	0.854042	0.24696	9.92	0.0001	
LBBA	<i>a</i>	23.91818	7.70821	3.1	0.0035	
	<i>b</i>	2.278922	0.08229	11	0.0001	
	<i>c</i>	0.905521	0.23598	9.66	0.0001	

Table 4— Fit statistics and parameter estimates for cone formulation 1 on the Brister Data for the coastal plain of North and South Carolina

	DF Model	DF Error	SSE	MSE	Root MSE	R-square
TNB	3	25	62661312	2506453	1583.2	0.8989
LBBA	3	25	2.61 E+08	10431197	3229.7	0.9307

	Parameter	Estimate	Standard Error	Aprox T ratio	Aprox Prob > T
TNB	<i>a</i>	60.92423	21.79016	2.8	0.0098
	<i>b</i>	2.4915	0.36447	6.84	0.0001
	<i>c</i>	2.238613	0.49897	4.49	0.0001
LBBA	<i>a</i>	253.6395	66.22154	3.83	0.0008
	<i>b</i>	1.989328	0.29422	6.76	0.0001
	<i>c</i>	1.942278	0.31365	6.19	0.0001

As density changes it seems logical that tree crown volume changes, adapting its form to the conditions of stand density and competition. Cone formulation 1 above implies that the cone is modified not only by the scale parameter but by changing the form of the arc defined by $(D/4)^b + L^c$. In cone formulation 2 parameter *d* generalizes the form of the relationship onto a more flexible structure.

The geometric formulations were used to estimate both LBBA and TNB with good results (table 2 and 3) on the spacing study trees. For both formulations the parameters are stable with an acceptable fit. As hypothesized the r-square for LBBA is higher than for the TNB. To correct for

effect of heteroscedasticity the following weight function was used

$$(3) \quad W = \frac{1}{D^2 L}$$

MODEL TEST FOR GEOMETRIC MODELS

To test the previous equations 28 trees measured by Brister were fitted with geometric equations to predict TNB and LBBA (table 3 and 4). It is also interesting to notice that estimation of LBBA is better than for TNB as foreseen for this kind of data (total foliage sampled per tree). Note however that the cone formulations show stability on the parameters for both TNB and LBBA fit. In this case no effect

Table 5— Fit statistics and parameter estimates for cone formulation 2 on the Brister Data for the coastal plain of North and South Carolina

	DF Model	DF Error	SSE	MSE	Root MSE	R-square
TNB	3	25	62471770	2498871	1580.8	0.8992
LBBA	3	25	2.61 E+08	10430419	3229.6	0.9307

	Parameter	Estimate	Standard Error	Aprox T ratio	Aprox Prob > T
TNB	<i>a</i>	55.99056	22.82834	2.45	0.0215
	<i>b</i>	1.708317	0.09752	6.48	0.0001
	<i>c</i>	0.632363	0.41849	4.08	0.0004
LBBA	<i>a</i>	255.5783	78.06633	3.27	0.0031
	<i>b</i>	1.957699	0.07653	6.48	0.0001
	<i>c</i>	0.495678	0.32397	6.04	0.0001

of heteroscedastisity is shown in the residual analysis, so the estimates of the model are the regular least squares.

Theoretically the models generated shall produce similar results on the same data range. However validation is needed to better qualify the model behavior at more operative levels using a wider more realistic range of variability.

CONCLUSIONS

The inherent hypothesis of the geometric models suggest that a better knowledge of the surface stem geometry at the crown level may produce better estimates of the TNB, crown biomass and LBBA at tree and stand level. The models generated show stability on their parameters and a predictive ability among the best for loblolly pine at the tree level. These structures should produce reliable and more site specific estimates of photosynthetic tissue.

REFERENCES

- Baldwin Jr.; V. C. 1989. Is sapwood a better predictor of loblolly pine crown biomass than bole diameter? *Biomass* 20: 177-185.
- Long J. M.; F. W. Smith. 1988. Leaf area-sapwood area relations of lodgepole pine as influenced by stand density and site index. *Canadian Journal of Forestry Reseach*. 18: 247-250.
- Long J. M.; F. W. Smith. 1989. Estimating leaf area of *Abies lasiocarpa* across ranges of stand density and site quality. *Canadian Journal of Forest Research*. 19: 930-932.
- McCrary, R. L.; E. J. Jokela. 1998. Canopy dynamics, light interception, and radiation use efficiency of selected loblolly pine families. *Forest Science*. 44(1): 64-72.
- O'Hara, K. L. 1989. Stand growth efficiency in a Douglas fir Thinning trial. *Forestry*. 62(4): 409-418.
- Pienaar, L. V.; B. D. Shiver; W. M. Harrison. 1997. Growth potential of loblolly pine plantations in the Georgia Piedmont: A spacing study example. PMRC Tech. Rep. 1997-1. Plantation Management Research Coop., University of Georgia. Athens, GA. 15 p.
- Seymour, R.; D. M. Smith. 1987. A new stocking guide formulation applied to eastern White pine. *Forest Science*. 33:469-484.
- Shinozaki, K.; K. Yoda; K Hozumi· T. Kira. 1964. A quantitative analysis of plant form · The pipe model theory. I. Basic Analyses. *Japanese Journal of Ecology*. 14: 133-139.
- Waring, R.H.; K. Newman; J. Bell. 1981. Efficiency of tree crowns and stemwood production at different canopy leaf densities. *Forestry*. 54: 129-137.
- Whitehead D.; Grace J.C.; Godfrey M.J.S. 1990. Architectural distribution of foliage on in individual *Pinus radiata* D. Don crowns and the effects of clumping on radiation interception. *Tree Physiology*. 7: 135-155
- Xu, M. 1997. Growth efficiency of individual loblolly pine (*Pinus taeda* L.) as affected by crown morphology, stand characteristics and microenvironment. Athens, GA: University of Georgia. 132p Ph.D. dissertation.
- Zhang, Y. 1997. A physiologically based simulation model for loblolly pine. Athens, GA: University of Georgia. 127p. Ph.D., dissertation.

A MODEL DESCRIBING GROWTH AND DEVELOPMENT OF LONGLEAF PINE PLANTATIONS: CONSEQUENCES OF OBSERVED STAND STRUCTURES ON STRUCTURE OF THE MODEL

J.C.G. Goelz and Daniel J. Leduc¹

Abstract-As longleaf pine (*Pinus palustris* Mill.) may currently represent as little as 1/30th of its former acreage, restoration within its former range in the southern coastal plain is active. Although the focus of these new plantings is aimed at ecosystem restoration, knowledge of the growth and development of longleaf plantations is essential to allow land managers to evaluate different management options. Stand development in longleaf plantations differs from development of plantations of other southern pines. Longleaf seedlings exist in a grass-stage for a varying period, and longleaf saplings and poles can often exist in an intermediate or suppressed crown class for long periods. Other southern pines do not exhibit this behavior. The consequence of these characteristics is that smooth, unimodal diameter distributions are inappropriate for characterizing longleaf pine stands. We will use alternative methods to describe the diameter distributions of longleaf pine. Depending upon viewpoint, the proposed model structure could be called a nonparametric diameter distribution model, or a diameter class model where a uniform distribution is not employed within a class. The model can also be implemented as an individual tree model, if the user desires. A neural net approach has proved promising for initially allocating trees to diameter classes for unthinned stands. A whole-stand basal area prediction equation ensures consistency between these components.

INTRODUCTION

Longleaf pine stands were once a major component of the southern coastal plain from North Carolina to Texas. Currently, longleaf pine may represent as little as 1/30th of its acreage in pre-colonial times (Franklin 1997). An aggressive planting program has developed to restore the longleaf ecosystem within its former range. Although the focus of that work is aimed at ecosystem restoration, knowledge about the growth and development of longleaf plantations is essential for sound management. Longleaf is well-suited for lower-intensity management, particularly longer rotation ages. Also, longleaf pine is less susceptible to most insect and disease problems than other southern pines (Boyer 1990).

Longleaf pine provides higher-value products, such as poles and pilings, more frequently than the more-abundant loblolly pine (*Pinus taeda* L.), and also has a higher specific gravity. Finally, longleaf pine is desirable because a forest of large, old, widely-spaced trees with a grassy understory is "parklike" and visually attractive to visitors.

Many of the older (30 years or older) plantations of longleaf pine arose in a restoration context that is different than the current situation. Often, longleaf plantations were established in cutover areas that had been repeatedly grazed and burned. The current context of restoration is afforesting agricultural fields or converting cutover stands formerly dominated by loblolly pine or mixed pine and hardwoods.

The silvics of longleaf pine distinguish it from other species in the U.S. (Boyer 1990). Three characteristics affect the stand structure of longleaf stands, and hence the structure of a model to describe longleaf plantations. First, longleaf seedlings exist in a "grass stage" for a varying period. The grass stage is a condition where the terminal bud is at or near ground level, and the needles appear similar to a bunchgrass. Although current management practices can often achieve active height growth of most seedlings in the second growing season, individual seedlings may reside in the grass stage for five or more years. Second, although longleaf is an intolerant species, saplings and poles can often exist in an intermediate or suppressed crown class for long periods. Other southern pines do not exhibit this behavior. Suppressed trees rarely respond to release, although trees with live crown ratios of 30 percent or more in the intermediate crown class do respond (Boyer 1990). Third, prescribed fire is an intrinsic part of longleaf pine management, although current practices restrict fire from plantations of other pine species. Interval between fires is often between 2 and 5 years. Prescribed fire ensures that mortality, though rare, will occur throughout the life of a stand, and will restrict ingrowth of volunteer hardwoods and loblolly pine.

There has been little growth and yield modeling done for plantation-grown longleaf pine. A relatively recent model for natural longleaf stands has been provided by Somers and Farrar (1991). The only existing model for plantation-grown

¹Principal Forest Biometrician and Computer Specialist, Southern Research Station, Alexandria Forestry Center, 2500 Shreveport Hwy, Pineville, LA 713605500, respectively.

Citation for *proceedings*: Outcalt, Kenneth W., ed. 2002. Proceedings of the eleventh biennial southern silvicultural research conference Gen. Tech. Rep. SRS-48. Asheville, NC: US. Department of Agriculture, Forest Service, Southern Research Station. 622 p.

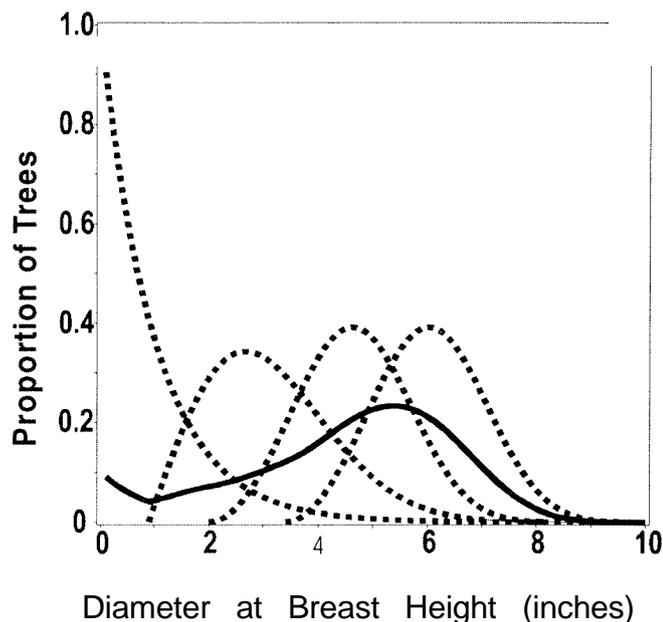


Figure 1-A theoretical diameter distribution represented as a mixture of four populations defined by length of time in the grass stage. The dotted lines represent the four populations, and the solid line is the mixture of these four populations. The proportion of the total is 0.1, 0.2, 0.3, and 0.4 for the four distributions proceeding from the distribution with the smallest mean to the distribution with the largest mean. For natural longleaf stands has been provided by Somers and Farrar (1991). The only existing model for plantation-grown longleaf is restricted to unthinned stands (Lohrey and Bailey 1977). Lohrey and Bailey's work is based on a part of the data available to us; most of those plots have been measured several additional times.

longleaf is restricted to unthinned stands (Lohrey and Bailey 1977). Lohrey and Bailey's work is based on a part of the data available to us; most of those plots have been measured several additional times.

A Theoretical Example

As individual seedlings reside in the grass stage for differing lengths of time, a diameter distribution for longleaf pine can be considered to be a mixture of distributions. The theoretical example in figure 1 suggests a mixture of four distributions: 40 percent of the trees resided in the grass stage for one year, 30 percent for two years, 20 percent for three or four years, and 10 percent for more than four years. Although the diameter distribution for each cohort is smooth, the pooled diameter distribution for the stand is not unimodal. Most commonly-used diameter distribution functions do poorly for stands that have a long, or heavy, left-hand tail, and are incapable of describing multimodality.

A Brief Primer on Alternative Model Structure

Growth and yield models are typically classified into convenient discrete classes. Although such simplistic polychotomies ignore functional similarities among models (see Goelz [in press] for a novel synthesis of modeling structures), we will describe model forms as discrete entities in this brief listing. One distinction is

whether growth of individual trees is projected, and whole stand growth is defined by the aggregation of the individual trees, or whether whole-stand variables are directly predicted. When whole stand variables are predicted, the model might disaggregate whole stand growth into a diameter distribution. Intermediate among these are the size class models that project the growth of trees from one size class to another (typically 1 or 2 inch wide dbh classes). Potentially, there are intermediate structures between these classes (Goelz [in press]).

OUR APPROACH TO MODEL STRUCTURE

We believe that model structure should be determined by the needs of the eventual users of the model, the idiosyncracies of the biology of the system to be modeled, and the data available for estimation of the model. For example, if all trees were of uniform value per unit volume, a whole stand type model would be appropriate. On the other hand, if value of the trees varied with species, size, and tree grade (and if these variables were not highly correlated), then an individual tree model might be suggested. If diameter tends to exhibit relatively smooth unimodal distributions, a diameter distribution model might be suggested; if diameter distributions tend to be irregular or multimodal, use of a parametric distribution function may be inappropriate.

Our Data

Our data are described in Goelz and Leduc [in press]. Over 250 plots are scattered from Texas to Alabama and each records over 20 years of stand dynamics. While technically arising following clearcutting natural stands, the areas were often repeatedly burned and grazed for many years before the plantations were planted. Thus, previous use for many of our plots was open-range grazing rather than forest or cropland. The oldest plantations in our database were last measured at age 65.

Example Diameter Distributions from our Data

We provide several diameter distributions from our plots in figure 2. The plots vary considerably. Some resemble the classical unimodal diameter distributions for even-aged stands (e.g. plot A). Others are very irregular, often being bi- or multi-modal (e.g. plot C, age 65). A distinct grass stage, or the vestige of trees that lingered in the grass stage, is evident in some of the graphs (e.g. plots B, C, D). In some cases, thinning encouraged bimodality as thinning was from below, but only merchantable (greater than 4 in. dbh) trees were removed. In other cases, thinning removed a long left-hand tail or subsidiary mode of the distribution. These example diameter distributions suggest that diameter distributions for longleaf plantations take various shapes, many of which do not comply with standard parametric distributions, and thus we will not use standard parametric distributions in our model for longleaf pine.

A Tentative Model Structure

The objectives for our model structure are to: (1) allow for varied diameter distributions, and potentially maintain those structures; (2) allow stand structure, rather than simply whole-stand variables, to influence growth projections; (3) allow relatively simple implementation (at least to the user); (4) allow the model to be invoked as an individual

tree model, diameter distribution model, or diameter class model to facilitate use by different clientele; (5) be applicable to inventory data tallied by diameter classes; (6) make extrapolations reasonable by being conditioned by a whole-stand basal area prediction equation; (7) be tractable for investigating optimal stand management. Regarding the use in extrapolation, although our oldest data are from 65 year old plantations, rotation age for longleaf may be as long as 150 years for some managers. To achieve these objectives, we suggest the following structure:

- (1) Initially allocate trees into fixed-width diameter classes.
- (2) Generate a diameter distribution that is a quadratic polynomial within a diameter class, but is discontinuous at the limits of each diameter class. Thus the diameter distribution consists of a number of pieces.
- (3) Adjust number of trees in each class using an individual tree mortality function.
- (4) Adjust/recalculate the parameters of the quadratic polynomial to reflect the effects of mortality.
- (5) Use an individual tree diameter growth equation to project the limits of the now varying-width diameter classes.
- (6) Adjust the growth in tree basal area to be consistent with a whole-stand basal area growth equation.
- (7) Adjust the parameters of the quadratic polynomial using a simple transformation.
- (8) Integrate (using appropriate limits of integration) the within-class diameter distributions to reconstitute a fixed-width diameter distribution.

These integrals are simple analytic integrals. The definite integrals will define movement ratios (or growth-index

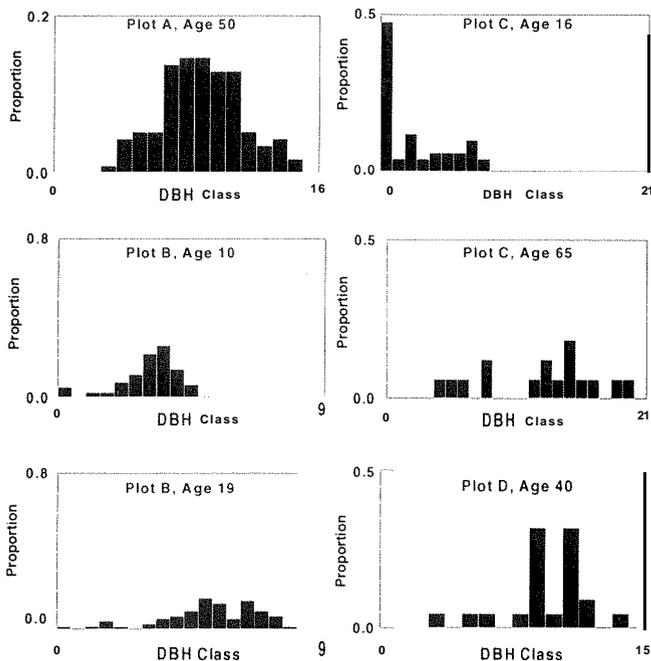


Figure 2-Six observed diameter distributions variability among diameter distribution shapes within the data.

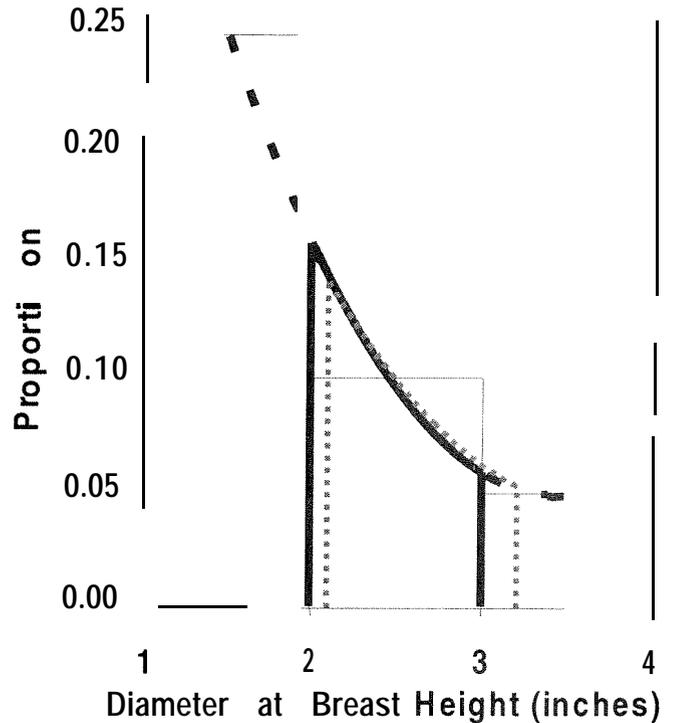


Figure 3-Example of projecting a diameter class into the future. The initial within-class distribution is a simple quadratic polynomial constrained to pass through the midpoints of the adjacent classes (solid lines) and integrate to the area of the histogram within the class. After projection (dotted line), the limits of the diameter class are changed, but the line still integrates to the same area.

ratios) in a diameter class (or stand table projection) model context.

Alternatively, the model could be implemented as an individual tree model, as the component parts are included in this model.

Figure 3 describes some of these steps. To avoid redundancy, we are starting with a diameter distribution that already reflects mortality. The quadratic polynomial ($Y = b_0 + b_1X + b_2X^2$; the math is easier if X is set equal to diameter minus the lower limit of the one-inch diameter class) is constrained to pass through the midpoint of the previous and succeeding diameter classes, and to integrate to the known probability within the diameter class. This is simple to ensure, as there are three parameters of the quadratic equation, and there are three pertinent known values, the proportion of trees in the subject, preceding, and succeeding diameter classes.

If we consider the lower limit of the diameter class to be 0, and the upper limit to be 1, and thus the midpoint of the previous diameter class to be -0.5 and the midpoint of the subsequent diameter class to be 1.5, we have three equations and three unknowns:

$$p_{i-1} = b_0 - \frac{b_1}{2} + \frac{b_2}{4} \quad [1]$$

$$p_{i+1} = b_0 + \frac{3b_1}{2} + \frac{9b_2}{4} \quad [2]$$

$$p_i = b_0 + \frac{b_1}{2} + \frac{b_2}{3} \quad [3]$$

where p_{i-1} is the proportion of trees in the previous diameter class, p_{i+1} is the proportion of trees in the succeeding diameter class, and p_i is the proportion of trees in the diameter class of interest. Equation [3] is the definite integral of the quadratic polynomial from 0 to 1. Equation [1] is obtained by setting Y of the quadratic polynomial equal to p_{i-1} and X equal to -0.5, and equation [2] is obtained by setting Y equal to p_{i+1} and X equal to 0.5. The parameters of the equation can be solved analytically to provide:

$$b_0 = \frac{9}{11} p_i - \frac{7}{44} p_{i+1} + \frac{15}{44} p_{i-1} \quad [4]$$

$$b_1 = \frac{12}{11} p_i + \frac{1}{22} p_{i+1} - \frac{23}{22} p_{i-1} \quad [5]$$

$$b_2 = -\frac{12}{11} p_i + \frac{6}{11} p_{i+1} + \frac{6}{11} p_{i-1} \quad [6]$$

In the case when the diameter class of interest is bounded by 0.0, then b_2 equals zero and equation [1] is not needed.

In that case, b_0 equals $\frac{3p_i - p_{i+1}}{2}$, and b_1 equals

$p_{i+1} - p_i$ if one-inch-wide diameter classes are employed.

When future condition of the diameter class is projected, the limits of the diameter class are predicted with an individual tree diameter growth equation. As larger trees grow more than smaller trees, the width of the diameter class expands. In our example, we used a diameter growth equation that was constrained to be consistent with whole-stand basal area growth, however this constraint could be invoked later. The parameters of the new within-class distribution are obtained by a simple transformation and another solution need not be calculated.

For example, if x is a given diameter within a diameter class with x_0 as the lower limit and x_1 as the upper limit, then the initial distribution might be:

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)^2 \quad [7]$$

and after projecting future conditions of the limits of the diameter class (indicated by the additional subscript, $_2$):

$$f(x_2) = \frac{b_0}{(x_{12} - x_{02})} + \frac{b_1}{(x_{12} - x_{02})} \left(\frac{x - x_0}{x_{12} - x_{02}} \right) + \frac{b_2}{(x_{12} - x_{02})} \left(\frac{x - x_0}{x_{12} - x_{02}} \right)^2 \quad [8]$$

Equation [8] is a simple transformation to ensure integration to the same proportion for that diameter class. To recover a fixed-width diameter distribution, the transformed equation is integrated from the lower level of the projected diameter class to the upper level of the previous fixed-width diameter class (3 inches in the example given in figure 3). That obtains the trees that remained in the same diameter class. Then, the number that moved up into the next diameter class may be obtained by subtraction, or by integration from the upper limit of the previous fixed-width class to the upper limit of the variable-width class. The method is applicable to situations when all trees of a diameter class move one or two classes, or even when trees of an initial fixed-width diameter class are projected to occur in three or more of the fixed-width classes at the end of the projection period. Although this procedure may seem somewhat involved, all of the math can be directly calculated without resorting to numerically solving for the parameters.

Initial Conditions

Although the preceding model structure can project the growth of stands of varying structure, there is no provision for initial conditions when the model will be applied to a "bare ground" starting point. Leduc and others [in press] has applied neural networks to predicting diameter distributions for longleaf pine plantations. We will also apply neural nets to provide the initial diameter distribution for a stand. This module of the model will be applicable to ages of 5 to 20 years. Although Leduc and others. applied neural networks to a much broader range of ages, the technique is less suited for the projection of future conditions, given some initial conditions, as it would be difficult to ensure that illogical behavior was avoided (such as abrupt shifts of diameter distributions within relatively short time periods). We will condition the neural net predictions of trees per acre in each diameter class to be consistent with the whole-stand basal area prediction equation that will also be used in projection. Thus, the basal area prediction equation will link the initial condition and projection components of the model, and will provide consistency.

CONCLUSION

This structure secures all of the objectives stated previously while no standard methodology does so. It could be considered to be an integration of standard diameter distribution models (although with a nonparametric distribution) and individual tree forest models, as well as evocative of "enhancements" to standard size class models (e.g. Cao and Baldwin 1999; Nepal and Somers 1992; Pienaar and Harrison 1988), and the "limitless" diameter class model of Clutter and Jones (1980). Thus, it

falls between classically-defined classes of models and incorporates an intermediate structure as discussed by Goelz [in press].

LITERATURE CITED

- Boyer, W.D.** 1990. *Pinus palustris* Mill. Longleaf Pine. In: R.M. Burns and B.H. Honkala, coord., *Silvics of North America*. Vol. 1, Conifers. USDA Forest Service, Agric. Handb. 654. Washington, DC. 405-412.
- Cao, Q.V.; V.C. Baldwin, Jr.** 1999. A new algorithm for stand table projection models. *Forest Science*. 45: 506-511.
- Clutter, J.L.; E.P. Jones, Jr. 1980. Prediction of growth after thinning of old-field slash pine plantations. USDA Forest Service, Research Paper SE-21 7. Southeast Forest Experiment Station, Asheville, NC. 19 p.
- Franklin, R.M. 1997. Stewardship of Longleaf Pine Forests: A Guide for Landowners. Longleaf Alliance Rept. No. 2. The Longleaf Alliance, Solon Dixon Forestry Education Center, Andalusia, AL. 44 p.
- Goelz, **J.C.G.** [in press]. Growth and yield models viewed from a diameter distribution perspective. *Journal of Mathematical Modelling and Scientific Computing*.
- Goelz, J.C.G.; D.J. Leduc. [in press]. Long-term studies on development of longleaf pine plantations. In J.L. Kush, ed., *Proceedings of the Third Longleaf Alliance Regional Conference. Forest for Our Future*. 16-18 October 2000, Alexandria, LA. The Longleaf Alliance and Auburn University, Auburn University, AL.
- Leduc, **D.J.; T.G. Matney; K.L. Belli; V.C. Baldwin, Jr.** Predicting diameter distributions of longleaf pine plantations: A comparison between artificial neural networks and other accepted methodologies. USDA Forest Service, Research Paper SRS-25. Southern Research Station, Asheville, NC. 18p.
- Lohrey, R.E.; R.L. Bailey.** 1977. Yield tables and stand structure for unthinned longleaf pine plantations in Louisiana and Texas. USDA Forest Service, Research Paper SO-133. Southern Forest Experiment Station, New Orleans, LA. 53 p.
- Nepal, S.K.; G.L. Somers.** 1992. A generalized approach to stand table projection. *Forest Science*. 38: 120-133.
- Pienaar, L.V.; W.M. Harrison.** 1988. A stand table projection approach to yield prediction in unthinned even-aged stands. *Forest Science*. 34: 804-808.
- Somers, G.L.; R.M. Farrar, Jr.** 1991. Biomathematical growth equations for natural longleaf pine stands. *Forest Science*. 37: 227-244.

ACCURACY OF EASTERN WHITE PINE SITE INDEX MODELS DEVELOPED IN THE SOUTHERN APPALACHIAN MOUNTAINS

W. Henry McNab¹

Abstract—Three older, anamorphic eastern white pine (*Pinus strobus* L.) site index models developed in the southern Appalachian Mountains between 1932 and 1962 were evaluated for accuracy and compared with a newer, polymorphic model developed in 1971. Accuracies of the older models were tested with data used in development of the 1971 model, in which actual site index had been determined by stem analysis. The 1971 model could not be evaluated for accuracy because independent data were unavailable. Evaluation statistics included prediction accuracy, bias, variance, mean square error, and tolerance interval. For one of the older models, prediction accuracy within 5 percent of observed site index was 100 percent, and other statistics compared favorably. Based on the premise that a polymorphic model best describes growth of eastern white pine over a range of site qualities, the site index model developed in 1932 performed surprisingly well.

INTRODUCTION

Eastern white pine (*Pinus strobus* L.) has long been recognized as one of the most valuable timber species in the southern Appalachian Mountains. This conifer is widely managed in natural and planted stands because of its desirable growth and yield characteristics, as well as the high value of its products. Site index (SI)—the average total height of the dominant and codominant trees of a stand at a specific standard age (Chapman and Myer 1949)—typically is used to measure the relative productivity of this species (Beck 1971). Site index relationships have been developed using various techniques, initially based on purely graphical methods and more recently based entirely on mathematical techniques (Chapman and Myer 1949). All types of SI relationships will be referred to as models in this paper.

Barrett first developed an SI model for eastern white pine (hereinafter white pine) in the southern Appalachian Mountains in 1932. Other models were developed as methods changed for quantifying the relationships that describe tree height increment over time. Five models based on data from the southern Appalachian Mountains are now available for white pine. The most recent model was developed by Beck (1971).

Potential problems associated with developing SI models are well known (Beck 1971, Beck and Trousdell 1973). Most problems are related to the inclusion of data from unrepresentative stands and inadequate methods of data analysis (Beck 1971). Each new SI study undoubtedly has reflected investigator intent to overcome perceived problems with earlier models. Therefore, a logical question might be: "Have white pine SI models evolved from less accuracy to greater accuracy over the past 70 years?" None of the southern Appalachian models has been tested for accuracy. This paper evaluates the accuracy of white pine SI models developed in the southern Appalachians.

METHODS

Site index Models

I examined the performance of four SI models that use a standard age of 50 years:

1. Barrett (1932) developed the first set of SI curves from "...measurements of 376 dominant and codominant trees growing in mixture with hardwoods..." He did not state his method for development of these curves, but likely based it on the guide-curve technique, where the age and height of individual trees throughout a region are measured, and one must assume that the population of site indices has been sampled adequately across all stand ages. The resulting SI model is derived from a single guide-curve that describes the average height increment relationship for the total set of sampled stands (Chapman and Meyer 1949). Site index models of this type are termed anamorphic because one curve shape describes the height-growth relationship over the entire range of site qualities sampled.
2. Doolittle and Vimmerstedt (1960) supplemented Barrett's data with additional observations from 105 plots in natural stands of pure white pine and mixed species composition in northern Georgia and western North Carolina. They, too, used the guide-curve method. However, recognizing that the rate of height growth varied with site quality, they attempted to correct for that effect using a mathematical technique based on the coefficient of variation (Chapman and Meyer 1949).
3. Vimmerstedt (1959, 1962) sampled 78 planted stands in North Carolina, Tennessee, and Georgia and established 111 plots for preparation of an SI model. Using linear regression, they developed an equation for predicting tree height at 25 years as a function of height and age, but they did not present statistics describing fit of the model. Vimmerstedt (1962) presented a conversion factor for

¹Research Forester, Bent Creek Experimental Forest, 1577 Brevard Road, Asheville, NC 28806.

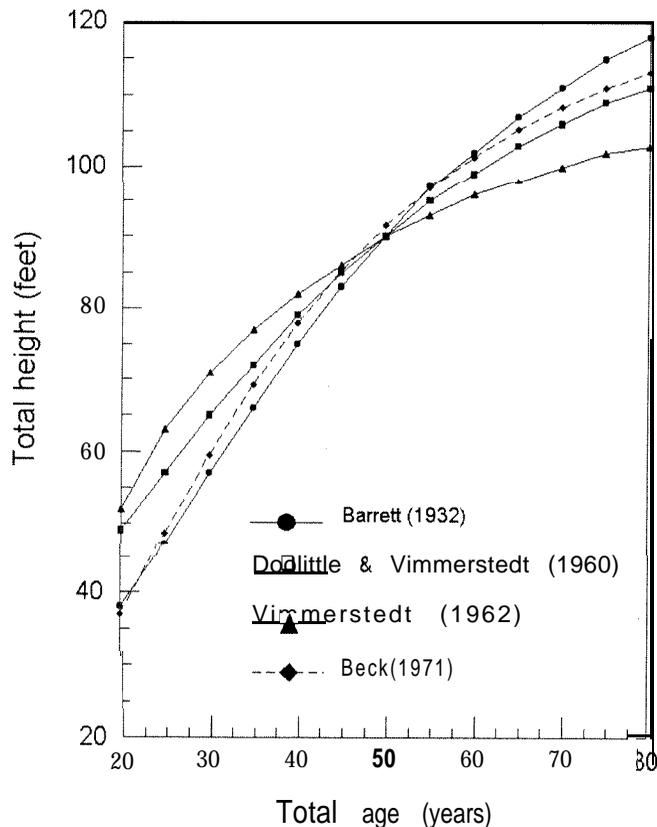


Figure 1—Comparison of eastern white pine site index curves developed in the southern Appalachian Mountains for site index 90 feet.

changing SI at a standard age of 25 to a standard age of 50 years.

4. Beck (1971) sampled 43 even-aged stands of naturally established white pine in western North Carolina, northern Georgia, eastern Tennessee, and southwestern Virginia. He used stem-analysis methods to determine the total height of each sample tree at successive ages, up to and including 50 years, which provided a direct measurement of observed SI for that site. He used a non-linear sigmoid function to derive a set of polymorphic curves whose shape varied in relation to site quality.

Summarized in table 1 are characteristics and ranges of total stand ages and site indices over which each of the four SI relationships can be applied. Predicted stand height over age for each of the models is presented in figure 1 for a SI of 90.

Independent Data Set

I used field data collected by Beck (1971) as an independent data set for evaluating each of the SI models. The SI of Beck's (1971) 43 stands averaged 92.7 feet (range 69 - 122), ages averaged 52.5 years (43 - 71), and total heights averaged 95.1 feet (70 - 119). About a quarter

of the stands were 48-years-old or less, a quarter were 49-51 years, and about half of the stands were 52-years or older (table 2). Additional information on field methods is described by Beck (1971). A deficiency of this independent data set is that it is not a random sample of the population of all site indices, but Beck (1971) selected it to represent certain conditions necessary for development of his model (Beck and Trousdell 1973).

I used each of the four models to predict SI of the 43 stands. I predicted SI to the nearest foot by reading directly from published age and height graphs for the models developed by Barrett (1932), and Doolittle and Vimmerstedt (1960). I obtained predicted SI by solving equations presented by Vimmerstedt (1962) and Beck (1971). However, because independent data were used in development of Beck's SI model, this data set cannot be used to validate his model. Performance results for the model developed by Beck (1971) are presented as a standard for comparison with the other models. The most recently developed SI model (Beck 1971) is referred to as the standard model: the other three are, collectively, the old models.

Model Performance Criteria

SI model performance is associated with and implies an unspecified accuracy of prediction. Accuracy is measured in terms of: bias and precision. Bias of a model is the average difference between predicted and the observed values. Precision is a measure of the scatter of predicted SI values around their mean value. Thus, an SI model may be characterized as: (1) unbiased and precise, (2) unbiased but imprecise, (3) biased but precise, or (4) biased and imprecise. An accurate model should have attributes of being both unbiased and precise. In some instances a model could have varying degrees of bias or imprecision

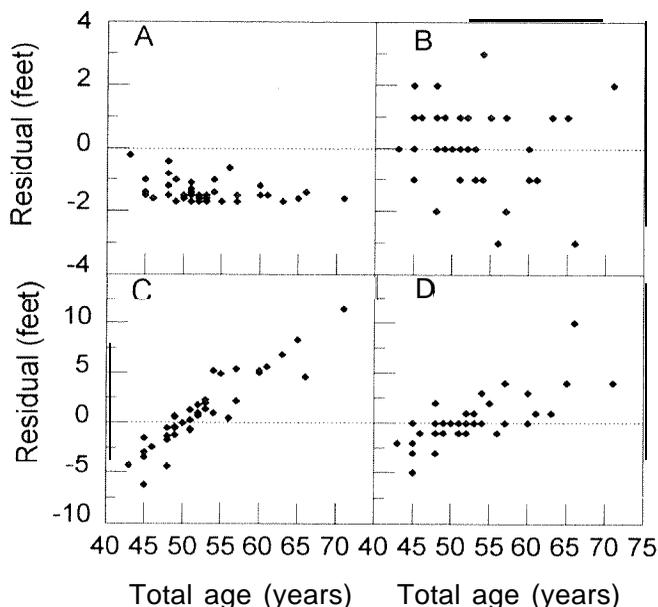


Figure 2—Residuals (predicted - observed) of eastern white pine site index resulting from application of a model used as a standard of comparison (A: Beck 1971) and three models evaluated for accuracy with an independent data set (B: Barrett, 1932; C: Vimmerstedt 1962; D: Doolittle and Vimmerstedt 1960).

¹Beck (1971) used only 42 of the 43 stands sampled to develop his SI model. The identity of and reason for excluding one stand is unknown.

Table I--Site index models for eastern white pine developed in the southern Appalachian Mountains

Model source	Stand type	Model format	Standard age (yrs)	Model Age (yrs)	ranges of SI (ft)
Barrett (1932)	Natural	Graph	50	20-120	50-130
Beck (1971)	Natural	Equation	50	5- 70	60-130
Doolittle and Vimmerstedt (1960)	Natural	Graph	50	20-100	50-130
Vimmerstedt (1959)	Planted	Equation	25	10- 35	40- 80
Vimmerstedt (1962)	Planted	Equation	50	10- 59	57-115

and still have acceptable accuracy. Each condition presents a different set of implications associated with model accuracy.

I evaluated the performance of each model using a number of statistics associated with accuracy. Because I was interested in learning the difference between observed and predicted SI values, I first determined the residual of each observation (stand):

$$\text{Residual} = (Y_i - \hat{Y}_i)$$

where \hat{Y}_i is the predicted SI for a stand, and Y_i is the observed SI value for the same stand. For many statistical comparisons, the standard method of calculating residuals is $(Y_i - \hat{Y}_i)$. However, I and others (Wiant 1993, Rauscher and others 2000) have used the reverse formulation because it provides results that are more easily comprehended: model overpredictions are positive errors and underpredictions are negative errors.

Bias is the mean of the residuals for all stands:

$$\text{Bias} = \Sigma (\text{Residual}_i)/n$$

where n is the number of sampled stands (here, 43).

The scatter of the residuals around the mean of observed SI for a model is a measure of its precision, which is quantified by the variance:

$$\text{Variance} = \Sigma (\bar{Y} - Y_i)^2/n-1$$

where \bar{Y} is the mean of all observed SI.

The bias and variance can be combined into single statistic, the mean square error:

$$\text{MSE} = \text{bias}^2 + \text{variance}$$

which provides a measure of the model accuracy and is an indication of the model that performs best overall for estimation of SI. A disadvantage of MSE is that it cannot be used to compare relative performance of models from other studies because it is dependent on the number of observations.

Two other statistics were used to overcome the limitations of MSE and provide a more easily understood measure of future prediction errors: prediction accuracy and tolerance interval. Rauscher and others (2000) used prediction accuracy (PA) to provide a measure of the proportion of predictions that occurred within a specified distance of the observed value. I used a PA value of ± 5 percent (e.g. PA-5), which is about equivalent to estimates within one SI class

Table P-Number of stands by total age and observed site index classes in the independent data set sampled by Beck (1971)

Age ^a (yrs)	Site index (ft)											
	70	75	80	85	90	95	100	105	110	115	120	Total
45				1	1	1	1		1		1	6
50	1	1	3	3	4	1	3	2	2		1	21
55			2	1	1	3			1	1		9
60					2			1				3
65	1	1					1					3
70		1										1
Total	2	3	5	5	8	5	5	3	4	1	2	43

^aMidpoints of age and site index classes (e.g., 45 = 43 through 47, 70 = 68 through 72).

Table 3-Error analysis statistics for three site index models developed for eastern white pine in the southern Appalachian Mountains compared to a model developed by Beck (1971) that was used as a standard of comparison

Category of site index model and source of model	Statistic				
	PA-5 ^a (pct)	Bias (ft)	Variance (ft)	MSE ^b (ft)	TI ^c (ft)
Site index models tested					
Barrett (1932)	100	0.17	1.56	1.59	3.03
Doolittle and Vimmerstedt (1960)	93	0.14	5.86	5.89	5.86
Vimmerstedt (1962)	77	1.14	12.22	13.53	8.47
Site index standard					
Beck (1971)	100	-1.47 ^d	0.13	2.30	0.89

^aPrecision accuracy = Percent of predicted site index values within 5 pct of actual.

^bMean square error = Bias² + variance.

^cTolerance interval = Bias ± limits of SI that will include 95 pct of future errors 95 pct of the time.

^dSignificantly different from zero at the 0.05 level of probability.

of 10 feet. Reynolds (1984) suggested calculation of tolerance interval as a means of determining the limits within which most errors will occur in an SI model. The tolerance interval is equal to the mean bias plus or minus the limits of predicted SI that will include 95 percent of future errors at a 0.95 level of probability. I calculated all statistics (except PA-5) using the computer program DOSATEST, which was developed by Rauscher (1986) and refined by Wiant (1993). DOSATEST calculates a trimmed mean and jackknife standard deviation for appropriate tolerance intervals if errors are not normally distributed (Wiant 1993). Accuracy testing using these five statistics (bias, precision, MSE, prediction accuracy, and tolerance interval) and the DOSATEST software has been reported by Wiant (1993) and Rauscher and others (2000).

RESULTS AND DISCUSSION

The PA-5 statistic was highest (100 percent) for two models, Barrett (1932) and the standard (table 3), indicating that all predicted values of SI were within 5 percent of observed. Only 77 percent of stand SI values predicted by the Vimmerstedt (1962) model were within these limits. The pattern of residuals of predicted and observed SI differed for each model (figure 2).

None of the three old SI equations was significantly biased (table 3). However, the equation developed by Beck (1971) exhibited a bias of -1.47 feet (see panel A in figure 2), which was significantly different from zero at the 0.05 level of probability. For example, on a plot with tree height 90 feet at 50 years, the standard model predicts SI as about 88.5 feet. The observed bias results from the model not being constrained, or adjusted, to pass through a value of SI equal to stand height at 50 years standard age (Personal communication T.Lloyd, Research Forester, USDA Forest Service, 1577 Brevard Road, Asheville, NC 28806), as is generally customary in most SI models. Constraining the model was not addressed by Beck (1971), but likely was not done in order to provide a model of greater overall accuracy. In contrast, Trousdell and others (1974) used a similar model formulation to develop SI curves for loblolly

pine (*Pinus taeda* L.) and adjusted the curves to pass through the indicated SI at age 50.

The tolerance interval was least for the standard model, which suggests a high degree of accuracy that is associated with small errors of prediction. Among the old models, tolerance interval was smallest (3.03 feet) for Barrett's (1932) and greatest (8.47 feet) for Vimmerstedt's (1962). For Barrett's model, which has a bias of zero (i.e. mean bias was 0.17, which was not significantly different from zero), the tolerance interval may be interpreted to indicate a 95 percent confidence that at least 95 percent of the population of future errors will occur within an interval of about ±3 feet of actual SI.

Mean square error, which combines the effects of bias and variance, was least for Barrett's model. The relatively large bias of the standard model (-1.47 feet) contributed to its large MSE. In many situations, however, a model with a large bias and small variance (e.g., Beck 1971) is preferable to a model with a small bias and large variance (e.g., Barrett, 1932). This is because prediction errors associated with bias can be easily corrected, but accounting for error arising from imprecision is problematic.

An explanation for the relatively poor performance of the Vimmerstedt (1962) model is likely due to several causes. First, unlike the other SI models evaluated, this one was developed in planted stands of white pine but tested using data from natural stands. Effects of stand establishment-method and species composition on SI relationships for white pine are not well known, although planted seedlings typically exhibit greater height growth than natural seedlings until about 5 years (Personal communication, Brian flitter, Forestry Supervisor, Biltmore Estate, One North Pack Square, Asheville, NC 28801). Second, over 80 percent of sample trees used in development of the Vimmerstedt model were less than 25 years of age, which tended to weight the curves away from height patterns at a standard age 50 years. Last, Vimmerstedt (1962)

presented without explanation a single factor for converting SI at base age 25 to base age 50. Application of the single factor suggests that total height at age 50 would be 1.4335 times that measured at age 25 on all sites. It seems likely that use of a single conversion factor would reduce accuracy of SI models at higher and lower site qualities. In comparison, Trousdell and others (1974) found that height of loblolly pine at 50 years ranged from about 1.4 to 1.7 times that at age 25, depending on site quality. The combination of these and other unknown factors likely contributed to reduced performance of the Vimmerstedt (1962) model.

The tests I conducted were restricted to stand ages 43-71 years, which covered only about half the age ranges applicable for most of the models. Tests of the models at younger ages were not possible due to lack of independent data. However, performance of the SI models for younger stand ages may be implied by their performance at the older ages. Assuming that the standard model offers the best representation of height for white pine at all ages, the model developed by Barrett (1932) probably would perform well in younger stands.

CONCLUSIONS

This study has shown that accuracy of eastern white pine SI models varies in the southern Appalachian Mountains. None of the three tested SI models exhibited performance superior to the most recently developed polymorphic model (Beck 1971), which, however, could not be evaluated because a satisfactory data set was not available. One of the anamorphic models (Barrett 1932) compared favorably to the standard model, and several components of its accuracy (bias and MSE) were slightly superior to the standard. The data presented in table 3 are statistics of fit for Beck's (1971) model, rather than independent tests of accuracy.

Results of this study should be useful to researchers for designing new studies and in helping managers decide which SI model to use. One reason I made this study was recognition of how little information is in the literature on the topic of SI validation testing. Site index models are one of the most commonly used forms of prediction equations in forestry; they typically are developed, presented, and used with no accompanying evaluation of performance. The DOSATEST program provides an easy-to-use tool for making tests of accuracy. The primary conclusions are that plantation SI curves seem to differ from natural stands, and that curves developed at two different times for the same region using very different model developmental techniques produced very similar results.

ACKNOWLEDGMENTS

I thank H. Michael Rauscher for providing the DOSATEST software. I also thank F. Thomas Lloyd and Mike Rauscher for many discussions on the topic of accuracy tests in forest measurements that led me to make this study. Tom Lloyd and Brian Ritter reviewed an early draft of this paper. Their suggestions for improvements are greatly appreciated.

LITERATURE CITED

- Barrett, L.I. 1932. Second growth white pine height growth classification. Tech. Notes 5. U.S. Department of Agriculture, Forest Service, Appalachian Forest Experiment Station. 1 p.
- Beck, D.E. 1971. Polymorphic site index curves for white pine in the Southern Appalachians. Res. Pap. SE-80. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 8 p.
- Beck, D.E.; Trousdell, K.B. 1973. Site index: accuracy of prediction. Res. Pap. SE-108. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 7 p.
- Chapman, H.H.; Meyer, W.H. 1949. Forest mensuration. New York: McGraw-Hill Book Co. 522 p.
- Doolittle, W.T.; Vimmerstedt, J.P. 1960. Site index curves for natural stands of white pine in the Southern Appalachians. Res. Note 141. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 2 p.
- Rauscher, H.M. 1986. The microcomputer scientific software series 4: testing prediction accuracy. Gen. Tech. Rep. NC-107. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 19 p.
- Rauscher, H.M.; Young, M.J.; Webb, C.D.; Robison, D.J. 2000. Testing the accuracy of growth and yield models for southern hardwood forests. Southern Journal of Applied Forestry. 24: 176-185.
- Reynolds, M.R. 1984. Estimating the error in model predictions. Forest Science. 30: 454-469.
- Trousdell, K.B.; Beck, D.E.; Lloyd, F.T. 1974. Site index for loblolly pine in the Atlantic Coastal Plain of the Carolinas and Virginia. Res. Pap. SE-115 Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 11 p.
- Vimmerstedt, J.P. 1959. Site index curves for Southern Appalachian white pine plantations. Res. Note 131. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 2 p.
- Vimmerstedt, J.P. 1962. Southern Appalachian white pine plantations: site, volume, and yield. Stn. Pap. 149. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 13 p.
- Wiant, H.V., Jr. 1993. DOSATEST for testing accuracy. Compiler. 11: 28-29.

NONLINEAR PROGRAMMING MODELS TO OPTIMIZE UNEVEN-AGED SHORTLEAF PINE MANAGEMENT

Benedict J. Schuiteand Joseph Buongiorno¹

Abstract-Nonlinear programming models of uneven-aged shortleaf pine (*Pinus echinata* Mill.) management were developed to identify sustainable management regimes that optimize soil expectation value (SEV) or annual sawtimber yields. The models recognize three species groups (shortleaf pine and other softwoods, soft hardwoods and hard hardwoods) and 13 2-inch diameter-at-breast-height size classes. Reproduction, growth and mortality rates are a function of tree diameter, stand density and site productivity. The optimal economic and production regimes each involve a guiding maximum diameter for softwoods and periodic hardwood control, with the optimal maximum diameter a function of site productivity.

INTRODUCTION

Growing public demand for non-commodity forest values such as biological diversity, scenic beauty, recreational opportunities and wildlife habitat has lead to increased interest in uneven-aged management. Yet models to predict the effects of specific management regimes on stand structure, species composition, timber production, economic returns and sustainability are not readily available for many forest types. This remains true for shortleaf pine, despite its economic importance and wide distribution. To help address this situation, we developed mathematical programming models to identify sustainable management regimes that maximize economic returns or annual sawtimber production for uneven-aged shortleaf pine.

GROWTH MODEL

To estimate stand growth, a site- and density-dependent matrix transition model was developed using data from 1047 naturally regenerated, shortleaf pine re-measurement

plots of the Southern Forest Inventory and Analysis (FIA) database (table 1, Hansen and others 1992). The average interval between inventories was 8.6 years. Observed upgrowth and mortality probabilities and ingrowth rates were converted to a one-year interval by exponential interpolation.

The model's structure follows Lin and others (1998). Trees are categorized into thirteen 2-inch diameter-at-breast height (DBH) size classes and three species groups: shortleaf pine and other softwoods, soft hardwoods and hard hardwoods. Size classes are denoted by their mid-point diameters and range from size class 2 to size class 26+, which contains all trees 25 inches DBH and larger. The model was calibrated on 838 plots (80 percent) chosen randomly from the 1047 available. The remaining 209 plots were used to test the accuracy of the model prior to re-estimating the parameters using data from all 1047 plots.

Table 1—Distribution of sample plots by state and inventory”

Inventory	State												
	AL	AL	AR	AR	LA	LA	MS	MS	OK	OK	TN	TX	TX
Current	'82	'90	'88	'95	'84	'91	'87	'94	'86	'93	'89	'86	'92
Previous	'72	'82	'78	'88	'74	'84	'77	'87	'76	'86	'80	'75	'86
Plots	108	47	192	174	34	21	85	42	82	85	14	a5	78

¹Inventories may span more than one year.

¹Research Assistant, Professor, Department of Forest Ecology and Management, 1630 Linden Drive, Madison, WI 53706-1598, respectively.

Citation for proceedings: Outcalt, Kenneth W., ed. 2002. Proceedings of the eleventh biennial southern silvicultural research conference Gen. Tech. Rep. SRS-48 Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 622 p.

Table 2-Equations for annual ingrowth (trees/ac/yr)

Sp. ^b	Stand	Site	Species		Con- stant	R ²	dF
	BA (ft ² /ac)	(ft ³ /ac /yr)	B	A			
SW	-0.597 **		0.41 **		41.9 **	0.12	1044
SH	-0.077 **	0.058 **	0.41 **		6.9 **	0.14	1043
HH	-0.091 **	0.059	0.13		12.7 **	0.02	1043

^aAsterisks denote level of significance: *, 0.01; **, 0.0001.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Ingrowth Rates

Table 2 gives the parameter estimates for the final ingrowth equations. Ingrowth rates were inversely proportional to total stand basal area and directly proportional to the basal area of the given species group, presumably reflecting the presence of more seed-producing trees. Site productivity had a significant, positive effect on the ingrowth of the soft hardwoods and hard hardwoods but not the shortleaf pine and other softwoods.

Upgrowth Probabilities

The upgrowth probability equations' parameters are in table 3. As expected, upgrowth probabilities were inversely proportional to stand density, directly proportional to site productivity, and a quadratic function of tree diameter for all three species groups. Upgrowth probabilities were lowest at small diameters, peaked at intermediate diameters, and declined again at large diameters.

Mortality Probabilities

The parameter estimates for the mortality equations are in table 4. All three species groups exhibit the expected convex relationship between diameter and mortality. Mortality probabilities were highest at small diameters, reached their lowest levels at intermediate diameters, and increased again at large diameters. For the shortleaf pine

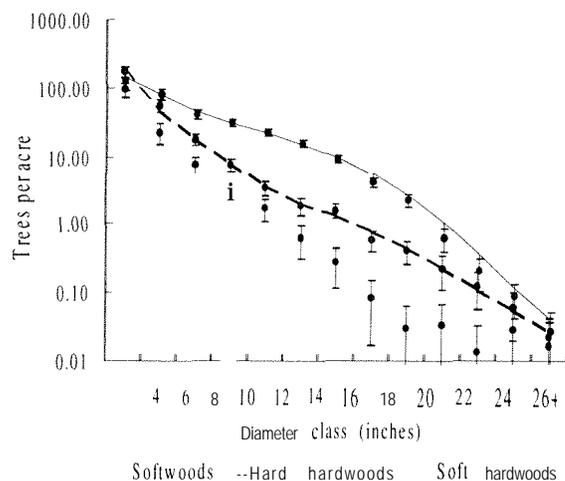


Figure 1-Average observed (dots, with 95 percent confidence intervals) and predicted (lines) distributions of shortleaf pine and other softwood, soft hardwood and hard hardwood trees on 209 post-sample plots after an average 8.6-years growth.

and other softwoods group, mortality probabilities were also significantly higher at higher stand densities and on more productive sites.

Projection Accuracy

To test the accuracy of the growth model for projections as long as the interval between two FIA inventories, the initial model, developed with data from the 838 estimation plots, was used to predict the diameter frequency distributions of the 209 validation plots at the current inventory given their distribution at the previous inventory and any intervening harvest. Figure 1 shows how the predicted distributions compared with the observed distributions. For most species-diameter categories, the average of the predicted number of trees was within the 95 percent confidence interval of average observed number of trees, though there was a slight tendency for the model to over predict the number of large shortleaf pine and other softwood trees.

YIELD MODEL

Cubic-foot sawlog and pulpwood volumes of individual trees are estimated using equations fitted to the stem

Table 3-Equations for probability of transition between size classes in 1 year^a

DF Spb	Stand BA (ft ² /ac)	Site (ft ³ /ac/yr)	DBH (in.)	DBH ² (in. ²)	Constant	R ²	DF
SW	-0.00034 **	0.00021 **	0.01000 **	0.00034 **	0.02740 **	0.09	5702
SH	-0.00018 **	0.00020 **	0.00830 **	-0.00032 **	0.00170	0.08	1234
HH	-0.00020 **	0.00022 **	0.00750 **	-0.00022 **	0.00470	0.10	2854

^aAsterisks denote level of significance: *, 0.01; **, 0.0001.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 4—Equations for probability of mortality in 1 year^a

Sp. ^b	Stand BA (ft ² /ac)	Site (ft ³ /ac /yr)	DBH (in.)	DBH ² (in. ²)	DBH ⁻¹ (l/in.)	Con-stant	R ²	dF
SW	0.0000560 **	0.00014 ***		0.000014 *	0.158 ***	-0.028 ***	0.22	5728
SH			-0.00470 ***	0.000160 ***		0.0360 ***	0.04	1244
HH			0.00085 **		0.092 ***	-0.007 ***	0.07	2878

^aAsterisks denote level of significance: *, 0.01; **, 0.001; ***, 0.0001.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 5—Equations for sawlog volume (cubic feet)

Sp. ^b	DBH ² (in. ²)	Sawlog (ft)	Con-stant	R ²	dF
SW	0.13	1.02	-36.1	0.94	299
SH	0.11	1.13	-23.6	0.96	229
HH	0.10	1.12	-31.6	0.96	225

^aFitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 6—Equations for pulpwood volume per tree (cubic feet)

Sp. ^b	Height (ft)	DBH ² (in. ²)	Con-stant	R ²	dF
SW	0.12	0.13	-36.1	0.97	47
SH	0.11	0.11	-23.6	0.97	80
HH	0.11	0.10	-31.6	0.97	80

^aFitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 7—Equations for top pulpwood volume per tree (cubic feet)

Sp. ^b	Height (ft)	Sawlog (ft)	DBH ² (in. ²)	Con-stant	R ²	dF
SW	0.067	-0.99	0.067	-21.5	0.89	298
SH	0.057	-1.06	0.057	-22.3	0.91	228
HH	0.056	-1.07	0.056	-17.0	0.91	224

^aFitted to the stem volume equations of Clark and Souter (1994). All coefficients are significant at the 0.0001 level.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 8—Equations for total tree height (feet)

Sp. ^b	Stand BA (ft ² /ac)	Site (ft ³ /ac /yr)	Site ² (ft ⁶ /ac ² /yr ²)	DBH (in.)	DBH ⁻¹ (l/in.)	Con-stant	R ²	dF
SW	0.090	0.42	-0.0010	1.04	-182	30.9	0.6617815	
SH	0.057	0.39	-0.0013		-274	56.4	0.55	1216
HH	0.071	0.44	-0.0013	0.75	-143	24.3	0.52	3654

^aAll coefficients are significant at the 0.0001 level.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 9—Equations for sawlog length (feet)

Sp. ^b	Height (ft)	DBH (in.)	DBH ⁻¹ (l/in.)	Con-stant	R ²	dF
SW	0.83	-2.2	-396	-36.1	0.75	11901
SH	0.49		-234	-32.6	0.44	291
HH	0.38	-1.9	-465	-31.6	0.37	1349

^aAll coefficients are significant at the 0.0001 level.

^bSpecies groups: SW, shortleaf pine and other softwoods; SH, soft hardwoods; HH, hard hardwoods.

Table 10—Stumpage prices^a

	Species group Pulpwood (\$/cord)	Species group Sawtimber (\$/Mbf)
Softwoods	21.88	324 ^b
Soft hardwoods	13.85	153 ^c
Hard hardwoods	13.85	291 ^c

Source: Timber Mart-South (Sept. 1999 – Aug. 2000).

^bScribner log rule; ^cDoyle log rule.

Table 11—Steady-state management regimes that maximize soil expectation value on low, medium and high productivity sites^a. Trees harvested each cutting cycle are denoted by asterisks

Size	--Low site-			-Medium site-			-High site-		
	SW	SH	HH	SW	SH	HH	SW	SH	HH
2	269.5	22.8*	46.6*	239.2	34.8*	58.0*	218.5	73.5*	101.5*
4	112.9	0.4*	1.9*	110.1	2.0*	3.6*	113.8	9.4*	14.1
6	72.7	0.0*	0.1*	71.2	0.1*	0.2*	76.1	1.0*	1.7*
8	56.8			54.3			58.0	0.1*	0.2
10	49.4			45.6			31.4*		0.0*
12	18.6			18.5			11.3*		
14	3.7			4.1			2.8*		
16	0.4			0.5*			0.5		
18	0.0			0.0*			0.1*		
20									
22									
24									
26+									

Statistics^b

Cycle	8	6	9
SEV	2093	2711	3430
Saw	81	109	119
H ¹ _{tree}	41	41	35

^aLow site, shortleaf pine site index 67 feet at age 50; medium site, 102 feet; high site, 142 feet.

^bCycle, optimal cutting cycle (years); SEV, soil expectation value (\$/acre); Saw, annual sawtimber production (ft³/acre/year); H¹_{tree}, percent of theoretical maximum tree diversity (pct), after harvest.

volume tables of Clark and Souter (1994). Pulpwood is potentially available from poletimber trees (softwoods 5 to less than 9 inches DBH or hardwoods 5 to less than 11 inches DBH) and from the tops of sawtimber trees (softwoods 9 inches DBH and larger or hardwoods 11 inches DBH and larger). Pulpwood volumes of poletimber trees (table 5) are a linear function of tree height and diameter squared; whereas pulpwood volumes from the tops of sawtimber trees (table 6) are a linear function of tree height, sawlog length and diameter squared. Sawlog volumes (table 7) are a linear function of sawlog length and diameter squared.

Total heights of the average tree in each size class of a particular species group are estimated using equations based on more than 22,000 trees on the 1047 plots used to develop the growth model. Table 8 gives the empirical tree height equations. For a given size class, trees were

Table 12—Steady-state management regimes that maximize annual sawtimber production on low, medium and high productivity sites^a. Trees harvested each cutting cycle are denoted by asterisks

Size	Low site			Medium site			High site		
	SW	SH	HH	SW	SH	HH	SW	SH	HH
2	251.5	20.0*	46.3*	234.9	5.7*	10.4*	223.7	18.2	27.7*
4	94.9	0.8*	1.9*	101.5			111.3	0.3	0.5*
6	58.3	0.0*	0.1*	63.5			72.6		
8	44.2			47.8			54.6		
10	37.7			39.3			44.6		
12	34.7			34.7			8.2*		
14	14.8*			3.0"			0.4*		
16	3.4*								
18	0.5*								
20	0.0*								
22									
24									
26+									

Statistic^b

Cycle	7	1	2
SEV	1595	486	2968
Saw	88	111	133
H ¹ _{tree}	45	46	41

^aLow site, shortleaf pine site index 67 feet at age 50; medium site, 102 feet; high site, 142 feet.

^bCycle, optimal cutting cycle (years); SEV, soil expectation value (\$/acre); Saw, annual sawtimber production (ft³/acre/year); H¹_{tree}, percent of theoretical maximum tree diversity (pct), after harvest.

taller in stands with more basal area and on more productive sites. Similarly, sawlog lengths are estimated using equations based on more than 13,000 trees from the same plots. The empirical sawlog length equations are in table 9. Sawlog length was a function of tree diameter and height.

OPTIMIZATION MODELS

Maximizing Soil Expectation Value

Knowing the maximum economic return that can be obtained from a particular site provides a useful measure for comparing the economic performance of alternative management regimes. The preferred measure of a management regime's economic performance, when applied to a stand of a given productivity, is the soil expectation value (SEV), the present value of all future harvests, net of all costs, including the opportunity cost of the growing

^aThe optimization models presented in this paper have non-concave response surfaces, thereby necessitating the use of nonlinear programming techniques. Consequently, the optimal regimes they identify are locally, though not necessarily globally, optimal. To improve the likelihood of finding globally optimal solutions, each problem was solved 50 times, each time beginning with different initial values.

stock. Because SEV is highly influenced by a stand's initial structure and to ensure sustainability, only steady-state management regimes, those in which the stand returns to the same pre-harvest diameter distribution each cutting cycle, are considered here. Consequently, the model to identify the sustainable, uneven-aged management regime that maximizes soil expectation value is:

$$\max_{y_0, h_0} SEV = \frac{s \mathbf{h}_0 - F}{(1+r)^C - 1} - s'(\mathbf{y}_0 - \mathbf{h}_0) \quad (1)$$

subject to:

$$\mathbf{y}_1 = \mathbf{G}_0(\mathbf{y}_0 - \mathbf{h}_0) + \mathbf{I}_0$$

$$\mathbf{y}_2 = \mathbf{G}_1(\mathbf{y}_1) + \mathbf{I}_1$$

$$\mathbf{y}_C = \mathbf{G}_{C-1}(\mathbf{y}_{C-1}) + \mathbf{I}_{C-1} \quad (2)$$

$$\mathbf{y}_C = \mathbf{y}_0 \quad (3)$$

$$\mathbf{y}_0 - \mathbf{h}_0 \geq \mathbf{0} \quad (4)$$

$$\mathbf{h}_0 \geq \mathbf{0} \quad (5)$$

where C is the cutting cycle, y_t is a vector containing the number of trees per acre of species group i and size class j at the start of year t , h_0 is a vector containing the number of live trees per acre of species group i and size class j harvested each cutting cycle, G_t is a matrix containing transition probabilities for year t , and I_t is a vector containing the ingrowth for year t (i.e., the number of trees entering the smallest size class of each species).

The stumpage values of individual trees, s , are obtained by multiplying their pulpwood (cords) and sawtimber (board-foot) volumes by their stumpage prices. The stumpage prices used in this analysis are 1999-2000 average prices, weighted by area, for the Southeastern United States (table 10, Timber Mart-South). Pulpwood cubic-foot volumes are converted to cords assuming 72 cubic feet per cord for softwoods and 79 cubic feet for hardwoods. Koch's conversion table (Koch 1972) is used to convert cubic-foot sawlog volumes to board-foot measures (Scribner log rule for softwoods and Doyle log rule for hardwoods). Costs not already reflected in the stumpage prices, F , such as administration and hardwood control, are assumed to total \$80.00 per acre, while the real rate of interest, r , is set at 4 percent.

Equations (2) are the growth equations². There is one equation for each year of the cutting cycle. Equation (3) is the steady-state constraint, which ensures sustainability by requiring the stand to return to the same pre-harvest distribution each cutting cycle. Equation (5) guarantees that the number of trees harvested from the stand does not exceed the number of trees present; whereas equations (4)

and (5) together ensure that the number of trees in, and harvested from, each species-size category is nonnegative.

Maximizing Annual Sawtimber Production

While economic concerns may be a key concern of many forest landowners and managers, others are likely to be

$$\max_{y_0, h_0} \text{Saw} = \frac{\mathbf{v}_s' \mathbf{h}_0}{C} \quad (6)$$

more interested in the volume of sawtimber that can be produced on a sustainable basis. The model to maximize annual sawtimber production is:

subject to:

(2), (3), (4) and (5)

where \mathbf{v}_s is a vector containing the cubic-foot sawtimber volumes of trees in each species-size category.

Measuring Tree Diversity

In addition to managing for economic returns and timber production, forest landowners are also increasingly interested in managing for biological diversity. Because the distribution of trees by species and size largely determines a stand's structure, and thus the ecological niches available to other organisms, tree diversity is a key component of a stand's overall diversity (Wilson 1974, Rice and others 1984). One of the most widely used and accepted diversity indices is Shannon's index (Pielou 1977, Magurran 1988). Here we define Shannon's index of tree diversity in terms of basal area, rather than number of individuals, to give added weight to larger trees:

$$H_{trees} = -\sum_{i=1}^m \frac{b_{ij}}{b + \epsilon} \ln \left(\frac{b_{ij} + \epsilon}{b + \epsilon} \right) \quad (7)$$

where b_{ij} is the residual basal area in species group i and size class j , b is the residual stand basal area and epsilon is a small, positive constant (0.001) used to avoid division by zero and natural logarithm of zero errors. As defined here, Shannon's index reaches its maximum value of 3.66 [$\ln(39)$] when the residual basal area is distributed evenly among each of the thirty-nine species-size categories. It provides a useful measure for comparing the tree diversity of the optimal economic and sawtimber regimes.

RESULTS AND DISCUSSION

Table 11 gives the steady-state management regimes that maximize SEV on low (shortleaf pine site index 67 at age 50 years), medium (site index 102), and high productivity

²Because the parameters of the growth and ingrowth matrices are derived from regression equations which contain negative coefficients for residual stand basal area, it is possible for the predicted transition probabilities and ingrowth rates to be negative when the residual basal area is sufficiently high. To avoid such biologically infeasible predictions, the right hand side of each applicable regression equation, call it "z", was replaced by the expression $[z + (z^2)^{1/2}]/2$. This expression returns the original value of "z" if it is positive and zero otherwise. This equation was also used, as needed, with regression equations for predicting sawtimber and pulpwood volumes.

(site index 142) sites. The optimal cutting cycles are 8, 6 and 11 years, respectively. In all three cases, the hardwoods are completely controlled at each harvest and the shortleaf pines and other softwoods are managed with a guiding maximum diameter of 11 inches DBH on low and medium sites and 9 inches DBH on high sites.

The optimal regimes give SEVs of \$2,093, \$2,711 and \$3,430 per acre, while producing 81, 109 and 119 cubic feet of shortleaf pine and other softwood sawtimber per acre per year, respectively. The small diameters of softwoods and the absence of hardwoods in the residual stands result in relatively low Shannon indices of tree diversity of 41 percent of the theoretical maximum value on low and medium sites and 35 percent on high sites.

Sawtimber Production

Table 12 shows the optimal management regimes for producing sawtimber on low, medium and high productivity sites. The optimal cutting cycles are 7, 1 and 2 years, respectively. As was the case for the SEV-maximizing regimes, the optimal sawtimber regimes each involve complete hardwood control at each harvest and a guiding maximum diameter for shortleaf pine and other softwoods: 13 inches DBH on low sites and medium site and 11 inches DBH on high sites.

These regimes have annual shortleaf pine and other softwood sawtimber production rates of 88, 111 and 133 cubic feet per acre on low, medium and high sites, respectively. By leaving more large diameter softwoods in the residual stand than the SEV-maximizing regimes, Shannon's index of tree diversity improves to 45, 46, and 41 percent of its theoretical maximum on low, medium and high productivity sites, respectively. In contrast, SEV drops to \$1595, \$486 and \$2968 per acre, respectively. This poorer economic performance is due, in part, to the shorter cutting cycles, which cause the fixed costs to be incurred more frequently.

CONCLUSION

Deciding how best to manage forestlands to meet specific objectives requires a clear understanding of what is possible on different sites. The nonlinear programming models presented here help define these limits for uneven-aged shortleaf pine by identifying sustainable steady-state management regimes that maximize either the soil expectation value or the average annual sawtimber production on low, medium and high productivity sites. In addition, the growth model developed for this study allows land managers to explore additional management strategies for meeting their own specific objectives.

Because tree growth, reproduction, and mortality are highly stochastic processes, our ability to model them accurately is limited. Therefore, the optimal regimes presented in this paper should be interpreted as tentative recommendations and not as proven strategies to be adopted unquestioningly. Likewise, simulation results obtained with the growth model should be interpreted as representing the expected average behavior of a number of similar stands, not as predicting the precise behavior of an individual stand.

ACKNOWLEDGEMENT

This work was supported in part by the Wood Utilization for Ecosystem Management Project, U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, by the USDA Forest Service, Southern Research Station and by the School of Natural Resources, University of Wisconsin, Madison. We thank Kenneth Skog and Jeff Prestemon for their collaboration.

REFERENCES

- Clark, A., III; Souter, R.A.** 1994. Stem cubic-foot volume tables for tree species in the South. Res. Pap. SE-290. U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station, Asheville, NC. 85 p.
- Hansen, M.H.; Frieswyk, T.; Glover, J.F.; Kelly, J.F.** 1992. The eastwide forest inventory database: Users manual. Gen. Tech. Rep. NC-151. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 48 p.
- Koch, P.** 1972. Utilization of the southern pines. Agricultural Handbook No. 420. Washington, DC: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 1,663 p.
- Lin, C.-R.; Buongiorno, J.; Prestemon, J.; Skog, K.** 1998. Growth model for uneven-aged loblolly pine stands-Simulations and management implications. Res. Pap. FPL-RP-569. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 13 p.
- Magurran, A.E.** 1988. Ecological diversity and its measurement. Princeton, NJ: Princeton University Press. 179 p.
- Pielou, E.C.** 1977. Mathematical ecology. Toronto: John Wiley and Sons. 385 p.
- Rice, J.; Anderson, B.W.; Ohmart, R.D.** 1984. Comparison of the importance of different habitat attributes to avian community organization. *Journal of Wildlife Management*. 49: 895-911.
- Timber Mart-South.** Highlands, NC: Timber Marts, Inc
- Wilson, M.F.** 1974. Avian community organization and habitat structure. *Ecology*. 55: 1017-1029.

A COMPATIBLE STEM TAPER-VOLUME-WEIGHT SYSTEM FOR INTENSIVELY MANAGED FAST GROWING LOBLOLLY PINE

Yuja Zhang, Bruce E. Borders, and Robert L. Bailey*

Abstract-Geometry-oriented methodology yielded a compatible taper-volume-weight system of models whose parameters were estimated using data from intensively managed loblolly pine (*Pinus taeda* L.) plantations in the lower coastal plain of Georgia. Data analysis showed that fertilization has significantly reduced taper (inside and outside bark) on the upper segment and augmented the stem merchantable volume there, which was modeled using an adjusted form factor. On the other hand, the unit-weights of fertilized trees were not significantly different from unfertilized trees. Finally, our analysis showed no significant impacts of complete vegetation control on taper, volume or weight characteristics.

INTRODUCTION

Geometry-oriented methodology constructs a theoretically sound and physically meaningful framework for taper prediction equation (Ormerod 1973, Forslund 1982, Newberry and others. 1989, Bailey 1994, Byrne and Reed 1986, Broad and Wake 1995, Parresol and Thomas 1996, Fang and Bailey 1999, Fang and others. 2000). The associated merchantable volume equation is mathematically compatible with the total volume equation, that is, it results from integration of the taper function (Demaerschalk 1972, Clutter 1980, Byrne and Reed 1986, McTague and Bailey 1987, Bailey 1994, Fang and Bailey 1999, Fang and others. 2000).

Recently, intensive management has been applied in the Southeastern U.S. (Jokela and Stearns-Smith 1993) to accelerate tree and stand growth and increase financial returns. Many of these silvicultural treatments have been shown to result in dramatic growth increases (Ford 1984, Gent and others. 1986, Allen and others. 1990, Stearns-Smith and others. 1992, Jokela and Stearns-Smith 1993, Borders and Bailey 1997). However, the effect of these treatments on individual tree stem taper, volume, and weight has not been fully studied. Borders and Bailey (1997) reported an extremely fast growth rate of loblolly pine (*Pinus taeda* L.) obtained from intensively managed stands in the Southeastern U.S. The thirteen-year growth of the most responsive stands yielded an average annual increment of 1.50 cm for quadratic mean diameter and 1.55 m for dominant height. Obviously, when such dramatic growth rate differences exist it is possible that stem taper and unit weight may be impacted as well.

STUDY MATERIALS

This investigation used data from the Consortium for Accelerated Pine Plantation Studies (CAPPS) initiated in 1987 and maintained by the Daniel B. Warnell School of

Forest Resources, University of Georgia. The treatments employed were: 1) Complete vegetation control throughout stand life-span using herbicide (H), 2) Annual fertilization (F), 3) Herbicide and Fertilization (HF), and 4) Check (C). In the winter of 1999, 192 trees with age 12, 10, and 6 years old were harvested from two study installations from the lower coastal plain of Georgia for wood quality research. Stem taper and weight measurements were made in field and disk analysis was done in the USDA Forest Service laboratory in Athens, GA. The impacts of cultural treatments and age on stem taper were investigated using the split-split plot design. The dependent variables employed are form quotients inside and outside bark at height proportion 0.25, 0.50, 0.60, 0.75, and 0.90, considering that the change of a specified quotient implies the change of stem form, which may be related to cultural treatments. Data analysis showed 1) no significant impacts from treatment H, 2) significant effects of treatment F are found only for quotients of 0.75 and 0.90, and 3) age is not a significant contributor.

MODEL STRUCTURE

Fang and others. (2000) proposed a system of compatible volume-taper models for traditionally managed loblolly pine and slash pine (*Pinus elliottii* Engelm) plantations, in which two inflection points (three segments) were employed. Screening the taper profile of stems in this study (figure 1), one inflection point seems adequate for our taper prediction equations. Followings are derived models for stem taper, volume, and weight.

Taper (Outside Bark)

The derivation of taper equation is similar to the method introduced by Fang and others. (2000) except 1) Newton's segment volume equation was employed in the derivation and 2) a boundary condition that $dob = dbh$ where stem

*Research Coordinator, Professor, Retired Faculty, D. B. Warnell School of Forest Resource, The University of Georgia, Athens, GA, USA, respectively.

Citation for proceedings: Outcalt, Kenneth W., ed. 2002. Proceedings of the eleventh biennial southern silvicultural research conference, Gen. Tech. Rep. SRS-48. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 622 p.

$$dob = dbhH^{\frac{k-\beta_1}{2\beta_1}} \left(a^\theta (1-p)^{\frac{k-\beta_1(1-\theta)\beta_2^\theta}{\beta_1^{1-\theta}\beta_2^\theta}} \right)^{\frac{1}{2}} \quad (1)$$

height = breast height.

where dob is the stem diameter outside bark, dbh the diameter at breast height, $p = \frac{h}{H}$, h the stem height,

H the total height, $H' = \frac{H}{H - 1.3716}$, $k = \frac{\pi}{8}$,

$a = (1 - p')^{\frac{\beta_2 - \beta_1}{\beta_1\beta_2}}$, p' the stem ratio at the inflection point, β_1 and β_2 the coefficients, and q the dummy variable with value zero for stem ratio $p = p'$ and one for $p > p'$,

Taper (Inside Bark)

Taper inside bark (dib) is similar to stem taper outside bark, except an extra coefficient c because of the effect of tree bark at breast height:

$$dib = c \cdot dbhH^{\frac{k-\gamma_1}{2\gamma_1}} \left(a'^\theta (1-p)^{\frac{k-\gamma_1(1-\theta)\gamma_2^\theta}{\gamma_1^{1-\theta}\gamma_2^\theta}} \right)^{\frac{1}{2}} \quad (2)$$

where $a' = (1 - p')^{\frac{\gamma_2 - \gamma_1}{\gamma_1\gamma_2}}$, and y_1 and y_2 the coefficients.

Volume (Outside and Inside bark)

Compatible stem volume equation can be readily obtained from integrating the taper function:

$$V_{sob} = \int_{h_0}^h \eta dob^2 dh \quad (3)$$

where h_0 is stump stem height and η a coefficient.

Using the defined relationship (Eq. (1)), integration of Eq. (3) results in:

$$V_{sob} = \xi dbh^2 HH^{\frac{k-\beta_1}{\beta_1}} \left\{ \beta_1(1-p_0)^{\frac{k}{\beta_1}} \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k}{\beta_1}} \right]^{1-\theta} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k}{\beta_1}} \right]^\theta \right\} + a\theta\beta_2(1-p')^{\frac{k}{\beta_2}} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k}{\beta_2}} \right]^\theta \quad (4)$$

where V_{sob} is the stem volume outside bark and ξ a coefficient.

Likewise, using dib in the integration results in the prediction equation of stem volume inside bark

$$(V_{sib}): V_{sib} = \zeta dbh^2 HH^{\frac{k-\gamma_1}{\gamma_1}} \left\{ \gamma_1(1-p_0)^{\frac{k}{\gamma_1}} \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k}{\gamma_1}} \right]^{1-\theta} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k}{\gamma_1}} \right]^\theta \right\} + a'\theta\gamma_2(1-p')^{\frac{k}{\gamma_2}} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k}{\gamma_2}} \right]^\theta \quad (5)$$

where V_{sib} is the stem volume inside bark and ζ a coefficient.

Weight

Let

$$D = f(h) \quad (6)$$

where D is the density of wood or bark and f some function then stem weight can be expressed as:

$$W = \int_{h_0}^h D dV = \int_{h_0}^h f(h) dV \quad \text{or} \quad \int_{p_0}^p f(p) dV \quad (7)$$

where h_0 is the stump height and $p = \frac{h}{H}$.

Parresol and Thomas (1996) proposed a linear model for D :

$$D = u_0 + u_1 h + u_3 \text{Age} \quad (8)$$

where u_1 , u_2 , and u_3 are coefficients.

Data analysis showed that age is a predictor of stem dry weight wood only. Similarly, we investigated the impacts of treatments H and F and found that both are not significant contributors.

The distribution of wood density along stem was screened on the individual tree base and we found that a quadratic equation form may better reflect the variation of wood density along stem:

$$DWD = d_0 + d_1 h + d_2 h^2 + d_3 \text{Age} \quad (9)$$

$$GWBD = g_0 + g_1 h + g_2 h^2 \quad (10)$$

$$GWD = w_0 + w_1 h + w_2 h^2 \quad (11)$$

where DWD , $GWBD$, and GWD are density of dry wood, green wood and bark, and green wood, respectively, and d_1 , d_2 , d_3 , g_0 , g_1 , g_2 , w_1 , w_2 , and w_3 some coefficients.

Using the functions of wood density and volume, the weight equations can be obtained upon integrating Eq. (24):

$$W_{dw} = (d_0 + d_1 H + d_2 H^2 + d_3 \text{Age}) V_{sib} - \zeta k d b h^2 H^2 H' \gamma_1^{k-\gamma_1}$$

$$\left[\gamma_1 (1-p_0)^{\frac{k+\gamma_1}{\gamma_1}} \left(\frac{d_1 k + 2\gamma_1 (d_1 + d_2 H)}{(k + \gamma_1)(k + 2\gamma_1)} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right. \right. \right.$$

$$\left. \left. - \frac{d_2 H (1+p_0)}{k + 2\gamma_1} \left[1 - \left(\frac{1+p'}{1+p_0} \right) \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1+p}{1+p_0} \right) \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right) \right.$$

$$\left. \left. + \alpha' \gamma_2 \theta (1-p')^{\frac{k+\gamma_2}{\gamma_2}} \left(\frac{d_1 k + 2\gamma_2 (d_1 + d_2 H)}{(k + \gamma_2)(k + 2\gamma_2)} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] - \frac{d_2 H (1+p')}{k + 2\gamma_2} \left[1 - \left(\frac{1+p}{1+p'} \right) \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] \right) \right] \right]$$

$$W_{gw} = (w_0 + w_1 H + w_2 H^2) V_{sib} - \zeta k d b h^2 H^2 H' \gamma_1^{k-\gamma_1}$$

$$\left[\gamma_1 (1-p_0)^{\frac{k+\gamma_1}{\gamma_1}} \left(\frac{w_1 k + 2\gamma_1 (w_1 + w_2 H)}{(k + \gamma_1)(k + 2\gamma_1)} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right. \right.$$

$$\left. \left. - \frac{w_2 H (1+p_0)}{k + 2\gamma_1} \left[1 - \left(\frac{1+p'}{1+p_0} \right) \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1+p}{1+p_0} \right) \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right) \right.$$

$$\left. \left. + \alpha' \gamma_2 \theta (1-p')^{\frac{k+\gamma_2}{\gamma_2}} \left(\frac{w_1 k + 2\gamma_2 (w_1 + w_2 H)}{(k + \gamma_2)(k + 2\gamma_2)} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] - \frac{w_2 H (1+p')}{k + 2\gamma_2} \left[1 - \left(\frac{1+p}{1+p'} \right) \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] \right) \right] \right]$$
(12)

$$W_{dw} = (d_0 + d_1 H + d_2 H^2 + d_3 \text{Age}) V_{sib} - \zeta k d b h^2 H^2 H' \gamma_1^{k-\gamma_1}$$

$$\left[\gamma_1 (1-p_0)^{\frac{k+\gamma_1}{\gamma_1}} \left(\frac{d_1 k + 2\gamma_1 (d_1 + d_2 H)}{(k + \gamma_1)(k + 2\gamma_1)} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right. \right.$$

$$\left. \left. - \frac{d_2 H (1+p_0)}{k + 2\gamma_1} \left[1 - \left(\frac{1+p'}{1+p_0} \right) \left(\frac{1-p'}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^\theta \left[1 - \left(\frac{1+p}{1+p_0} \right) \left(\frac{1-p}{1-p_0} \right)^{\frac{k+\gamma_1}{\gamma_1}} \right]^{1-\theta} \right) \right.$$

$$\left. \left. + \alpha' \gamma_2 \theta (1-p')^{\frac{k+\gamma_2}{\gamma_2}} \left(\frac{d_1 k + 2\gamma_2 (d_1 + d_2 H)}{(k + \gamma_2)(k + 2\gamma_2)} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] - \frac{d_2 H (1+p')}{k + 2\gamma_2} \left[1 - \left(\frac{1+p}{1+p'} \right) \left(\frac{1-p}{1-p'} \right)^{\frac{k+\gamma_2}{\gamma_2}} \right] \right) \right] \right]$$
(13)

where W_{dw} , W_{gwb} , and W_{gw} are stem weight of dry wood, stem weight of green wood and bark, and stem weight of green wood, respectively.

RESULTS

Treatment Effect

There is evidence that treatment F has impacted the upper segment taper, though treatment H did not affect these variables very much. To reflect this fact, the value of the upper segment form factor (b_2 or g_2) should be different for fertilized trees and unfertilized trees.

Estimation of Coefficients

Note that the derived stem taper and volume equations share not only independent variables such as p , dbh , etc. but also coefficients like b_1 and b_2 . System modeling is required for obtaining efficient estimates of parameters because the estimation of shared coefficients needs the information from all associated dependent variables, i.e., both taper and volume. Note that V_{sob} and V_{sib} are endogenous variables because they appear on both sides of volume and weight equations. To eliminate simultaneous

Table I-Estimates of parameters with standard errors (in second line)

dob	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	ρ'
	0.1569	0.1441	0.1561	0.6025
	0.0005	0.0007	0.0007	0.0092
dib	c	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_{2f}$
	0.8625	0.1724	0.1428	0.1588
	0.0035	0.0011	0.0008	0.0008
V_{sob}	ξ			
	1.99E-4			
	3.60E-7			
V_{sib}	ζ			
	1.59E-4			
	6.19E-7			
W_{dw}	d_0	d_1	d_2	d_3
	367.63	-5.95	-5.70E-2	12.82
	13.73	7.82E-1	1.31 E-2	1.69
W_{gwb}	g_0	g_1	g_2	
	767.38	21.96	5.23E-2	
	5.72	4.50E-1	7.42E-3	
W_{gw}	w_0	w_1	w_2	
	784.88	21.68	1.86E-1	
	7.04	5.72E-1	8.97E-3	

Table S-Fit statistics of each equation in the taper-volume-weight equation system, where MB is the mean bias, RMSE the root mean square error, and EF the modeling efficiency

Equations	MB	RMS	EF
dob	0.2667 (cm)	1.1841 (cm)	0.9621
dib	0.4736 (cm)	1.1042 (cm)	0.9545
V_{sob}	6.30E-4 (m ³)	0.0092 (m ³)	0.9900
V_{sib}	8.20E-4 (m ³)	0.0099 (m ³)	0.9835
W_{dw}	-5.34E-1 (kg)	2.17 (kg)	0.9966
W_{gwb}	3.68E-1 (kg)	9.17 (kg)	0.9890
W_{gw}	-3.10E-1 (kg)	9.12 (kg)	0.9870

equation bias, predicted rather than observed V_{sob} and V_{sib} values were used as regressors in weight equations during parameter estimation (Borders and Bailey, 1986).

The mixed-effects systematic modeling technique was applied for obtaining unbiased and consistent estimates of parameters. The modeling efficiency (EF), root mean square error (RMSE), and mean bias (MB) (Loague and Green, 1991, Mayer and Butler 1993) were applied as fit statistics. The estimates of coefficients involved are listed in table 1 with the fit statistics in table 2.

DISCUSSION AND CONCLUSION

Compared with an empirical taper equation, the one derived from geometric relationships is more theoretically sound and physically meaningful and reduces the parameters dramatically, which simplifies model structure and helps parameter estimation in nonlinear regression.

Resultant taper equations showed that two segments well depict the relationship between stem diameter and height for trees in this study. The above conclusion does not go with Fang and others.'s (2000) where three segments are required. A plausible explanation for this disagreement might be the fact that the stems used in this work are relatively young and do not exhibit much butt swell.

Fig. (2) shows the profiles of *dob* and *dib* using a 18 meters long stem with *dbh* 20 centimeters for unfertilized and fertilized trees, implying a significant fertilizer impact for both inside and outside bark diameters on upper segment. Specifically, fertilized trees have less taper than unfertilized trees, implying more volume and woody materials on the upper stem of fertilized trees.

In this study, we derived segmented stem weight equations by integrating wood density and segmented volume. This approach provides logical estimates of wood density for any segment along the stem. This is especially noteworthy since previously derived equations overpredict wood density in the upper part of stems. Yet, the fact that fertilization did not significantly affect the unit-weight of stem wood agrees with the results of data analysis and conclusions derived from the investigation done by Clark using the same data (personal communication).

It should be noted that this system of stem taper-volume-weight equations was fitted to a small database from a specific geographic location. Thus, any use of these functions should first be validated on independent data. The objective here was not to produce equations that will be widely used by practitioners but to develop a modeling framework that is flexible enough to reflect the impacts of various silvicultural treatments. As such, these equations provide researchers a useful tool for simulating the impact that fertilization may have on stem form, volume, and weight.

ACKNOWLEDGMENTS

Authors wish to gratefully acknowledge the support from the sponsors of the Consortium for Accelerated Pine Plantation Studies, Boise Cascade Corporation, Champion International Corporation, Gilman Paper Company, International Paper Company, Jefferson Smurfit Group, Mead Coated Board, Rayonier Industry,

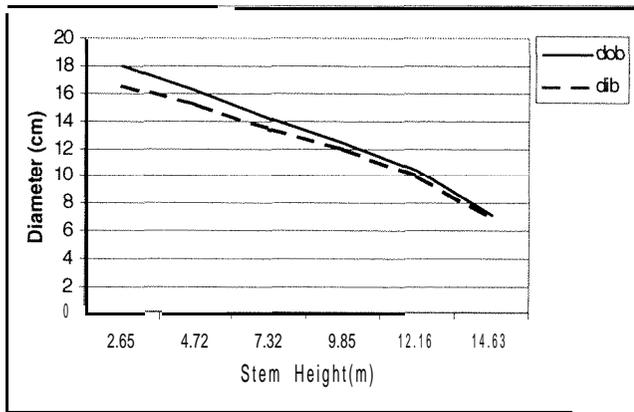


Figure 1—Stem taper profile of a tree at age 12, where *dob* is the stem diameter outside bark and *dib* the stem diameter inside bark.

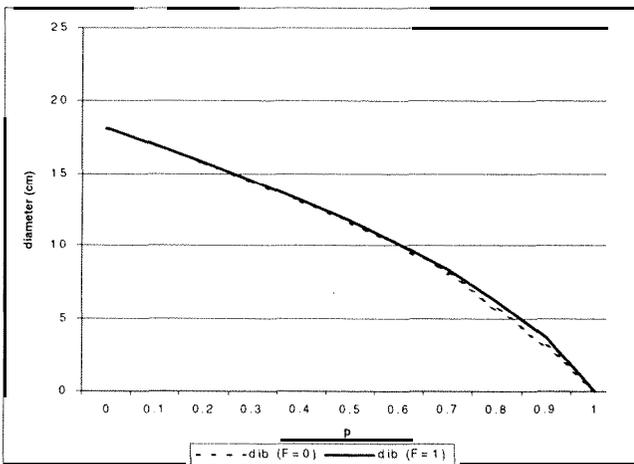


Figure 2a—Stem taper profiles of diameter outside bark *dob* for fertilized ($F = 1$) and unfertilized ($F = 0$) trees with total height 18 meters and *dbh* 20 centimeters, where p is the ratio of stem height to total height.

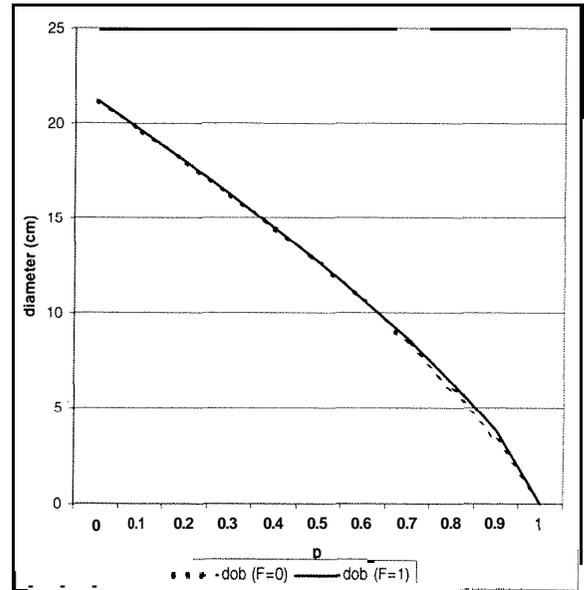


Figure 2b—Stem taper profiles of diameter inside bark (*dib*) for fertilized ($F = 1$) and unfertilized ($F = 0$) trees with total height 18 meters and *dbh* 20 centimeters.

Temple-Inland Industry, The Timber Company, Union Camp Corporation, US Alliance, Westvaco Corporation, and Weyerhaeuser Company.

REFERENCES

- Allen, H.L.; P.M. Dougherty; R.G. Campbell. 1990. Manipulation of water and nutrients-practice and opportunity in southern U.S. pine forests. *Forest Ecology Management*. 30: 437-453.
- Bailey, R.L. 1994. A compatible volume-taper model based on the Schumacher and Hall generalized constant form factor volume equation. *Forest Science*. 40: 303-313.
- Borders, B.E.; R.L. Bailey. 1986. A compatible system of growth and yield equations for slash pine fitted with restricted three-stage least squares. *Forest Science*. 32: 185-201.
- Borders, B.E.; R.L. Bailey. 1997. Loblolly pine: Pushing the limits of growth. Consortium on Accelerated Pine Production Studies. Technical Report CAPPS 1997-I. The University of Georgia, Cooperative Extension Service, Forest Resources Publication. January 1997.
- Broad, L.R.; G.C. Wake. 1995. Derivative based methods for constructing volume-ratio and taper equations. *Forest Science*. 41: 157-167.
- Byrne, J.C.; D.D. Reed. 1986. Complex compatible taper and volume estimation systems for red and loblolly pine. *Forest Science*. 32: 423-443.
- Clutter, J.L. 1980. Development of taper functions from variable-top merchantable volume equations. *Forest Science*. 26: 117-120.
- Demaerschalk, J.P. 1972. Converting volume equations to compatible taper equations. *Forest Science*. 18: 241-245.
- Fang, Z.; R.L. Bailey. 1999. Compatible volume and taper models with coefficients for tropical species on Hainan Island in southern China. *Forest Science*. 45: 85-100.

- Fang, Z.; B.E. Borders; R.L. Bailey.** 2000. Compatible volume-taper models for loblolly and slash pine based on a system with segmented-stem form factors. *Forest Science*. 46:1-12.
- Ford, E.D.** 1984. The dynamics of plantation growth. In: *Nutrition of plantation forests*. Bowen, G.D.; E.K.S. Nambiar, eds. London: Academic Press: 17-52.
- Forslund, R.R.** 1982. A geometrical volume model based on the location of the center of gravity of the bole. *Canadian Journal of Forest Research*. 12: 215-221.
- Gent, J.A.; H.L. Allen; R.G. Campbell.** 1986. Phosphorous and nitrogen plus phosphorous fertilization in loblolly pine stands at establishment. *Southern Journal of Applied Forestry* 10(2): 114-117.
- Jokela, E.L.; S.C. Stearns-Smith.** 1993. Fertilization of established southern pine stands: effects of single and split nitrogen treatments. *Southern Journal of Applied Forestry* 17(3): 135-138.
- Loague, K.; R.E. Green.** 1991. Statistical and graphical methods for evaluating solute transport models: overview and application. *J. Contam. Hydrol.* 7: 51-73.
- Mayer, D.G.; D.G. Butler.** 1993. Statistical validation. *Ecol. Model.* 68: 21-32.
- McTague, J.P.; R.L. Bailey.** 1987. Simultaneous total and merchantable volume equations and a compatible taper function for loblolly pine. *Canadian Journal of Forest Research*. 17: 87-92.
- Newberry, J.D.; H.E. Burkhart; R. L. Amateis.** 1989. Individual tree merchantable estimating system. *Canadian Journal of Forest Research*. 19: 679-683.
- Ormerod, D.W.** 1973. A simple bole model. *For. Chron.* 59: 136-138.
- Parresol, B.R.; Thomas, C.C.** 1996. A simultaneous density-integral system for estimating stem profile and biomass: slash pine and willow oak. *Canadian Journal of Forest Research*. 26: 773-781.
- Stearns-Smith, S.C.; E.J. Jokeia; R.C. Abt.** 1992. Thinning and fertilizing southern pine stands of the lower coastal plain: biological and economical effects. *Southern Journal of Applied Forestry*. 16(4): 186-193.

A MODEL FOR DEFINING AND PREDICTING THE URBAN-WILDLAND INTERFACE FOR THE PIEDMONT OF SOUTH CAROLINA

Mary L. Webb Marek and Lawrence R. Gering¹

Abstract- Resource managers continue to experience a deluge of management conflicts as urban population centers expand into areas that were formerly wildland settings. Traditional forest management practices, fire suppression, recreational opportunities and wildlife management are activities that have become contentious in many locales. A better understanding of the interface zones between these two types of land use is important if managers are to successfully maintain the values of such lands. A model for defining the urban-wildland interface for the Piedmont of South Carolina (Anderson, Oconee, and Pickens Counties) was developed, allowing identification of these transitional zones. Landsat TM and SPOT images provided a description of the current land cover and land use of the study area. Census data were used to obtain information on housing densities, population densities, and other social and cultural activities. Additional data (such as digital road maps) were processed and added to the ArcView-based GIS structure. On-site ground truthing was also conducted. This procedure created a snapshot view of current interface zones and provides a foundation for developing a dynamic model designed to predict future change.

INTRODUCTION

The objective of this study is to develop a process for determining the urban-wildland interface within Anderson, Oconee, and Pickens Counties in South Carolina using social, economic, and land use/land cover data through the use of a desktop geographic information system (GIS).

The idea of urban-wildland interface is a relatively new problem in natural resource management. The growth of the United States as a nation over the past two centuries is intricately tied to the concept of the conflict between urban development and pre-existing wildland. Between 1970 and 1990, the United States population increased by 22.5 percent with 21 million acres being converted to more urban land uses (Garkovich 2000).

According to a recent Sierra Club report, South Carolina lags behind the rest of the nation in terms of open space protection, ranking third to the last among the fifty states (Romain 2000). This situation has sparked valid concerns of urban sprawl in the Upstate of South Carolina. As the metropolitan areas of Greenville, SC; Atlanta, GA; and Charlotte, NC continue to expand, the counties located just outside of and between these cities will continue to provide evidence of this rural to urban transition. Concerned residents and politicians have formed groups focused on the protection of specific open spaces. Some of such groups include Upstate Forever, Friends of the Reedy River, and a Committee of Changing Land Use and Environment (CLUE). While the concept of multiple use is widely acknowledged and often practiced, the problem of externalities on the

urban-wildland interface presents a particularly difficult management challenge because of the concentrated nature of the activities. These changes in land use have sparked many conflicts, making natural resource management more difficult and creating the need for identification of these urban-wildland interface areas. Management of these areas requires that one must understand the various land cover, economic, social, political, and historical factors that are involved in the make up of these zones. From this information, a definition can then be developed and the existence of these interface zones may be predicted and appropriate management concepts applied.

METHODS

A GIS database can be a vital tool within many different professions and especially in the management of natural resources because of its integral spatial component. Predictive modeling, as demonstrated in this study, is a common application of GIS technology. Within ArcView GIS, queries can easily be built and the results visually displayed. The query tool in ArcView allows an analyst to select features and records in a table that relate to attributes of the map data.

For this study, data from the 1990 United States Census (median household income, housing density, roads and municipalities) and 1992 satellite imagery (Landsat Thematic Mapper imagery of land use/land cover) were combined to get a definition of the urban-wildland interface.

¹Graduate Student, Assistant Professor, Department of Forest Resources, Clemson University, Clemson, SC, respectively.

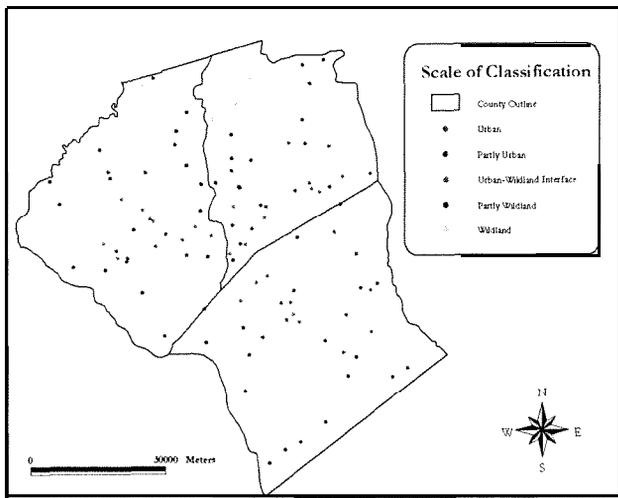


Figure 1—GPS Data from Preliminary Sample

In order to develop a reasonable query for urban-wildland interface, a preliminary sample of global positioning system (GPS) data was taken using a Trimble GeoExplorer II. Points were collected randomly along roadsides within the tri-county area. Each point was classified along a scale of urban to wildland based on observation. Although this appears to be a subjective task, much of the process was based on the Anderson Level II Land Use Classification System (Anderson 1976). The map in figure 1 depicts the scale of classification used and the preliminary sample of GPS data.

The “GPSed” points were examined to determine statistical trends in each classification. Queries were then conducted to identify where urban-wildland interface areas were located. Once the queries were completed, a verification sample of 125 points was collected, again using GPS technology, to compare the queried data with on-site observations (ground-truthing).

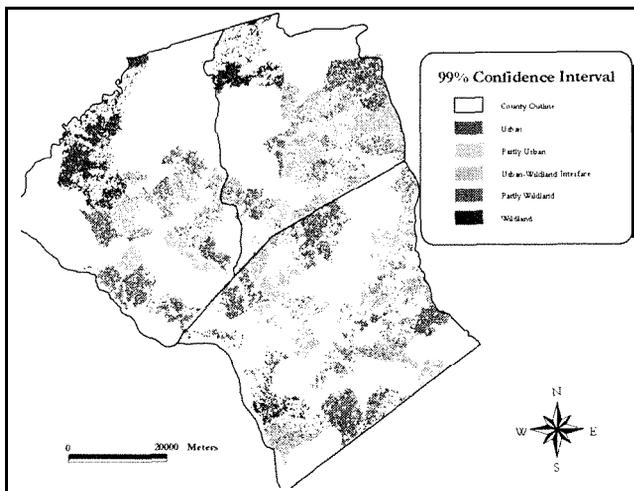


Figure 2—Query Results from 99 Percent Confidence Interval on Housing Density and Median Household Income.

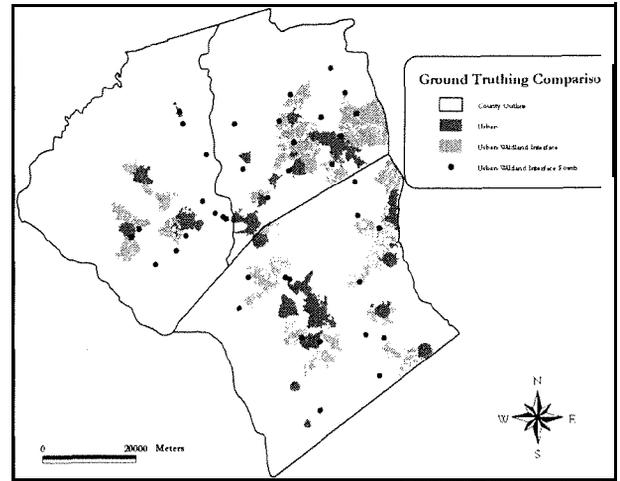


Figure 3—Urban-Wildland Interface Verification Sample Compared with 99 Percent Confidence Interval Query.

RESULTS

The initial map queries of each classification included 90 percent, 95 percent, 99 percent confidence intervals based on median household income and housing density because these were the quantitative variables used in the study. These queries also specified land use/land cover types which occurred 30 percent or more of the time in each urban to wildland classification. Figure 2 depicts the results from the 99 percent confidence interval query.

Figure 3 focuses on the query results for urban-wildland interface, comparing those areas with urban-wildland interface points collected in the verification sample. Local municipalities were displayed for location reference. Here, a total land area of 635 square kilometers meets the criteria from the query for urban-wildland interface. Points from the verification sample allow one to examine the veracity of the query results. Some points lay directly on urban-wildland interface, indicating agreement between the query criteria and on-site observations. Other points lay just

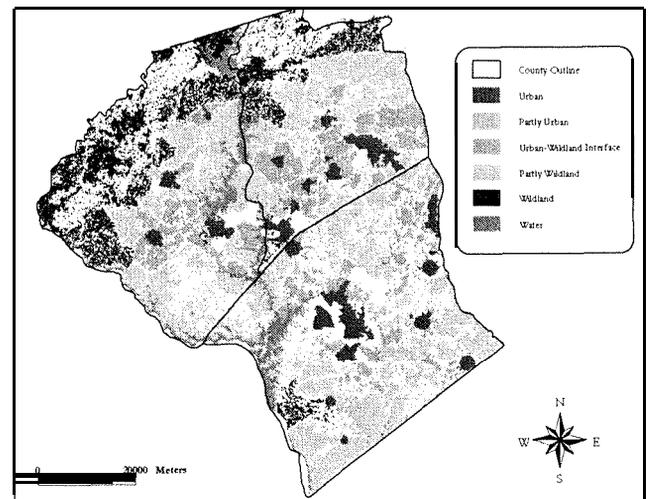


Figure 4—Extended Query Results.

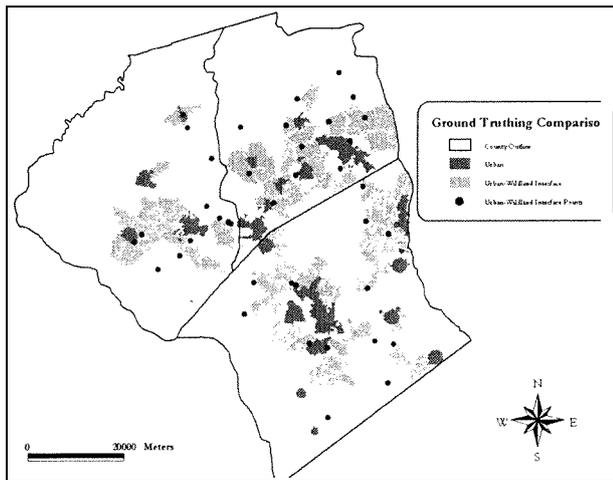


Figure 5—Urban-Wildland Interface Verification Sample Compared with Extended Query.

outside of or along the outskirts of urban-wildland interface, indicating partial agreement between the query data and on-site observations. Still other points remain in question, lying on areas for which census data were not available, and therefore, not answered by the urban-wildland interface query.

Because of overlapping data from the initial queries, each classification was separated based on housing, with higher densities depicting more urban areas. The queries were further refined by extending the ranges within each classification and by requesting urban areas to reflect municipality boundaries. Figure 4 depicts the results from this extended query. This map shows a more logical pattern for locations of each classification, where urban areas blend into interface areas, which in turn, blend into wildland areas.

Figure 5 focuses on the results from the extended query for urban-wildland interface, as compared with the verification points for urban-wildland interface. This query indicates

that a total area of 1061 square kilometers meet the criteria for urban-wildland interface, nearly 1/5th of the entire tri-county area. This map shows more agreement between the verification points and the query results. That agreement reflects accuracy within the queries, and depicts the locations of urban-wildland interface. However, points of partial agreement and points of question still exist. It is yet to be determined what may be occurring in these areas. Perhaps they have outlying values in the housing density or median household income variables, which would prevent these regions from meeting the query criteria for urban-wildland interface. A revisit to these points may answer some of these questions.

CONCLUSION

This study reveals that using social, economic, land use/land cover data may define urban-wildland interface locations. Maps like these could be used as a tool for development and land management planning, especially where conflicts exist or may arise. These maps allow land managers to easily identify and focus on regions of concern. They provide information for not only land managers and developers, but also for interested community members and local residents within areas experiencing land use changes.

This study could be further expanded to examine interface in larger regions using additional social and economic variables. The added information would provide an even more detailed definition, applicable to more communities. The model could also be adjusted to depict future change, by including a variable representing time.

REFERENCES

- Anderson; Hardy; Roach; Witmer. 1976. "A Land Use and Land Cover Classification System for Use with Remote Sensor Data." USGS Professional Paper. No. 964. 26 p.
- Garkovich; Lorraine. 2000. Land Use at the Edge: The Challenges of Urban Growth for the South. The Southern Rural Development Center, No. 13, August.
- Romain, L. 2000. Greenville rec chief: Buy Land Now or Miss Out. The Greenville News, March 25, 13A.

THE EFFECT OF DENSITY ON THE HEIGHT-DIAMETER RELATIONSHIP

Boris Zeide and Curtis Vanderschaaf¹

Abstract-Using stand density along with mean diameter to predict average height increases the proportion of explained variance. This result, obtained from permanent plots established in a loblolly pine plantation thinned to different levels, makes sense. We know that due to competition, trees with the same diameter are taller in denser stands. Diameter and density are not only necessary, but may be sufficient for determining tree height because other factors affecting height are reflected by diameter and density. In the process of developing the proposed model we found that height increases monotonically with density and that this increase is not bounded by an asymptote. Contrary to our expectations, the inclusion of density did not bring the allometric parameter of diameter closer to the theoretical value of 2/3.

INTRODUCTION

By relating height and diameter we can express height from diameter, which can be measured easier and more reliably. This relationship also informs us about stem taper and, therefore, volume. As a result, the height-diameter relationship is one of the most studied in forestry. Although diameter is a good predictor of height, we may advance further by using other available information. Diameter explains a lot of variation in height. After all, it is designed to support the load that depends on tree height. Still, there are other factors determining the load that may modify the height-diameter relationship. The most obvious among these factors is stand density.

APPROACH AND BASIC ASSUMPTIONS

Theoretical and empirical studies of the height-diameter relationship suggest that it is an allometric function with the power of diameter, b , equal to 2/3 (Greenhill 1881, McMahon 1973, Norberg 1988, O'Brien and others 1995):

$$H = aD^b \quad (1)$$

This relationship describes a column of equal resistance to bending and buckling, which is a reasonable assumption for tree stems exposed, in addition to the force of gravity, to wind (O'Brien and others 1995, Schniewind 1962) and snow (King and Loucks 1978, McMahon and Bonner 1983). Such a column maintains elastic similarity along the stem (Rich 1986, Rich and others 1986). Elastic similarity leads to $b=2/3$ and allows the tree to maintain a constant safety factor against both buckling and bending due to tree weight and wind force (McMahon and Bonner 1983, Norberg 1988, Rich and others 1986).

Besides purely structural considerations, there is a biological component. Trees have evolved to equalize not so much the strength along the stem as to equalize the

damage to its survival. Below the crown this biological requirement coincides with the mechanical one because at any point breakage dooms the tree. The situation inside the crown is different. Trees may survive the loss of a third of the crown and more. Therefore, it would not pay to invest into equal strength of the upper stem. Indeed, trees often lose tree tops, most frequently within the upper third of the crown.

Equation (1) assumes that height depends exclusively on diameter. This is not true: in dense stands trees with the same diameter are taller than those in less dense stands. Therefore, stand density should be included as the second predictor of average height. Out of many ways to incorporate density into the predicting equation, we tested several asymptotic and non-asymptotic density modules (table 3).

As the measure of density we used Reineke's Stand Density index (SDI) (Reineke 1933):

$$SDI = N*(D/10)^{1.7} \quad (2)$$

where: N = number of trees per acre, D = quadratic mean diameter of a stand. The power of 1.7 was provided by MacKinney and others (1937) who reanalyzed the data used by Reineke (1933) with standard statistical methods. Sometimes it is convenient to normalize the index by dividing it by the maximum value of 450 which was reported by Reineke for loblolly pine:

$$I = (N*(D/10)^{1.7})/450 \quad (3)$$

Density does not affect height prior to the onset of competition, which happens, according to our observations when Reineke's index is 34. This minimal level of density, denoted as $I_0 = 34/450$, is used in the following models to set the initial effective density to 1.

¹Professor and Research Specialist, School of Forest Resources, University of Arkansas, Monticello, AR 71656-3468, respectively.

Table I-Quadratic mean diameter (D - in inches) and average total tree height (H - in feet) by TBA (target basal area in square feet per acre) treatment from the Monticello thinning and pruning study. No measurements were conducted prior to age 27 for the Control TBA, and no height measurements were conducted at age 37 for any density. The Control TBA had an average basal area of 137 square feet per acre across all

Age	TBA										
	30		50		70		90		Control		
	D	H	D	H	D	H	D	H	D	H	
12	6.0	36.4	6.9	37.2	6.6	36.6	6.6	3	5	.	9
15	9.1	43.6	8.7	44.2	8.0	43.7	7.8	4	3	.	3
16	9.7	46.3	9.4	48.1	8.6	46.8	8.2	4	6	.	0
19	11.7	51.8	11.1	51.5	9.9	51.2	9.4	4	9	.	0
24	14.4	61.2	13.5	64.3	12.0	62.8	11.1	61.0	.	.	.
27	16.2	67.2	15.0	69.4	13.4	68.7	12.3	67.1	9.9	58.2	.
30	18.4	73.5	16.8	75.5	14.9	74.5	13.7	74.2	10.8	65.7	.
35	21.1	77.5	19.1	80.3	16.9	79.8	15.4	79.7	11.6	72.8	.
37	21.8	.	19.7	.	17.4	.	16.0	.	12.6	.	.
40	22.7	84.7	20.6	86.6	18.3	86.0	16.7	86.5	12.8	78.6	.

DATA

We used data collected during ten measurements on 40 permanent plots (table 1) established in 1970 by the Southern Forest Experiment Station in a typical 12-year-old loblolly pine (*Pinus taeda* L.) plantation in southeast Arkansas (Burton 1981). This is the second oldest active thinning and pruning study in loblolly pine stands. What makes these data particularly suitable for this research is the wide range of density. Plots were initially thinned at age 12 to 40, 60, 80 and 100 feet²/ac of basal area. After the second inventory at age 15, basal areas were reduced to 30, 50, 70 and 90 feet²/ac (TBA) and maintained at these levels by subsequent thinnings at ages 24, 27, 30, 35, and 40. The density variation was further enhanced by three severe ice storms. At the age of 27 five control plots (without thinning) were established on the adjacent untreated portion of the plantation.

DOES DENSITY HELP TO PREDICT HEIGHT WHEN DIAMETER IS KNOWN?

Before designing a model to predict height using diameter and density, we would like to make sure that the density effect is significant. Two methods were used for this purpose. First, we fitted the traditional allometric model relating height and diameter (equation (1)) to five groups of plots differing in density. The equation was linearized by log-transforming the variables. We found that predicted average heights of the stands with the same diameter (average quadratic mean diameter across all treatments and ages) increased with stand density level (table 2). The height difference between the extreme levels of density is 21 feet. Parameter *b* also showed an increasing trend in managed stands. Its pooled value is 0.7374, which is slightly greater than 2/3, probably because of the unaccounted effects of density.

The second method is to test several models including density as a predictor along with diameter (table 3). In all

tested models the parameter of density was significantly different from 0, which indicates that, regardless of equation form, density does help to predict height when diameter is known. The inclusion of density increased the proportion of explained variation in height from 0.88 to 0.93.

IS THERE AN OPTIMAL DENSITY FOR HEIGHT GROWTH?

Now that we are sure that density is an important predictor of height, we want to know whether there is a density at which height reaches its maximum for a given diameter. Discovering such an optimal density would be of help to foresters who are interested in maximizing height growth.

To solve this question, we used a model flexible enough to locate a possible culmination of height. To this end, our model includes two density terms, driven by density (*I*), and density squared (*I*²):

$$H = aD^b e^{cI + g(I^2)} \quad (4)$$

If *c* and *g* are both positive, there is no maximum height. If *c* and *g* are both negative, then our logic and analytical procedures are entirely incorrect because this would mean that height decreases when density increases. But, if *c* is positive and *g* is negative then there is a maximum height.

The results (*c* = 0.9412 *g* = -0.6180) show that there is an optimal density, that is the density at which height reaches a maximum. This conclusion contradicts our understanding of the involved processes. We believe that when diameters are equal, average height increases with increasing density. Should we trust the parameter values obtained from a limited data set or our reasoning? Fortunately, this contradiction can be resolved by calculating the value of the

Table 2—Comparison of the relationship between height and diameter by density treatment fitted to data from the Monticello thinning and pruning study. Where D = quadratic mean diameter in inch&s, H = average height in feet (height corresponding to D), Obs. = number of observations, Den = square feet of basal area per acre, SEE = standard error of the estimate, Hest = average height in feet estimation of a stand with a QMD of 13 inches (average size of D across all treatments and ages), SEE = standard error of the estimate, Adj. R^2 = is the adjusted R-squared value. Variables were log-transformed prior to fitting. The number after-e represents the single standard error

Equation	Obs.	Den	a	b	SEE	Adj. R^2	Hest
$H = aD^b$	90	32	9.7834 ± 0.4197	0.6855 ± 0.0160	0.0596	0.9538	57
$H = aD^b$	90	51	8.3875 ± 0.3441	0.7730 ± 0.0157	0.0533	0.9646	61
$H = aD^b$	90	69	7.8677 ± 0.2680	0.8259 ± 0.0136	0.0436	0.9764	65
$H = aD^b$	90	85	6.5471 ± 0.2638	0.9194 ± 0.0165	0.0486	0.9720	69
$H = aD^b$	19	137	9.3161 ± 2.8621	0.8263 ± 0.1105	0.0598	0.7532	78

optimal density, l' , which can be obtained from the following equation:

$$dH/dl = H(c+2gl) = 0 \quad (5)$$

Hence $l' = -c/2g = 0.7615$. This value is beyond the data range: the actual maximum density of the data is 0.7017. This means that the discovered optimum is illusory. The negative term indicates that the relationship between height and density is not linear but concave down.

IS THE RELATIONSHIP BETWEEN DENSITY AND HEIGHT ASYMPTOTIC?

The next question is: does the discovered concave form approach a finite maximum height or is the height increase unlimited? The asymptotic form means that when density is high further increase will produce practically no increase in height, which is not likely. We believe that the non-asymptotic form is more biologically reasonable. Besides this somewhat intuitive reasoning, we tested both asymptotic and non-asymptotic log-transformed models to estimate height using diameter and density as predictors. As it turned out, the non-asymptotic models are slightly more precise. To make sure that this result is not an artifact of a specific

equation form, we tested models of each form (table 3). For practical use we recommend the most precise model, the last in table 3.

CONCLUSIONS

Diameter and height provide us information about stem taper and ultimately tree volume. Often height is estimated using the easier obtained diameter. However, prediction of height using only diameter does not account for differences in stem taper associated with changes in density for stands of the same diameter. Density helps to explain variation in height and therefore needs to be included into the height-diameter relationship. The relationship between height of trees with the same diameter and density is concave down. Yet, it is not bounded by an asymptote. The model we recommend (table 3) satisfies all the considered requirements. It is also the most precise.

Still, we are not totally happy with our results. We expected that the introduction of density as a predictor would bring the value of parameter b closer to its theoretical value of 0.67. We failed in this respect: the excess of parameter b over 0.67 increased from 0.07 to 0.15 (table 3). Further studies need to be conducted to develop a density module that is both efficient in explaining variation in height and provides b with a value close to that predicted theoretically.

Table 3-Comparison of the relationships between height, diameter, and density fitted to 379 obs. from the Monticello thinning and pruning study. D = quadratic mean diameter in inches, H = average height in feet (height corresponding to D), SDI = Reineke's stand density index, SDI0 = minimum value of SDI (onset of competition) equal to 34.03, SEE = standard error of the estimate, Adj. R² = is the adjusted R-squared value. Variables were log-transformed prior to fitting. The number after ± represents the single standard error

Equation	a	b	c	SEE	Adj. R ²
Normal height-diameter relationship					
H = aD ^b	9.4734 ± 0.3443	0.7374 ±0.0142		0.0981	0.8763
Height-diameter relationship with an asymptotic density module					
H = aD ^{b*} (2 - e ^{-cSDI/SDI0}) ^k	6.4200 0.2567	0.8196 ±0.0121	0.0723 ±0.0074	0.0756	0.9266
Height-diameter relationship with a non-asymptotic density module					
H = aD ^{b*} (1+SDI/SDI0) ^c	5.8751 ±0.2392	0.8210 ±0.0120	0.1945 ±0.0118	0.0750	0.9278
H = aD ^{b*} (SDI/SDI0) ^c	6.5875 ±0.2353	0.8223 ±0.0120	0.1422 ±0.0086	0.0749	0.9280

REFERENCES

- Burton, J.D.** 1981. Thinning and pruning influence glaze damage in a young loblolly pine plantation. Research Note SO-264. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station.
- Greenhill, G.** 1881. Determination of the greatest height consistent with stability that a vertical pole or mast can be made, and of the greatest height to which a tree of given proportions can grow. Proc. Cambridge Philosophical Society. 4: 65-73.
- King, D.A., and O.L. Loucks.** 1978. The theory of tree bole and branch form. Radiation and Environmental Biophysics. 15: 141-165.
- MacKinney, A.L., F.X. Schumacher, and L.E. Chaiken.** 1937. Construction of yield tables for nonnormal loblolly pine stands. Journal of Agriculture Research. 54: 531-545.
- McMahon, T.A.** 1973. Size and shape in biology. Science. 179: 1201-1204.
- McMahon, T.A. and J.T. Bonner.** 1983. On size and life. New York: Scientific American Books.
- Norberg, R.A.** 1988. Theory of growth geometry of plants and self-thinning of plant populations: geometric similarity, elastic similarity, and different growth modes of plants. American Naturalist. 131: 220-256.
- O'Brien, S.T., S.P. Hubbell, P. Spiro, R. Condit, R.B. Foster.** 1995. Diameter, height, crown, and age relationships in eight neotropical tree species. Ecology. 76: 1926-1939.
- Reineke, L.H.** 1933. Perfecting a stand-density index for even-aged forests. Journal of Agricultural Resources. 46: 627-638.
- Rich, P.M.** 1986. Mechanical architecture of arborescent rain forest palms. Principes. 30:117-131.
- Rich, P.M., K. Helenurm, D. Kearns, S.R. Morse, M.W. Palmer, L. Short.** 1986. Height and stem diameter relationships for dicotyledonous trees and arborescent palms of Costa Rican tropical wet forest. Journal of the Torrey Botanical Club. 113: 241-246.
- Schniewind, A.P.** 1962. Horizontal specific gravity variation in tree stems in relation to their support function. Forest Science. 8: 111-118.

INTERIM TAPER AND CUBIC-FOOT VOLUME EQUATIONS FOR YOUNG LONGLEAF PINE PLANTATIONS IN SOUTHWEST GEORGIA

John R. Brooks, Stacey Martin, Jeff Jordan, and Chris Sewell¹

Abstract- Outside bark diameter measurements were taken at 0, 0.5, 2.0, 4.5, 6.0, 16.6 and 4 foot height intervals above 6 foot to a 2 inch dob top diameter on 42 longleaf pine trees selected from intensively managed longleaf pine (*Pinus palustris Mill.*) plantations in Dougherty and Worth Counties in southwest Georgia. Trees were sampled from unthinned, cutover stands in their 11th and 14th growing season that are currently part of an existing growth and yield study. Sample trees ranged from 2 to 7 inches in diameter and from 18 to 40 feet in total height. Parameters for a segmented polynomial taper and compatible cubic foot volume equation were simultaneously estimated using a seemingly unrelated nonlinear fitting procedure to volumes based on a generalized Newton formula and an overlapping bolt methodology. Resultant taper and volume functions were compared to published equations for longleaf plantations in the West Gulf physiographic region.

INTRODUCTION

Over the past decade there has been an increased interest in planting longleaf pine (*Pinus palustris Mill.*) in southwest Georgia. This interest is based on a historical, as well as an emotional relationship with this species, the existence of cost sharing programs, and the ability to consistently establish well stocked, uniform plantings that generally do not exhibit a "grass-stage". The ability to establish these types of plantings is based on major advancements in seedling care, planting techniques, more intensive site preparation methods and inclusion of post planting herbaceous weed control.

Little is known regarding the growth and yield of longleaf plantations in the Southeast, especially for these more intensively managed plantations. Most of the published mensurational information on planted longleaf stands has been for cutover sites in the West Gulf physiographic region. Compatible taper and volume functions have been published for outside bark diameters (Baldwin and Polmer 1981) and inside bark diameters (Thomas and others 1995) for plantations in central Louisiana and east Texas. A total and merchantable cubic foot volume equation has also been developed from plantations in this same region by Baldwin and Saucier (1983). Whether these equations accurately model the taper and volume of trees in southwest Georgia has never been examined.

The purpose of this project was to develop compatible taper and cubic foot volume functions as part of a growth and yield study for unthinned longleaf pine plantations on cutover sites in southwest Georgia and to compare the resulting equations with those that have been developed for longleaf pine plantations in the West Gulf.

METHODS

Sample trees were selected during the summer of 2000 from three unthinned plantations in Dougherty and Worth Counties, Georgia that are part of an existing growth and yield study. Plantations were established on cutover stands that received mechanical as well as chemical site preparation. Plantations ranged in age from 12 to 14 years and were established on sandy loam soils using bare root seedlings. A description of these plantations is presented in table 1.

Approximately 15 sample trees were selected from the interior of each plantation from the area buffering existing permanent growth and yield plots. An attempt was made to stratify the sample by diameter class without leaving holes in the existing stand. Sample tree distribution by height and diameter class is displayed in table 2. Trees possessing multiple stems, broken tops, obvious cankers or crooked boles were not included in the sample. Each sample tree was felled at ground level and total tree height recorded to the nearest 0.1 foot. One inch sample disks were removed from the base, 0.5 foot, 2.0 feet, 4.5 feet, 6.0 feet and repeatedly along the stem at 4 foot intervals until reaching a 2 inch dob top diameter. An additional disk was also removed at 16.6 feet to represent Girard form class height. Diameter outside bark to the nearest 0.01 inch was measured for each disk using a diameter tape. The data set included 456 outside bark measurements on 42 sample trees. Cubic foot volume outside bark was calculated for each bolt utilizing an overlapping bolt method (Bailey, 1995) and a generalized Newton formula described by Wiant and others (1992).

¹Biometrician, F&W Forestry Services, Inc., Albany, GA; Research Coordinator, University of Georgia, Athens, GA; Inventory Supervisor; Appraiser, F&W Forestry Services, Inc., respectively.

DATA ANALYSIS

The Max and Burkhart (1976) segmented polynomial taper function was selected as the first candidate taper model, which has the form:

$$\frac{d^2}{D^2} = \beta_1(Z_u - 1) + \beta_2(Z_u^2 - 1) + \beta_3(\alpha_1 - Z_u)^2 I_1 + \beta_4(\alpha_2 - Z_u)^2 I_2 \quad (1)$$

Where d is diameter outside bark (in.) at some given height h (feet), D is diameter outside bark (in.) at breast height, Z_u is the ratio of the upper bolt height to total height, α_1 and α_2 represent the joint points estimated during the fitting procedure, and the β 's are model parameters. The I_i 's are indicator variables and are defined as:

$$I_i = \begin{cases} 1, & \text{if } Z_u \leq \alpha_i \\ 0, & \text{if } Z_u > \alpha_i \end{cases}$$

Integration of the taper function over height results in the volume model:

$$V = kD^2H \left\{ \begin{array}{l} \frac{\beta_2}{3}(Z_u^3 - Z_l^3) + \frac{\beta_1}{2}(Z_u^2 - Z_l^2) - (\beta_1 + \beta_2)(Z_u - Z_l) \\ - \frac{\beta_3}{3}[(\alpha_1 - Z_u)^3 J_1 - (\alpha_1 - Z_l)^3 K_1] \\ - \frac{\beta_4}{3}[(\alpha_2 - Z_u)^3 J_2 - (\alpha_2 - Z_l)^3 K_2] \end{array} \right\} \quad (2)$$

Where V is volume outside bark in ft^3 , k is $\pi/576$, H is total height in feet, Z_u is the ratio of upper bolt height to H , Z_l is the ratio of lower bolt height to H , and the α 's and β 's are as previously defined. The J 's and K 's are indicator variables and are defined as:

$$J_i = \begin{cases} 1, & \text{if } Z_u \leq \alpha_i \\ 0, & \text{if } Z_u > \alpha_i \end{cases}$$

$$K_i = \begin{cases} 1, & \text{if } Z_l \leq \alpha_i \\ 0, & \text{if } Z_l > \alpha_i \end{cases}$$

Traditional development of compatible taper and volume functions involves parameter estimation for the taper function, which is then integrated to provide volume. This approach will minimize the error associated with stem diameter estimation but does not ensure minimal error in volume estimation. In an attempt to simultaneously minimize the error associated with taper and volume, Equation (1) and Equation (2) were simultaneously fit as

seemingly unrelated regressions (SUR) using SAS/ETS Model Procedure (SAS Institute Inc. 1993).

RESULTS

Statistics of fit and parameter estimates from the SUR fitting procedure for Equation (1) and Equation (2) are presented in tables 3 and 4, respectively.

Taper

The proposed taper function was compared to the equation published by Baldwin and Polmer (1981) for planted longleaf in the West Gulf region. Residuals for diameter outside bark at several relative height classes were compared using statistics similar to those applied by Parresol and others (1987). These included: (1) the Sum of squared relative residuals (SSRR); (2) Mean absolute residual (AbsD); (3) Bias (D); and (4) Standard deviation of residuals (Sd) (table 5). The Baldwin and Polmer (1981) model was superior only in relative height class 1 (relative height of 0.06 to 0.15) and to a lesser extent, in relative height class 2 (table 6). The superiority in this part of the stem is due to the fact that for the tree sizes evaluated in this study, relative height class 1 reflects the relative height at dbh and the Bennett and others (1978) model employed by Baldwin and Polmer (1981) constrains the model to equal dbh at 4.5 feet. A review of the residual plot for the Baldwin and Polmer model indicated an over estimation of stem diameter at the base of the tree, an under estimation of stem diameter between relative heights of 0.2 and 0.5, and an over estimation of stem diameter between relative heights of 0.5 and 0.9. Both models are constrained to a 0 inch top diameter at total tree height. No irregularities were detected from the residual plot for the proposed model.

Volume

The proposed compatible cubic foot volume function was compared to the total cubic foot volume estimates from the Baldwin and Polmer (1981) and Baldwin and Saucier (1983) models using their published parameter estimates. In terms of total stem cubic foot volume (ob), the Baldwin and Polmer equation was superior to the Baldwin and Saucier equation and the proposed equation was superior to the Baldwin and Polmer equation. The same statistics used to evaluate the differences in stem diameter

Table i-Description of sampled longleaf pine plantations

Plantation	Planting Spacing	Age	T P A	BA/AC	QMD _(in)	DHT* (ft)
1	6*8	14	516	66.4	4.9	39.0
2	6*8	12	798	87.7	4.5	35.2
3	6*8	12	695	92.8	4.9	33.7

* Where DHT equals the average total height of dominant and codominant trees

Table 2-Distribution of felled longleaf pine sample trees by diameter and total height class

Dbh (in.)	Total					
	20	25	30	35	40	
2	5	3				8
3		4	1			5
4		2	5	5		12
5			1	4	2	7
6				5	1	6
7				1	3	4
Total	5	9	7	15	6	42

Table 3-Nonlinear SUR summary of residual errors

Equation	Model	DF Error	SSE	MSE	R-Square	DF
						Adj R-Square
1	3	465	5.2521	0.0113	0.9579	0.9577
2	3	465	0.4426	0.000952	0.9684	0.9683

Table 4-Nonlinear SUR parameter estimates

Parameter	Estimate	Std Err	t value	P> t
B1	-3.0544	0.2902	-10.53	0.0001
B2	1.349745	0.1727	7.84	0.0001
B3	-1.36556	0.1662	-8.21	0.0001
B4	154.0197	22.8409	6.74	0.0001
A1	0.606008	0.0504	12.02	0.0001
A2	0.057371	0.00416	13.78	0.0001

Table 6-Statistics of fit for 10 relative height classes based on planted longleaf pine taper data

Model*	Statistic	Relative Height Class									
		0	1	2	3	4	5	6	7	8	9
1	SSRR	0.608	0.047	0.090	0.216	0.320	0.365	0.949	0.697	0.695	0.007
	AbsD	0.060	0.017	0.027	0.067	0.069	0.068	0.105	0.140	0.155	0.086
	D	-0.048	0.006	0.022	0.066	0.052	-0.007	-0.081	-0.132	-0.147	0.086
	Sd	0.339	0.101	0.122	0.168	0.231	0.287	0.330	0.228	0.207	
2	SSRR	0.313	0.084	0.071	0.058	0.131	0.230	0.518	0.257	0.234	0.002
	AbsD	0.043	0.026	0.031	0.031	0.043	0.050	0.080	0.072	0.085	0.039
	D	0.006	0.010	0.007	0.011	0.015	0.003	0.016	0.054	0.054	0.039
	Sd	0.249	0.136	0.128	0.156	0.188	0.236	0.275	0.175	0.177	

* (1) Baldwin & Polmer (2) Brooks and others

Table 5—Statistics used to evaluate predicted diameters (ob) and total cubic foot volume (ob)

Sum of Squared Relative Residuals (SSRR)

$$\sum \left(\frac{y_i - \hat{y}_i}{y_i} \right)^2$$

Mean Absolute Residual (AbsD)

$$\sum ABS(y_i - \hat{y}_i)$$

Bias (D) ⁿ

$$\sum (y_i - \hat{y}_i)$$

Standard ⁿ Deviation of Residuals (Sd)

$$\left[\frac{\sum (y_i - \hat{y}_i)^2 - \frac{(\sum (y_i - \hat{y}_i))^2}{n}}{n-1} \right]^{0.5}$$

Where: y_i represents either the observed diameter or volume (ob) and \hat{y}_i represents either the predicted diameter or volume (ob).

estimates were applied to total cubic foot volume differences (table 7). The average residual (D) for the proposed model was 85 percent smaller than that for the Baldwin and Polmer equation. Differences between the standard deviation of the residuals were minute. A review of the residual plots indicated that the Baldwin and Saucier equation underestimated volume for all trees greater than 4 inches dbh. This bias increased directly with dbh with residuals ranging from -0.4 to 0.6 cubic foot. The Baldwin and Polmer equation underestimated volume for 78 percent of the trees with residuals ranging from -0.5 to 0.3 cubic foot. The proposed model was biased for trees in the 2 and 3 inch diameter class, however, this bias was small (< 0.1 cubic foot). Residuals ranged from -0.5 to 0.2 cubic foot.

Table 7-Statistics to evaluate predicted total cubic foot volume for 42 longleaf pine trees

Statistics	Baldwin & Saucier (1983)	Baldwin & Polmer (1981)	Brooks and others
SSRR	0.46073	0.47951	0.36356
AbsD	0.12583	0.11511	0.09566
D	0.07009	0.06385	0.00984
Sd	0.16215	0.13627	0.13479

CONCLUSIONS

The objective of the study was to compare existing taper and cubic foot volume equations for planted longleaf pine in the West Gulf to an equation fit to 42 sample trees from plantations in southwest Georgia. It is not surprising that the proposed model had the smallest residuals since it was fit to the test data. How the existing equations predicted taper and/or volume compared to the proposed model was of primary interest. The Baldwin and Saucier (1983) volume equation possessed residual trends that would make it an unlikely candidate for use in these young plantations. The Baldwin and Polmer (1981) equations provided reasonable estimates of volume but was limited in its ability to accurately predict stem diameter. Further analysis is planned to fit the Bennett and others (1978) model and other nonlinear segmented polynomial models to this data set in an attempt to further reduce volume and taper estimation errors.

REFERENCES

- Bailey, R.L.** 1995. Upper stem volumes from stem analysis data: an overlapping bolts method. *Canadian Journal of Forest Research*. 25: 170-173.
- Baldwin, V.C. Jr.; Polmer, B.H.; 1981. Taper functions for unthinned longleaf pine plantations on cutover West Gulf sites. In: Barnett, J.P., ed. *Proceedings of the First biennial southern silvicultural research conference*; 1980 November 6-7, Atlanta, GA. Gen. Tech. Rep. SO-34. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station: 156-163.
- Baldwin, V.C.; Jr. Saucier, J.R.** 1983. Above ground weight and volume of unthinned, planted longleaf pine on west gulf forest sites. Res. Pap. SO-191. New Orleans: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station, 8p.
- Bennett, F.A.; Lloyd, F.T.; Swindel, B.F.; E.W. Whitehorne.** 1978. Yields of veneer and associated products from unthinned, old-field plantations of slash pine in the North Florida and South Georgia flatwoods. Res. Pap. SE-176. Ashville: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 80 p.
- Max, T.A.; Burkhardt, H.E.** 1976. Segmented polynomial regression applied to taper equations. *Forest Science*. 22(3): 283-289.
- Parresol, B.A.; Hotvedt, J.E.; Q.V. Cao.** 1987. A volume and taper prediction system for bald cypress. *Canadian Journal of Forest Research*. 17: 250-259.
- SAS Institute.** 1993. *SAS/ETS User's Guide, Version 6, Second Edition*. SAS Institute, Inc. Cary, NC.
- Wiant, H.V., Jr.; Wood, G.B.; G.M. Furnival.** 1992. Estimated log volume using the centroid position. *Forest Science*. 38(1): 187-191.

ECONOMIC EVALUATION OF RESTORING THE SHORTLEAF PINE-BLUESTEM GRASS ECOSYSTEM ON THE OUACHITA NATIONAL FOREST

**Michael M. Huebschmann, Daniel S. Tilley, Thomas B. Lynch,
David K. Lewis, and James M. Guldin**

POSTER SUMMARY

The USDA Forest Service is restoring pre-European settlement forest conditions on about 10 percent (155,000 acres) of the Ouachita National Forest in western Arkansas. These conditions are characterized by large, scattered shortleaf pine and hardwoods maintained on 120-year rotations, with bluestem grass and associated herbaceous vegetation in the understory. These are expected to re-establish a broad habitat type missing from the landscape, one suited to supporting the recovery of the endangered red-cockaded woodpecker (USDA Forest Service 1996a, 1996b). This study was designed to forecast the amount of timber harvest volume and revenue the Ouachita National Forest may forego by adopting the shortleaf pine-bluestem grass (or pine-bluestem) management system in lieu of traditional, even-aged management.

Published growth and yield models were used to predict volumes available for harvest during a 100-year-long period in the pine-bluestem restoration area under both scenarios. Table 1 contrasts the rotation lengths and other significant characteristics of the two management scenarios. A model for predicting timber sale value was developed, and then applied to the predicted volumes in order to compare the respective revenue streams (Huebschmann 2000, Huebschmann and others 2000).

During the 100-year simulation period, the pine-bluestem scenario produces 26 percent less pine sawlog volume in the restoration area. Timber sale revenue from the area also declines by 51 percent in present-value terms. Because the pine-bluestem area covers only a small portion of the Ouachita National Forest, however, this decline translates into a Forest-wide revenue reduction of between 2 and 5 percent.

As a result of restoring the pine-bluestem ecosystem, the Forest Service expects to provide habitat capable of eventually supporting 400 breeding pairs of red-cockaded woodpeckers. By foregoing the revenue that could be

Table 1-Characteristics of the traditional even-aged and pine-bluestem management scenarios compared in this study

Characteristic	Management scenario	
	Traditional	Pine-Bluestem
Rotation length (yr)	80	120
Stand BA (ft ² /ac)	60 ≤ pine ≤ 90 10 ≤ hdwd ≤ 15	60 ≤ pine ≤ 80 10 ≤ hdwd ≤ 15
Post-harvest residual overstory BA (ft ² /ac)	20 pine 10 hdwd	40 pine 10 hdwd
Burning interval (yr)	4	3

generated under even-aged management, the Forest Service places an implicit value of about \$1,700 per year (in present-value terms) on each woodpecker.

The pine-bluestem management regime requires successful silvicultural treatments and growth and yield forecasts outside the range of general experience in the region. Thus, additional monitoring will be needed to validate the conditions and estimates used in this analysis.

REFERENCES

- Huebschmann, M.M. 2000. Restoring the shortleaf pine-bluestem grass ecosystem on the Ouachita National Forest: An economic evaluation. Ph.D. dissertation. Oklahoma State Univ. Stillwater, OK. 141 p.
- Huebschmann, M.M.; T.B. Lynch; D.K. Lewis. 2000. Restoring the shortleaf pine-bluestem grass ecosystem on the Ouachita National Forest: An economic evaluation. Final project report to the USDA Forest Service - Southern Research Station. 51 p.

*Natural Resource Economist, Boise Cascade Corporation, Boise, ID 83728; Professor, Dept. of Agricultural Economics, and Associate Professors, Dept. of Forestry, Oklahoma State University, Stillwater, OK 74078; Project Leader/Research Forest Ecologist, USDA Forest Service, Southern Research Station, Hot Springs, AR 71902, respectively.

Citation for proceedings: Outcalt, Kenneth W., ed. 2002. Proceedings of the eleventh biennial southern silvicultural research conference. Gen. Tech. Rep. SRS-48. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station, 622 p.

USDA Forest Service. 1996a. Renewal of the shortleaf pine/bluestem grass ecosystem and recovery of the red-cockaded woodpecker: Final environmental impact statement for an amendment to the land and resource management plan, Ouachita National Forest. 94 p.

USDA Forest Service. 1996b. Record of decision: Final environmental impact statement. Amendment 22: Renewal of the shortleaf pine/bluestem grass ecosystem and recovery of the red-cockaded woodpecker. Ouachita National Forest land and resource management plan, Scott and Polk counties, Arkansas. USDA For. Serv. - Southern Region. 13 p.

ESTIMATING THE PROBABILITY OF ACHIEVING SHORTLEAF PINE REGENERATION AT VARIABLE SPECIFIED LEVELS

Thomas B. Lynch, Jean Nkouka, Michael M. Huebschmann, and
James M. Guldin¹

POSTER SUMMARY

A model was developed that can be used to estimate the probability of achieving regeneration at a variety of specified stem density levels. The model was fitted to shortleaf pine (*Pinus echinata* Mill.) regeneration data, and can be used to estimate the probability of achieving desired levels of regeneration between 300 and 700 stems per acre 9-10 years after thinning to a specified level of overstory basal area per acre. The level of regeneration to be achieved was used to modify a logistic model to estimate probability of obtaining regeneration at the desired level. Variables used in the model to predict probability of achieving the desired regeneration level were site index for shortleaf pine (base age 50), overstory basal area, age at time of thinning and a dummy (0 or 1) variable representing year of plot establishment.

The data consisted of measurements made on 5-milacre plots located within 182 circular permanent plots, 0.2-acre in size. These plots were established in natural even-aged pure shortleaf pine stands thinned to predetermined residual overstory basal area levels in one of four density categories: 30, 60, 90 or 120 square feet per acre. Plots were established in four age categories (20, 40, 60 and 80 years) and in four site index classes (50, 60, 70, 80 feet at 50 years). At the time of plot establishment hardwoods were treated with herbicide by tree injection or girdling. Each 0.2-acre plot was surrounded by a 33-foot buffer strip, which received the same thinning and herbicide treatment. Shortleaf pine regeneration stems were tallied on two 5-milacre plots located due north and south midway between the plot center and the boundary of each 0.2-acre shortleaf pine overstory plot. The regeneration sample occurred 9-10 years after plot establishment.

Larson and others (1997) used a logistic model to predict the probability of achieving specified levels of oak regeneration. Target density levels could not be varied within a particular equation for the models developed by Larson and others (1997). However, they fitted several equations independently to different target density

levels. For the current study, varying levels of shortleaf pine regeneration density levels were obtained by using the natural logarithm of the specified density level as an independent variable in a logistic model. To obtain a satisfactory fit for the model it was necessary to form an independent variable by multiplying the natural logarithm of density level by the square of site index.

Parameter estimation for this model with common logistic regression software is problematic because the level of regeneration success is variable. Therefore, a maximum likelihood procedure was developed and used to estimate parameters in this model. Three levels of regeneration success were specified for the purpose of parameter estimation: 300, 500 and 700 stems per acre. These levels were used to define regeneration categories such that each plot could be assigned to one of these categories, for example, more than 300 but less than 500 stems per acre. A multinomial distribution based on these categories was used to develop a likelihood function. The modified logistic function was used to represent probabilities in the likelihood function. The LOGDEN function in SHAZAM (White 1993) econometric software was used to maximize the likelihood function with respect to equation parameters so that maximum likelihood estimates were obtained.

Parameter estimates from the model indicate that adequate shortleaf pine regeneration is less likely on good sites than on poor sites as measured by shortleaf pine site index. Adequate shortleaf pine regeneration becomes less likely as overstory basal area per acre increases, and is less likely at young overstory stand ages. The parameter estimate associated with year of thinning and herbicide treatment indicates that the probability of obtaining adequate regeneration can vary substantially due to the year in which thinning and herbicide treatment occurred. This could be due to variability in shortleaf pine seed crops and/or conditions for seedling establishment and survival. The resulting model for prediction of probability for regeneration success should be applicable for levels of shortleaf pine regeneration between 300 and 700 stems per acre.

¹Associate Professor and former Graduate Research Assistant, Department of Forestry, Oklahoma State University, Stillwater, OK 74078-6013; Natural Resource Economist, Boise Cascade Corporation, P.O. Box 50, Boise, ID 83728-0001; Project Leader/Research Forest Ecologist, USDA Forest Service Southern Research Station, Monticello, AR 71656-3516, respectively.

REFERENCES

- Larson, D.R.; Metzger, M.A.; Johnson, P.S.** 1997. Oak regeneration and overstory density in the Missouri Ozarks. *Canadian Journal of Forest Research*. 27: 869-875.
- White, K.J.** 1993. SHAZAM user's reference manual version 7.0 . second printing. New York: McGraw-Hill. 482 p.