EMPIRICALLY DERIVED OPTIMAL GROWTH EQUATIONS FOR HARDWOODS AND SOFTWOODS IN ARKANSAS

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Abstract—Accurate growth projections are critical to reliable forest models, and ecologically based simulators can improve silvicultural predictions because of their sensitivity to change and their capacity to produce long-term forecasts. Potential relative increment (PRI) optimal diameter growth equations for loblolly pine, shortleaf pine, sweetgum, and white oak were fit to data from the Arkansas portion of the Eastwide Forest Inventory Data Base (EFIDB). Large sample sizes are necessary for successful application of the PRI methodology, and in aggregate almost 29,000 trees were used to develop these models. In the final model versions, only a handful (< 30 per species) of the fastest growing trees given their species, size, and growing conditions were retained from the Arkansas EFIDB. Shortleaf pine, sweetgum, and white oak all generated skewed model curves, while loblolly pine produced a monotonically declining curve. Comparison of these optimal increment models across tree size indicated that loblolly pine had higher potential than the other species until ~ 10 cm in diameter at breast height (d.b.h.), after which sweetgum and white oak overtook it at intermediate sizes. However, loblolly pine optimal performance decreased at a lesser rate than any of the other species, so that by 60 cm d.b.h. it once again had the greatest potential. The other taxa outperformed shortleaf pine throughout most of the diameter range considered, while sweetgum proved intermediate between shortleaf and white oak. These optimal diameter functions are a valuable first step in the development of forest simulators.

INTRODUCTION

Foresters have increasingly used models to predict long-term stand dynamics. Empirically based growth and yield models, e.g., Lynch and others (1999), Wykoff and others (1982), are popular because they are relatively easy to parameterize. However, the rigid nature of these designs, their finite analysis options, and their lack of ecological mechanism have limited their applicability beyond short-term growth-and-yield prediction. Ecological process models are becoming more widespread, e.g., Botkin and others (1972), Bragg (1999), Pacala and others (1993), in part because of their greater complexity and flexibility. However, these models often lack an empirical foundation and sometimes rely upon questionable assumptions. Blending the positive features of empirical and ecological models should improve the reliability of long-term forecasts of forest dynamics.

Most forest simulators include some kind of individual tree growth model. A fundamental goal of this increment model is to predict realized growth accurately, and there are at least two different ways to approach this problem. Most empirical models use a fitted statistical response where increment is either added or subtracted from a standard level, depending on how favorable conditions are for growth, e.g., Wykoff and others (1982). While commonly applied, this design limits the growth function to a specified set of modifiers, thus restricting its adaptability. The other primary approach employs a potential increment function that is rescaled downward based on departures from optimal growth conditions, e.g., Botkin and others (1972), Bragg (2001). Thus, one predicts realized growth from its departure from optimal growth using appropriate modifier function(s). In principle, this strategy has greater flexibility for ecological modeling because environmental response functions can be more sophisticated and mechanistic. However, one of the biggest challenges to optimal growth modeling lies in the development of an acceptable response curve.

Researchers have developed and evaluated numerous designs of potential growth equations (Botkin and others 1972, Moore 1989, Pacala and others 1993, Zeide 1993). Most recently, Bragg (2001) developed the Potential Relative Increment (PRI) methodology to fit inventory data to an ecologically robust function, thus linking desirable theoretical and statistical properties. This paper presents optimal PRI increment models for loblolly pine (Pinus taeda L.), shortleaf pine (P. echinata Mill.), sweetgum (Liquidambar styraciflua L.), and white oak (Quercus alba L.) in Arkansas using data from the Eastwide Forest Inventory Data Base (EFIDB) (Hansen and others 1992).

METHODS

The details of the PRI method are beyond the scope of this paper (see Bragg 2001). Briefly, all records of the species of interest with positive growth were selected for processing. After identifying this initial group, those individuals growing at the greatest rate for each 2-cm diameter at breast height (d.b.h.) class (one tree per size class) were segregated into a maximal actual increment

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Citation for proceedings: Outcalt, Kenneth W., ed. 2002. Proceedings of the eleventh biennial southern silvicultural research conference. Gen. Tech. Rep. SRS–48. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 622 p.

Table 1—Statistics on the four species extracted from the Arkansas portion of the Eastwide Forest Inventory Data Base

Species	Original sample size	Minimum d.b.h.	Maximum d.b.h.	Standard d.b.h. deviation
		---------- *Centimeters* ----------		
Loblolly pine	11,340	2.8	88.6	13.52
Shortleaf pine	7,587	2.8	70.1	10.57
Sweetgum	3,906	2.8	111.3	14.13
White oak	6,089	2.8	100.8	13.70

Table 2—Final model regression coefficients (b_1, b_2, and b_3), goodness-of-fit, and final sample sizes

Species	b_1	b_2	b_3	R^2	Loss[a]	Final n[b]
Loblolly pine	2.708480	-1.033813	0.993497	0.9975	0.00284	29
Shortleaf pine	1.171747	-.623995	.962244	.9953	.00278	25
Sweetgum	.439226	-.053773	.944559	.9942	.00109	16
White oak	.273683	.086414	.955673	.9960	.00028	12

[a] Loss = Σ(observed - predicted)^2.

[b] Final number of points used to fit the optimal potential relative increment curves and generate the R^2 values in this table.

pool. From these, a best subset was identified to fit the PRI growth function:

$$PRI = b_1 D_{MAX}^{b_2} b_3^{D_{MAX}} \quad (1)$$

where

D_{MAX} = the d.b.h. of the maximally performing individual (by size class) and
b_1 to b_3 = species-specific nonlinear ordinary least squares regression coefficients.
Optimal increment is the product of PRI and current d.b.h., while realized increment can be estimated by multiplying optimal increment with limiting environmental scalar(s) (Bragg 2001). In this final step, factors such as competition and site quality come into play.

The spatially extensive sample found in the EFIDB covers most of the possible variation in the environment. However, the odds of finding a Forest Inventory and Analysis plot with the perfect combination of site quality, stand density, and genetics to produce a truly optimal growth environment are negligible. Therefore, the PRI methodology is a conservative representation of potential diameter growth (Bragg 2001). The inference that optimal conditions can be approximated from inventory data requires a large sample of trees of the desired species from an extensive area. To ensure adequate representation, almost 29,000 individuals from the taxa of interest were selected from the Arkansas portion of the EFIDB (11,340 loblolly pines, 7,587 shortleaf pines, 3,906 sweetgums, and 6,089 white oaks) (table 1).

Only a small fraction (< 30 per species) of the records were retained for the final models (table 1). Loblolly pine provides an example of the iterative fitting process. Originally, over 11,000 records were considered usable, covering most of the range of possible size and increment with little apparent measurement error (fig. 1A). The exception is an outlier identified by the arrow in figures 1A and 1B. This tree apparently grew from 61.0 cm d.b.h. to 90.2 cm d.b.h. in 7.2 years (an average of 4.2 cm annually), a highly dubious rate given the size of the tree. Of the initial multitude of records, 42 loblolly were chosen, one for each respective diameter class (fig. 1B). Since the objective of the methodology was to identify an optimal growth curve, individuals within the d.b.h. class structure that did not maximize this function were removed (including the outlier). Thus, a final subset of 29 loblolly pines was retained for curve fitting (fig. 1C). This process was repeated for the other species until a suite of models was developed.

RESULTS AND DISCUSSION

Figure 2A illustrates that optimal increment performance is a distinct function of species and size. Translated into the more interpretable measure of potential annual d.b.h.

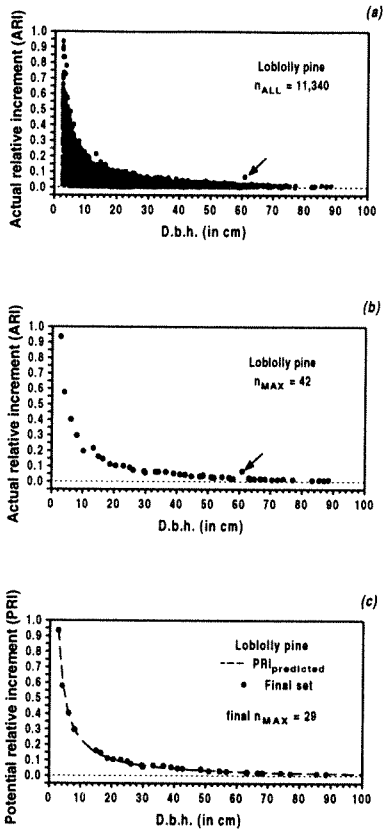

Figure 1—Graphical demonstration of the potential relative increment (PRI) methodology for loblolly pine in Arkansas. After the initial sample (n_{ALL}) of 11,340 loblolly pines was chosen (A), only the points (n_{MAX}) representing the highest growth performance within each 2-cm d.b.h. class remained after an initial filtering (B). Further discrimination resulted in a subset (final n_{MAX} = 29) used to develop the PRI model (C). Note the arrows in (A) and (B) that identify the outlier removed before final model fitting.

growth (fig. 2B), differences in performance become even more marked. Shortleaf pine, sweetgum, and white oak all produced skewed model curves with different local maxima and trajectories, while loblolly pine yielded a monotonically declining curve with a maximum at the smallest d.b.h. class. Thus, for the smaller diameters (< 12 cm), loblolly pine had the potential to outgrow any of the other species in this sample, especially shortleaf pine and sweetgum. However, between 12 and 30 cm d.b.h., both sweetgum and white oak were predicted to have higher potential performance than loblolly pine, with white oak continuing this trend to 60 cm. From 60 centimeters on, loblolly regained its dominance over the other species.

Shortleaf pine failed to approach the maximal performance of sweetgum and white oak until very large diameters, and never matched loblolly's potential. Sweetgum performed at an intermediate level until larger diameters were reached, upon which its optimal performance decreased noticeably. Note that these results are for predictions of potential increment, not those realized in the field: actual diameter growth will be a function of factors such as

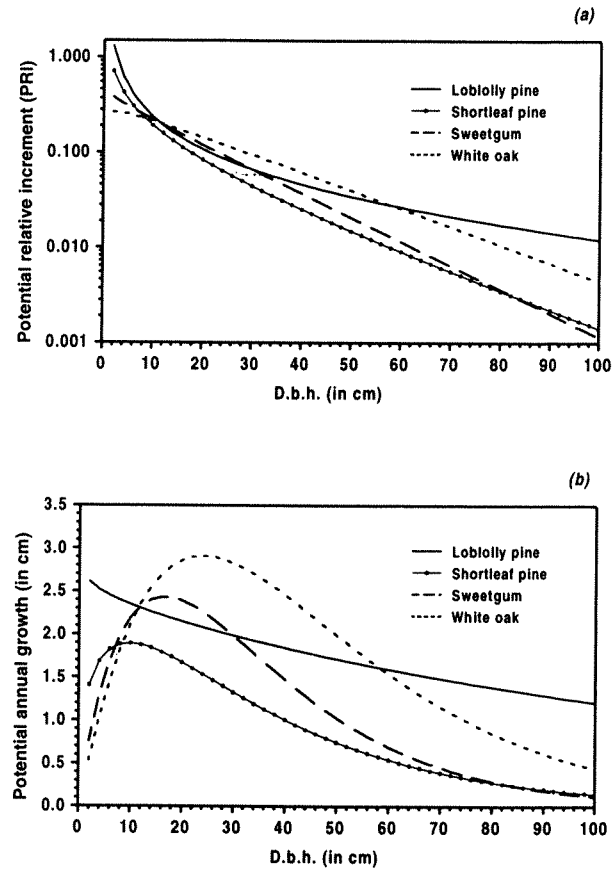

Figure 2—Annual potential relative increment (PRI) (A) and potential annual growth (B) for loblolly pine, shortleaf pine, sweetgum, and white oak. The differences in the PRI curves yield dramatic differences in optimal growth performance between species.

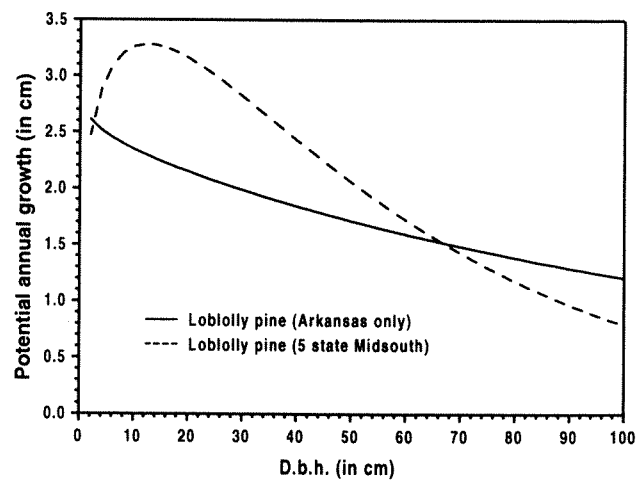

Figure 3—Comparison of the Arkansas potential relative increment model of loblolly pine (solid line) and the Midsouth model (Unpublished manuscript. D.C. Bragg, Research Forester, USFS Southern Research Station, P.O. Box 3516 UAM, Monticello, AR 71656) that included Arkansas (dashed line). The Midsouth model benefited from supplemented and extended size classes, thus resulting in a noticeably different prediction of potential tree performance.

localized edaphic, climatic, and competitive conditions, photosynthetic surface area, tree moisture status, genetic predisposition, or the presence of pathogens.

From these data, it appears loblolly pine has the potential to add the greatest diameter increment (2.6 cm annually) at the smallest size, while shortleaf pine peaks (~ 1.9 cm annually) at approximately 8 to 10 cm d.b.h., sweetgum reaches a maximum (~2.4 cm annually) at 15 to 18 cm d.b.h., and white oak crests (~2.9 cm annually) at approximately 25 cm d.b.h.. These results differ from a more extensive set of PRI curves fit to an inventory pool for the Midsouth (Arkansas, Louisiana, Missouri, Oklahoma, and Texas).[2] Using loblolly pine as an example, noticeable differences in potential increment are apparent at both small and large diameters (fig. 3).

Under the Midsouth model, a skewed model form replaces the monotonically declining model of the Arkansas-only data, with a new, higher maximal annual growth peak of > 3.2 cm now found at ~ 15 cm d.b.h. Optimal growth potential remains higher until loblolly pine reaches > 65 cm d.b.h., after which it drops below the Arkansas model. The data used for the Midsouth model changed the curve shape dramatically by adding points at small diameters that produced more optimistic optimal performance while simultaneously contributing new observations in the larger diameter classes. Pooling can increase confidence in results by supplementing and/or extending the range of sample data. In some cases, though, pooling may overestimate local growth potential if environmental and genetic conditions are significantly different from the more limited study area. Because of similarities in the environmental and loblolly genetic conditions in the Midsouth, the increase in potential optimal performance noted in figure 3 should not cause major problems when applied in Arkansas.

CONCLUSIONS

Optimal tree diameter growth performance is a function of both species and size. In this Arkansas sample, loblolly pine and white oak outperformed sweetgum and shortleaf pine. However, all species considered in this paper can potentially add 2 to 3 cm of diameter annually. The ability to differentiate species performance based on standardized growth functions should help the forest research community, especially if the inventory information is widely available.

Large public databases like the EFIDB can assist the development of silvicultural and mensurational applications. Their considerable spatial extent, rigorous sampling design, and broad range of species and size classes also favor their use in other fields, especially ecological modeling. The development of empirically derived optimal growth models provides the basis for forest simulators grounded in both theory and reality.

[2] Bragg, D.C. Unpublished manuscript.

ACKNOWLEDGMENTS

I thank the following people and institutions for their support of this work: Dave Roberts (Utah State University), Tom Crow (USDA Forest Service, North Central Research Station), and Jim Guldin (USDA Forest Service, Southern Research Station). Reviews from the following improved the quality of this manuscript: Jim Guldin, Boris Zeide (University of Arkansas at Monticello), Bernie Parresol (USDA Forest Service, Southern Research Station), and David Larsen (University of Missouri at Columbia).

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