A COMPATIBLE STEM TAPER-VOLUME-WEIGHT SYSTEM FOR INTENSIVELY MANAGED FAST GROWING LOBLOLLY PINE

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Abstract-Geometry-oriented methodology yielded a compatible taper-volume-weight system of models whose parameters were estimated using data from intensively managed loblolly pine (*Pinus taeda* L.) plantations in the lower coastal plain of Georgia. Data analysis showed that fertilization has significantly reduced taper (inside and outside bark) on the upper segment and augmented the stem merchantable volume there, which was modeled using an adjusted form factor. On the other hand, the unit-weights of fertilized trees were not significantly different from unfertilized trees. Finally, our analysis showed no significant impacts of complete vegetation control on taper, volume or weight characteristics.

INTRODUCTION

Geometry-oriented methodology constructs a theoretically sound and physically meaningful framework for taper prediction equation (Ormerod 1973, Forslund 1982, Newberry and others. 1989, Bailey 1994, Byrne and Reed 1986, Broad and Wake 1995, Parresol and Thomas 1996, Fang and Bailey 1999, Fang and others. 2000). The associated merchantable volume equation is mathematically compatible with the total volume equation, that is, it results from integration of the taper function (Demaerschalk 1972, Clutter 1980, Byrne and Reed 1986, McTague and Bailey 1987, Bailey 1994, Fang and Bailey 1999, Fang and others. 2000).

Recently, intensive management has been applied in the Southeastern U.S. (Jokela and Stearns-Smith 1993) to accelerate tree and stand growth and increase financial returns. Many of these silvicultural treatments have been shown to result in dramatic growth increases (Ford 1984, Gent and others. 1986, Allen and others. 1990, Stearns-Smith and others. 1992, Jokela and Stearns-Smith 1993, Borders and Bailey 1997). However, the effect of these treatments on individual tree stem taper, volume, and weight has not been fully studied. Borders and Bailey (1997) reported an extremely fast growth rate of loblolly pine (*Pinus taeda* L.) obtained from intensively managed stands in the Southeastern U.S. The thirteen-year growth of the most responsive stands yielded an average annual increment of 1.50 cm for quadratic mean diameter and 1.55 m for dominant height. Obviously, when such dramatic growth rate differences exist it is possible that stem taper and unit weight may be impacted as well.

STUDY MATERIALS

This investigation used data from the Consortium for Accelerated Pine Plantation Studies (CAPPS) initiated in 1987 and maintained by the Daniel B. Warnell School of Forest Resources, University of Georgia. The treatments employed were: l)Complete vegetation control throughout stand life-span using herbicide (H), 2)Annual fertilization (F), 3)Herbicide and Fertilization (HF), and 4)Check (C). In the winter of 1999, 192 trees with age 12, 10, and 6 years old were harvested from two study installations from the lower coastal plain of Georgia for wood quality research. Stem taper and weight measurements were made in field and disk analysis was done in the USDA Forest Service laboratory in Athens, GA. The impacts of cultural treatments and age on stem taper were investigated using the split-split plot design. The dependent variables employed are form quotients inside and outside bark at height proportion 0.25, 0.50, 0.60, 0.75, and 0.90, considering that the change of a specified quotient implies the change of stem form, which may be related to cultural treatments. Data analysis showed 1) no significant impacts from treatment H, 2) significant effects of treatment F are found only for quotients of 0.75 and 0.90, and 3) age is not a significant contributor.

MODEL STRUCTURE

Fang and others. (2000) proposed a system of compatible volume-taper models for traditionally managed loblolly pine and slash pine (*Pinus elliottii* Engelm) plantations, in which two inflection points (three segments) were employed. Screening the taper profile of stems in this study (figure 1), one inflection point seems adequate for our taper prediction equations. Followings are derived models for stem taper, volume, and weight.

Taper (Outside Bark)

The derivation of taper equation is similar to the method introduced by Fang and others. (2000) except 1) Newton's segment volume equation was employed in the derivation and 2) a boundary condition that dob = dbh where stem

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$$dob = dbhH'^{\frac{k-\beta_1}{2\beta_1}} \left(a^{\theta} (1-p)^{\frac{k-\beta_1^{1-\theta}\beta_2^{\theta}}{\beta_1^{1-\theta}\beta_2^{\theta}}} \right)^{\frac{1}{2}} \qquad (1)$$

height = breast height.

where *dob* is the stem diameter outside bark, *dbh* the diameter at breast height, $p = \frac{h}{H}$, h the stem height, H the total height, $H' = \frac{H}{H - 1.3716}$, $k = \frac{\pi}{8}$,

$a = (1 - p')^{k\frac{\beta_2 - \beta_1}{\beta_1 \beta_2}}$, p'the stem ration at the inflection point, β_1 and β_2 the coefficients, and q the dummy variable with value zero for stem ratio $p = p'$ and one for $p > p'$,

Taper (Inside Bark)
Taper inside bark *(dib)* is similar to stem taper outside bark, except an extra coefficient c because of the effect of tree bark at breast height:

$$dib = c \cdot dbhH'^{\frac{k-\gamma_1}{2\gamma_1}} \left(a'^{\theta} (1-p)^{\frac{k-\gamma_1^{1-\theta}\gamma_2^{\theta}}{\gamma_1^{1-\theta}\gamma_2^{\theta}}} \right)^{\frac{1}{2}} \qquad (2)$$

where $a' = (1 - p')^{k\frac{\gamma_2 - \gamma_1}{\gamma_1 \gamma_2}}$, and y, and y_2 the coefficients.

Volume (Outside and Inside bark)
Compatible stem volume equation can be readily obtained from integrating the taper function:

$$V_{sob} = \int_{h_0}^{h} \eta\, dob^2 dh \qquad (3)$$

where h_0 is stump stem height and η a coefficient. Using the defined relationship (Eq. (1)), integration of Eq. (3) results in:

$$V_{sob} = \xi dbh^2 HH'^{\frac{k-\beta_1}{\beta_1}} \left\{ \begin{array}{l} \beta_1 (1-p_0)^{\frac{k}{\beta_1}} \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k}{\beta_1}} \right]^{1-\theta} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k}{\beta_1}} \right]^{\theta} \\ + a\theta\beta_2 (1-p')^{\frac{k}{\beta_2}} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k}{\beta_2}} \right] \end{array} \right\} \qquad (4)$$

where V_{sob} is the stem volume outside bark and ξ a coefficient.

Likewise, using *dib* in the integration results in the prediction equation of stem volume inside bark

$$(V_{sib}): V_{sib} = \zeta dbh^2 HH'^{\frac{k-\gamma_1}{\gamma_1}} \left\{ \begin{array}{l} \gamma_1 (1-p_0)^{\frac{k}{\gamma_1}} \left[1 - \left(\frac{1-p}{1-p_0} \right)^{\frac{k}{\gamma_1}} \right]^{1-\theta} \left[1 - \left(\frac{1-p'}{1-p_0} \right)^{\frac{k}{\gamma_1}} \right]^{\theta} \\ + a'\theta\gamma_2 (1-p')^{\frac{k}{\gamma_2}} \left[1 - \left(\frac{1-p}{1-p'} \right)^{\frac{k}{\gamma_2}} \right] \end{array} \right\} \qquad (5)$$

where V_{sib} is the stem volume inside bark and ζ a coefficient.

Weight
Let

$$D = f(h) \qquad (6)$$

where D is the density of wood or bark and f some function then stem weight can be expressed as:

$$W = \int_{h_0}^{h} D\, dV = \int_{h_0}^{h} f(h)\, dV \quad or \quad \int_{p_0}^{p} f(p)\, dV \qquad (7)$$

where h_0 is the stump height and $p = \frac{h}{H}$.

Parresol and Thomas (1996) proposed a linear model for D:

$$D = u_0 + u_1 h + u_3 Age \qquad (8)$$

where u_1, u_2 and u_3 are coefficients.

Data analysis showed that age is a predictor of stem dry weight wood only. Similarly, we investigated the impacts of treatments H and F and found that both are not significant contributors.

The distribution of wood density along stem was screened on the individual tree base and we found that a quadratic equation form may better reflect the variation of wood density along stem:

$$DWD = d_0 + d_1 h + d_2 h^2 + d_3 Age \qquad (9)$$

$$GWBD = g_0 + g_1 h + g_2 h^2 \qquad (10)$$

$$GWD = w_0 + w_1 h + w_2 h^2 \qquad (11)$$

where *DWD, GWBD,* and *GWD* are density of dry wood, green wood and bark, and green wood, respectively, and d,, d,, d_2, d_3, g_0, g,, g_2, w,, w,, and w_2 some coefficients.

Using the functions of wood density and volume, the weight equations can be obtained upon integrating Eq. (24):

$$
W_{dw}=(d_0+d_1H+d_2H^2+d_3Age)V_{sib}-\zeta kdbh^2H^2H'^{\frac{k-\gamma_1}{\gamma_1}}
\left\{\begin{array}{l}
\gamma_1(1-p_0)^{\frac{k+\gamma_1}{\gamma_1}}\left(\begin{array}{l}
\frac{d_1k+2\gamma_1(d_1+d_2H)}{(k+\gamma_1)(k+2\gamma_1)}\left[1-\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta} \\
-\frac{d_2H(1+p_0)}{k+2\gamma_1}\left[1-\left(\frac{1+p'}{1+p_0}\right)\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1+p}{1+p_0}\right)\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta}
\end{array}\right) \\
+\alpha'\gamma_2\theta(1-p')^{\frac{k+\gamma_2}{\gamma_2}}\left(\frac{d_1k+2\gamma_2(d_1+d_2H)}{(k+\gamma_2)(k+2\gamma_2)}\left[1-\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]-\frac{d_2H(1+p')}{k+2\gamma_2}\left[1-\left(\frac{1+p}{1+p'}\right)\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]\right)
\end{array}\right\}
$$

$$
W_{gw}=(w_0+w_1H+w_2H^2)V_{sib}-\zeta kdbh^2H^2H'^{\frac{k-\gamma_1}{\gamma_1}}
\left\{\begin{array}{l}
\gamma_1(1-p_0)^{\frac{k+\gamma_1}{\gamma_1}}\left(\begin{array}{l}
\frac{w_1k+2\gamma_1(w_1+w_2H)}{(k+\gamma_1)(k+2\gamma_1)}\left[1-\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta} \\
-\frac{w_2H(1+p_0)}{k+2\gamma_1}\left[1-\left(\frac{1+p'}{1+p_0}\right)\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1+p}{1+p_0}\right)\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta}
\end{array}\right) \\
+\alpha'\gamma_2\theta(1-p')^{\frac{k+\gamma_2}{\gamma_2}}\left(\left(\frac{w_1k+2\gamma_2(w_1+w_2H)}{(k+\gamma_2)(k+2\gamma_2)}\right)\left[1-\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]-\frac{w_2H(1+p')}{k+2\gamma_2}\left[1-\left(\frac{1+p}{1+p'}\right)\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]\right)
\end{array}\right\} \quad (12)
$$

$$
W_{dw}=(d_0+d_1H+d_2H^2+d_3Age)V_{sib}-\zeta kdbh^2H^2H'^{\frac{k-\gamma_1}{\gamma_1}}
\left\{\begin{array}{l}
\gamma_1(1-p_0)^{\frac{k+\gamma}{\gamma_1}}\left(\begin{array}{l}
\frac{d_1k+2\gamma_1(d_1+d_2H)}{(k+\gamma_1)(k+2\gamma_1)}\left[1-\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta} \\
-\frac{d_2H(1+p_0)}{k+2\gamma_1}\left[1-\left(\frac{1+p'}{1+p_0}\right)\left(\frac{1-p'}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{\theta}\left[1-\left(\frac{1+p}{1+p_0}\right)\left(\frac{1-p}{1-p_0}\right)^{\frac{k+\gamma_1}{\gamma_1}}\right]^{1-\theta}
\end{array}\right) \\
+\alpha'\gamma_2\theta(1-p')^{\frac{k+\gamma_2}{\gamma_2}}\left(\frac{d_1k+2\gamma_2(d_1+d_2H)}{(k+\gamma_2)(k+2\gamma_2)}\left[1-\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]-\frac{d_2H(1+p')}{k+2\gamma_2}\left[1-\left(\frac{1+p}{1+p'}\right)\left(\frac{1-p}{1-p'}\right)^{\frac{k+\gamma_2}{\gamma_2}}\right]\right)
\end{array}\right\} \quad (13)
$$

where W_{dw}, W_{gwb}, and W_{gw} are stem weight of dry wood, stem weight of green wood and bark, and stem weight of green wood, respectively.

RESULTS

Treatment Effect

There is evidence that treatment F has impacted the upper segment taper, though treatment H did not affect these variables very much. To reflect this fact, the value of the upper segment form factor (b_2 or g_2) should be different for fertilized trees and unfertilized trees.

Estimation of Coefficients

Note that the derived stem taper and volume equations share not only independent variables such as *p*, *dbh,* etc. but also coefficients like b_1 and b_2. System modeling is required for obtaining efficient estimates of parameters because the estimation of shared coefficients needs the information from all associated dependent variables, i.e., both taper and volume. Note that V_{sob} and V_{sib} are endogenous variables because they appear on both sides of volume and weight equations. To eliminate simultaneous equation bias, predicted rather than observed V_{sob} and V_{sib} values were used as regressors in weight equations during parameter estimation (Borders and Bailey, 1986).

Table I-Estimates of parameters with standard errors (in second line)

dob	^ 1	^ 2	^ 2f	p'
	0.1569 0.0005	0.1441 0.0007	0.1561 0.0007	0.6025 0.0092
dib	c	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_{2f}$
	0.8625 0.0035	0.1724 0.0011	0.1428 0.0008	0.1588 0.0008
V_{sob}	ξ			
	1.99E-4 3.60E-7			
V_{sib}	ζ			
	1.59E-4 6.19E-7			
W_{dw}	d_0	d_1	d_2	d_3
	367.63 13.73	-5.95 7.82E-1	-5.70E-2 1.31 E-2	12.82 1.69
W_{gwb}	g_0	g_1	g_2	
	767.38 5.72	21.96 4.50E-1	5.23E-2 7.42E-3	
W_{gw}	w_0	w_1	w_2	
	784.88 7.04	21.68 5.72E- 1	1.86E-1 8.97E-3	

Table S-Fit statistics of each equation in the taper-volume-weight equation system, where MB is the mean bias, RMSE the root mean square error, and EF the modeling efficiency

Equations	MB	RMS	EF
dob	0.2667 (cm)	1 .1841 (cm)	0.9621
dib	0.4736 (cm)	1.1042 (cm)	0.9545
V_{sob}	6.30E-4 (m^3)	0.0092 (m^3)	0.9900
V_{sib}	8.20E-4 (m^3)	0.0099 (m^3)	0.9835
W_{dw}	-5.34E-1 (kg)	2.17 (kg)	0.9966
W_{gwb}	3.68E-1 (kg)	9.17 (kg)	0.9890
W_{gw}	-3.10E-1 (kg)	9.12 (kg)	0.9870

The mixed-effects systematic modeling technique was applied for obtaining unbiased and consistent estimates of parameters. The modeling efficiency (EF), root mean square error (RMSE), and mean bias (MB) (Loague and Green, 1991, Mayer and Butler 1993) were applied as fit statistics. The estimates of coefficients involved are listed in table 1 with the fit statistics in table 2.

DISCUSSION AND CONCLUSION

Compared with an empirical taper equation, the one derived from geometric relationships is more theoretically sound and physically meaningful and reduces the parameters dramatically, which simplifies model structure and helps parameter estimation in nonlinear regression.

Resultant taper equations showed that two segments well depict the relationship between stem diameter and height for trees in this study. The above conclusion does not go with Fang and others.'s (2000) where three segments are required. A plausible explanation for this disagreement might be the fact that the stems used in this work are relatively young and do not exhibit much butt swell.

Fig. (2) shows the profiles of *dob* and *dib* using a 18 meters long stem with *dbh* 20 centimeters for unfertilized and fertilized trees, implying a significant fertilizer impact for both inside and outside bark diameters on upper segment. Specifically, fertilized trees have less taper than unfertilized trees, implying more volume and woody materials on the upper stem of fertilized trees.

In this study, we derived segmented stem weight equations by integrating wood density and segmented volume. This approach provides logical estimates of wood density for any segment along the stem. This is especially noteworthy since previously derived equations overpredict wood density in the upper part of stems. Yet, the fact that fertilization did not significantly affect the unit-weight of stem wood agrees with the results of data analysis and conclusions derived from the investigation done by 'Clark using the same data (personal communication).

It should be noted that this system of stem taper-volume-weight equations was fitted to a small database from a specific geographic location. Thus, any use of these functions should first be validated on independent data. The objective here was not to produce equations that will be widely used by practitioners but to develop a modeling framework that is flexible enough to reflect the impacts of various silvicultural treatments. As such, these equations provide researchers a useful tool for simulating the impact that fertilization may have on stem form, volume, and weight.

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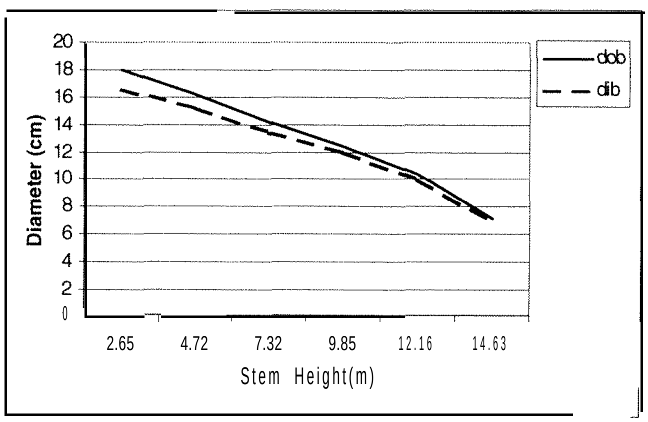

Figure 1—Stem taper profile of a tree at age 12, where *dob* is the stem diameter outside bark and *dib* the stem diameter inside bark.

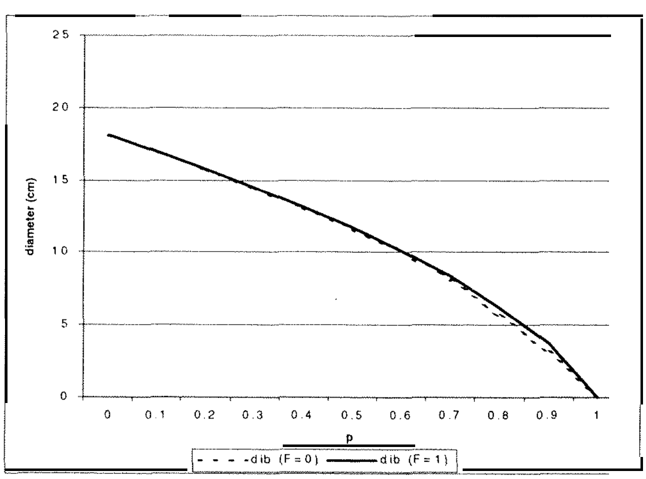

Figure 2a—Stem taper profiles of diameter outside bark *dob)* for fertilized (F = 1) and unfertilized (F = 0) trees with total height 18 meters and *dbh* 20 centimeters, where *p* is the ratio of stem height to total height.

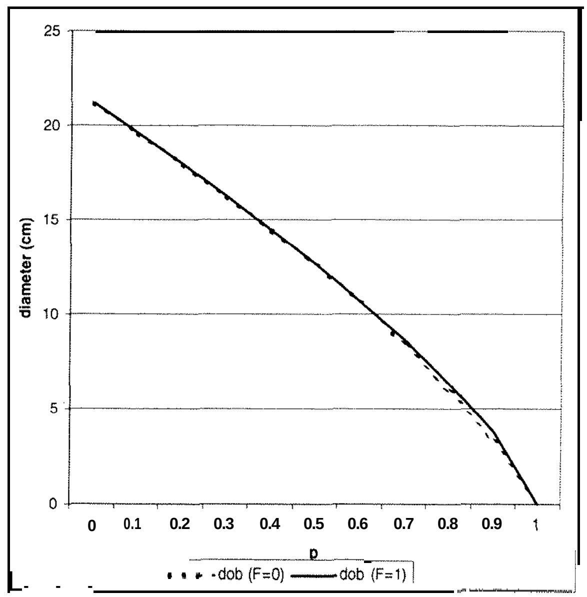

Figure 2b—Stem taper profiles of diameter inside bark (*dib*) for fertilized (F = 1) and unfertilized (F = 0) trees with total height 18 meters and *dbh* 20 centimeters.

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