A MATRIX TRANSITION MODEL FOR AN UNEVEN-AGED, OAK-HICKORY FOREST IN THE MISSOURI OZARK HIGHLANDS[1]

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Abstract-We present a matrix growth model for an uneven-aged, oak-hickory forest in the Ozark Highlands of Missouri. The model was developed to predict ingrowth, growth of surviving trees, and mortality by diameter class for a five-year period. Tree removal from management activities is accounted for in the model. We evaluated a progression of models from a static, fixed-parameter to a dynamic, function form. The model was based on data from 366 0.2-acre permanent plots, measured over seven five-year periods from 1957 to 1992. Variables used in development of the dynamic model included basal area of larger trees (an index of competition) and number of trees per acre. Models were evaluated using 92 reserved plots by comparing predicted and actual diameter-distributions over five- and thirty-five-year periods. Application of this matrix model requires only a DBH distribution in trees per acre.

INTRODUCTION

In recent years, there has been renewed interest in uneven-aged forest management (Guldin 1996, Stout 1998). To effectively manage our forests with uneven-aged systems, quantitative tools, such as growth models, are needed. In the state of Missouri, the most widely used forest growth model is TWIGS, an individual-tree, distance-independent model (Miner and others 1988). While the Central States version of TWIGS has been shown to produce reliable results on a region-wide basis (Indiana, Illinois, and Missouri), (Kowalski and Gertner 1987, Miner and others 1988) there has been much interest in developing a growth model specifically designed for application in uneven-aged forests in the Missouri Ozark Highlands. Compared to traditional, single-tree, distance-independent models, whole-stand solutions have been suggested as more appropriate for use in the application of uneven-aged management (Moser, 1972, Hann and Bare 1979). Matrix transition models have advantages over other whole-stand methods because they are conceptually simple and relatively easy to develop (Harrison and Michie 1985). Matrix-transition models have been developed for uneven-aged forests in many other regions of the United States (Buongiomo and Michie 1980, Solomon and Hosmer 1986, Michie and McCandless 1986, Lin and Buongiorno 1997, Lin and others 1998). This paper presents a whole-stand, matrix growth model developed for a large, privately owned forest under uneven-aged management in Southeast Missouri.

DATA

Beginning in the early 1950's, a system of continuous forest inventory (CFI) plots were installed on the Pioneer Forest, a 160,000-acre privately owned forest located in the Ozark Highlands of Missouri. By 1957, permanent, fifth-acre, circular plots had been laid out, at a density of approximately one plot for every 320 acres on the forest. Plots were remeasured on a five-year schedule; only trees five-inches and larger in diameter at breast height (DBH) were included in the inventory. Trees were permanently numbered and information was recorded on a number of variables including species and DBH. At subsequent inventories, mortality and harvest removals since the last inventory were also recorded. Ingrowth trees received a new, permanent number once they crossed the threshold diameter of 5 inches[3].

During the period 1957-1992, there were 366 plots that were consistently remeasured. These plots comprised the database for model fitting and validation. Twenty-five percent of the plots were randomly selected and resewed to validate the final version of the model. The remaining plots were used to parameterize the models. As the plots were remeasured at five-year intervals, one five-year period from each plot was selected at random for model development. This was done to insure independence.

For every plot in the modeling data set, basal area per acre (BA), number of trees per acre (TPA), and basal area of conifers per acre (BAC) were calculated. Basal area of conifers was the only species-specific variable used to develop the model. BAC was included to account for the differential growing space requirements of shortleaf pine (*Pinus echinata* Mill.) and upland hardwoods. To illustrate, in 1992 the quadratic mean diameter (QMD) on the Pioneer Forest was 8.72 inches (Loewenstein and others 1995). For upland hardwood stands with a QMD of nine inches, 100 percent stocking occurs at approximately 112 ft^2 of basal area per acre (Gingrich 1967). In comparison, a pine stand with the same QMD reaches 100 percent stocking at 150 ft^2 of BA (Rogers 1983). A similar relationship exists across all levels of stocking.

For each tree in the modeling data set, a binary variable was used to indicate if the tree survived or died over the selected five-year period. For each surviving tree, an additional binary variable was coded to specify whether it remained in the same two-inch diameter class over the five-year period, or grew into a larger diameter class. An indicator of competition, basal area of larger trees (BAL) was calculated for each tree. BAL is the sum of the BA for all trees with a larger diameter than the subject tree. This value is a

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[3] Loewenstein. 1996. An analysis of the size- and age-structure of a managed unevenaged oak forest. Unpublished Ph.D. Diss., University of Missouri. 167 p.

surrogate for social position of the subject tree and estimates the volume of growing space available.

METHODS

Static Model

The simple, fixed parameter model was developed first. For each two-inch diameter class, probabilities for survival and growth were calculated. The transition matrix is given in Table 1. Probabilities are listed for a tree moving from the DBH class indicated by the column label, to the DBH class indicated by the row label. For example, the probability that a tree in the 1 P-inch DBH class would move to the 14-inch class is 0.2562 and its probability for remaining in the 12-inch class is 0.6860. The probability of mortality is derived by subtracting the sum of each column from one. Ingrowth was estimated on a grand average of stems crossing the threshold diameter during the five-year period.

Table I-Probabilities obtained from the fixed-parameter model

D.b.h.	6	8	IO	12	14	16	18	20	22+
6	.7093	0	0	0	0	0	0	0	0
8	.2581	.6975	0	0	0	0	0		0
10	0	.2765	.7052	0	0	0	0	0	0
12	0	0	.2712	.6860	0	0	0	0	0
14	0	0	0	.2562	.7111	0	0	0	0
16	0	0	0	0	.2089	.6640	0	0	0
18	0	0	0	0	0	.2800	.6071	0	0
20	0	0	0	0	0	0	.3393	.5000	0
22+	0	0	0	0	0	0	0	.4000	.9444

Dynamic Model

The variable-parameter model was fit with a two-step, logistic regression model.

Mortality Model

The first model predicted tree mortality. Separate models were fit for each two-inch diameter class using the glm function in S-PLUS (Venables and Ripley 1997). Independent variables considered were basal area per acre, conifer basal area per acre, number of trees per acre, and basal area of larger trees. The most significant independent variables were determined to be basal area of larger trees and trees per acre. The model form is shown by equation 1, where in this instance "Y" is mortality.

$$E(Y|BAL,TPA)=\frac{EXP(b_0+b_1(BAL)+b_2(TPA))}{(1+EXP(b_0+b_1(BAL)+b_2(TPA)))} \qquad (1)$$

Growth Model

The second model predicted five-year diameter growth. Specifically the model predicted if a tree would move to the larger, adjacent diameter class, or remain in the same two-inch diameter class. Using trees that survived the five-year time step, the models were fit by diameter. Again, the same variables were considered as in the mortality model, and the same variables were used, BAL and TPA. The model form is shown by equation 1, where "Y" in this case is movement to the larger diameter class. Ingrowth for this model also used the grand average of stems crossing the threshold diameter during the five-year period.

Probability

After parameterization, probabilities obtained from the mortality and growth models were compared to the actual outcomes of mortality and growth.

RESULTS AND DISCUSSION

There was no statistically significant correlation between the actual mortality and growth values and the predicted values. Thus it was impossible to assign a critical probability for mortality or growth. The low correlation values, along with the minimal removal of error by the model suggested that there would be little improvement in using the variable-parameter model.

From this point on, we will examine the performance of the static, fixed parameter model. Using the transition probabilities obtained from the fixed-parameter analysis, the reserved validation plots were "grown" for periods of five and 35 years. In the event of a harvesting activity, the appropriate number of trees was removed from the diameter distributions, and harvesting activity was assumed to occur immediately prior to the inventory. CFI data from the 1957 inventory was utilized for this evaluation. Corresponding CFI data was used to compare the output of the model projections. The CFI data for a specific plot in 1962 was compared to the five-year model predictions for that plot, and the same procedure was repeated for the 35-year period using the 1992 data.

Model Evaluation

Five-year results-Output was evaluated using a chi-square goodness of fit test. Diameter distributions of the CFI data were compared to the modeled diameter distributions. Of the 92 validation plots, 87 (95 percent) of the plots were found to be statistically similar to the CFI data at an alpha level of 0.05. This suggests that for short periods of time, such as a five-year period, the model predicts diameter structure well.

Thirty-five-year results-After seven periods of simulation, (35 years), when modeled output was compared to the 1992 CFI data, only 11 of the plots did not differ from the CFI data (alpha = 0.05). The area of greatest divergence occurred in the smallest diameter classes, the six- and eight-inch classes. This was likely due to changing levels of recruitment, or ingrowth into the six-inch class over time (Loewenstein 1996).

CONCLUSIONS

Conditions on the Pioneer Forest were not static for the period of 1957-l 992. Average number of trees and stocking increased while the average diameter of trees went down (Loewenstein 1996). The parameters for this model were based on averages across this time period. However, these averages apparently do not explain all of the variation in stand dynamics on the forest. Another event that may have caused problems in prediction occurred in the early-1980's. The Ozark region of Missouri suffered extreme drought in 1980 and again in 1983 (Law and Gott 1987). Mortality from these drought events and subsequent oak decline (Dwyer and others 1995) dramatically affected trees on the forest and thus, the representative inventory. The model does not have a mechanism to account for such catastrophic mortality.

For short periods of time, specifically five years, the model performs well. However, when time periods are lengthened, the model's predictive ability declines. While the model predicts fairly well in larger diameter classes, those above the 14-inch class, it does not fair as well in the smaller diameter classes. This may be attributed to the lack information available for small diameter trees. It is difficult to predict recruitment into the threshold diameter class, especially with a relatively large threshold diameter of five inches DBH (Shifley and others 1993). With additional information on the dynamics of small diameter trees, and an improved methodology for ingrovvth prediction, overall model prediction should improve.

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