

Growth and Yield

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AN INDIVIDUAL-TREE SURVIVAL FUNCTION FOR LOBLOLLY PINE MANAGED UNDER SINGLE-TREE SELECTION

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Abstract—Aspects of the growth and stand dynamics of uneven-aged loblolly pine (*Pinus taeda* L.) managed under single-tree selection were investigated as follows. Eighty-one plots were installed in south Arkansas and north Louisiana. Treatments were three basal areas (40, 60, and 80 ft²/acre), three maximum diameters (12, 16, and 20 inches), and three site classes (site index <81, 81-90, and >90 ft at base age 50 years, loblolly pine). Each treatment combination was replicated three times. The logistic function was selected for the individual-tree survival equation, and tree, stand, and site factors were tested as explanatory variables. Initial individual-tree diameter, ratio of initial individual-tree diameter to quadratic mean diameter, and site index were the best variables. The resulting survival equation was used to examine survival patterns for different uneven-aged stand structures. Implications for management are discussed.

INTRODUCTION

The major components of uneven-aged stand dynamics are recruitment, growth, and mortality of trees. Trees in even-aged stands compete only with members of their own cohort, but trees in uneven-aged stands not only compete with members of their own cohort but also compete with members of older and younger ones. An uneven-aged loblolly pine stand is made up of clumps of trees and looks understocked. The clumps vary in size, but each is made up of trees of generally similar size. Following a disturbance, where a single mature tree or clump of trees is cut or dies, a patch of pine reproduction regenerates in the opened space. These pine seedlings compete with their peers and are also affected by the surrounding overstory trees. As a new cohort develops, some of its members inevitably die as a result of competition or other causes. When the cohort becomes merchantable, cutting reduces its numbers further. Finally, only a few trees of the original patch remain, and these tower over their neighbors and are little affected by competition. However, their surviving counterparts in even-aged stands still have significant peer competition. Thus, in comparison with trees in even-aged stands, those in uneven-aged stands have more competition earlier in life and less competition as they grow.

Vanclay (1994) gives a succinct summary of the various approaches to modeling tree survival. Most survival models relate to natural even-aged stands or plantations; relatively few have been developed for uneven-aged conditions.

Hamilton and Edwards (1976) and Monserud (1976) explain how to use the logistic function to model individual-tree survival. Hann (1980) developed a stand simulator for even-aged and uneven-aged ponderosa pine (*Pinus ponderosa* Laws.). Buchman and others (1983) used a variant of the logistic function to develop survival equations for Lake State tree species. An alternative approach is to model changes in the number of trees in the stand resulting from both ingrowth and mortality. For example, Lynch and Moser (1986) and Moser (1974) estimated change in number of trees by means of a differential equation that was a component of a system of differential equations

describing stand growth. More recently, McTague and Stansfield (1995) used a nonlinear projection equation to describe change in the number of trees.

Our intent here was to develop a survival function that could ultimately be a part of an individual-tree simulator for uneven-aged loblolly pine stands. We chose the logistic function for model development because other workers have used it successfully in similar applications and because the methodology is well developed.

METHODS

Treatment Variables

Uneven-aged stand structures are typically defined in terms of basal area, maximum diameter, and a quotient, q . This quotient is the ratio of the number of trees in a diameter class to the number of trees in the next larger diameter class. For example, if there are 12 trees per acre in the 12-inch class and 10 trees per acre in the 13-inch class, the q value is 1.2. The width of the diameter classes affects q ; for example, a q value of 1.2 for 1-inch classes is equivalent to a q value of 1.44 for 2-inch classes. Several published guides explain how to use these three variables to describe stand structure (Brender 1973, Moser 1976, Murphy and Farrar 1982).

Although q is one of the variables most often used to describe uneven-aged stand structure, experience has shown that it is not very amenable to management, at least initially. Therefore, the other variables were selected for manipulation in this first effort and q was fixed at 1.2. Reynolds (1959, 1969) and Reynolds and others (1984) have observed and used this value of q in managing loblolly-shortleaf pine stands by uneven-aged methods.

For treatments, we selected target basal areas of 40, 60, and 80 ft²/acre in trees with d.b.h. >3.5 inches; maximum diameters of 12, 16, and 20 inches d.b.h.; and site index ranges of <81, 81 to 90, and >90 ft (loblolly pine base age 50). The site index classes in this study adequately cover the range of site quality that is encountered in the west gulf

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coastal plain. Each treatment combination was replicated three times, and there are 81 plots in all.

Basal areas are kept lower than in even-aged stands to favor the development of pine reproduction. Reynolds (1959) recommended cutting when stand basal area reaches 75 ft²/acre to allow pine regeneration to develop. Basal area in uneven-aged loblolly pine stands, therefore, should probably not be much above this level at any time during a cutting cycle. A slightly higher basal area (80 ft²/acre) was chosen so that we could investigate its long-term effects on loblolly pine growth and regeneration. The lowest basal area treatment level, 40 ft², probably represents the lower acceptable density limit for management. Densities lower than this approach understocked conditions in which growth is lost without any offsetting gain in regeneration.

Maximum d.b.h. in uneven-aged management is somewhat akin to rotation age in even-aged management. Selection of a larger maximum d.b.h. implies a longer term investment. A residual maximum d.b.h. of 20 inches probably represents an upper limit for both economic and product-size goals. A 12-inch maximum d.b.h. represents a lower limit for an adequate seed source.

Field Installation and Measurements

Each of the stands chosen for plot installation had at least 70 percent of its basal area in loblolly pine; no evidence of cutting within the last 10 years; no evidence of catastrophic loss caused by insects, disease, weather, or fire; and a site index that did not vary more than 10 ft over a plot. Stands that exhibited a reverse J-shaped stand structure were preferred if available.

The stands represented a gamut of structures: some already exhibited a reverse J-shaped stand structure, while others had a mound-shaped structure more typical of even-aged stands. Most stands had more than one plot installed in them. All 81 study plots are on the Coastal Plain in southern Arkansas and northern Louisiana (fig. 1).

Each square 1.6-acre gross plot included an interior square 0.5-acre net plot. Before harvest, all loblolly pine trees with d.b.h. >3.5 inches were inventoried by 1-inch d.b.h. classes for the 0.5-acre net plots and 1.1-acre isolation strips. Plots were then marked for harvest. Marking was designed to give each plot its assigned residual structure as defined by residual basal area, maximum d.b.h., and a q of 1.2 for 1-inch d.b.h. classes. Any shortleaf pines occurring in the plots were cut. All hardwoods with a groundline diameter ≥ 1 inch or larger were injected with herbicide prior to harvest, if possible, but no later than the first growing season after treatment. Plot installation and harvesting were carried out over a 3-year period beginning in the fall of 1983. All cutting was completed during the early part of the dormant season of each year, and about one-third of the plots were established each year.

After harvesting, all residual loblolly pine trees with d.b.h. >3.5 inches on the net plot were numbered, mapped, and



Figure 1—Study plot locations (locations have more than one plot).

measured. D.b.h. was measured to the nearest 0.1 inch using a tape. A d.b.h. mark was painted on each tree to ensure consistency in subsequent measurements. Total height and height to the crown base were measured to the nearest foot on a sample of 20 percent of the trees in each 1-inch d.b.h. class. Five to 10 height-sample trees suitable for site index calculation were identified and their age determined by increment coring. Trees whose ring widths and growth patterns were indicative of past suppression were not used for site index computation. Site index was computed as suggested by Farrar (1973).

The plot trees were remeasured after 4 to 5 years of growth. With the exception of tree age, the same measurements were taken for both surviving trees and ingrowth trees. In addition, tree status (living, dead, ingrowth) was recorded for all trees in both inventories. Cause of death was determined where possible.

Data Summary and Analysis

Trees that were alive at the time of the first inventory were recorded as alive (1) or dead (0) at the second inventory. The following variables were calculated for trees with d.b.h. ≥ 3.6 inches:

P_i^t = probability of survival of the i th tree for t years

D_{max} = maximum d.b.h. (inches) for the plot,

N = stems per acre,

B = stand basal area (ft²/acre),

D_q = quadratic mean d.b.h. (inches),

S = site index (ft at 50 years, loblolly pine),

D_i = initial d.b.h. (inches) of i th tree,

L = basal area (ft²/acre) in trees with d.b.h. equal to or larger than that of the subject tree,

D_i/D_q = ratio of the subject tree's d.b.h. to the quadratic mean d.b.h. for the plot's trees, and

t = length (years) of the growth period.

Means and ranges for the data are presented in table 1. Although the planned treatment basal areas were 40, 60, and 80 ft²/acre, the actual residual densities ranged from 32 to 89 ft²/acre because of logging damage and failure to harvest all marked trees.

In this study, analysis started with the following full logistic model:

$$P_i^t = [1 + \exp(b_0 + b_1 D_{max} + b_2 B + b_3 N + b_4 D_q + b_5 S + b_6 D_i + b_7 L + b_8 D_i/D_q)]^{-t} \quad (1)$$

where P_i is the annual survival probability, the b_i 's are coefficients to be determined, and the other variables are as previously defined.

The procedure LOGISTIC (SAS Institute 1989) was used to initially screen variables by modeling growth period survival probabilities. This procedure uses a stepwise process for variable selection that is based on the adjusted chi-square statistic at the 0.05 probability level. After the initial screening, reduced models for annual survival were fitted by iteratively reweighted nonlinear least squares and compared with each other. A final model was selected based upon a compromise between model parsimony and adequate depiction of survival patterns using the chi-square goodness-of-fit test described by Hamilton and Edwards (1976).

Table 1—Descriptive statistics for tree, stand, and site variables at beginning of growth period

Variable ^a	Mean	Minimum	Maximum
Maximum d.b.h. (inches)	16.2	11.6	22.0
Trees/acre	135	50	310
Basal area (ft ² /acre)	59.2	31.7	88.6
Quadratic mean d.b.h. (inches)	9.3	6.1	13.7
Site index (ft)	83	56	97
Basal area in larger trees (ft ² /acre)	44.2	31.7	88.6
D.b.h. of subject tree (inches)	8.3	3.6	22.0
D.b.h. ratio	0.93	0.28	2.66

^a Tree variables (basal area in larger trees, d.b.h., and d.b.h. ratio) are based on 5,465 sample trees; stand variables (maximum d.b.h., trees per acre, basal area, quadratic mean d.b.h., and site index) are based on data from 81 0.5-acre plots.

RESULTS AND DISCUSSION

The final model has three independent variables—initial tree d.b.h., ratio of tree d.b.h. to quadratic mean d.b.h., and site index:

$$P_i^t = [1 + \exp(-7.28518 - 0.173095 D_i - 1.46790 D_i/D_q + 0.0644407 S)]^{-t} \quad (2)$$

This model yielded close agreement between observed and predicted survival probabilities for 1-inch d.b.h. classes (table 2). The calculated chi-square statistic for the equation using 1-inch d.b.h. classes is 0.343 with 15 degrees of freedom, which has a probability level of >0.999. Thus, the model predicts d.b.h. class survival rates that are not significantly different from actual rates.

The negative coefficients for initial d.b.h. and the d.b.h. ratio in equation (2) indicate that an increase in these variables results in an increase in predicted survival. However, an increase in site index decreases predicted survival. Vanclay (1991) noted the same negative effect of site index on tree survival in north Queensland rainforests. The reverse-J structure characteristic of uneven-aged stands influences the survival probability calculated by equation (2) through the d.b.h.-ratio term, because a stand's quadratic mean diameter is functionally defined by maximum diameter and q (Murphy and Farrar 1982). A q value of 1.2 for 1-inch d.b.h. classes was assumed, and equation (2) was used to

Table 2—Goodness-of-fit statistics for the logistic survival function

D.b.h. class	Observed survival	Observed survival proportion	Predicted survival proportion	Chi-square
<i>Stems/ac</i>				
4	592	0.866	0.870	0.015
5	572	0.883	0.892	0.066
6	615	0.923	0.914	0.062
7	576	0.937	0.936	0.001
8	560	0.954	0.951	0.007
9	480	0.968	0.965	0.006
10	435	0.982	0.971	0.047
11	393	0.978	0.980	0.003
12	296	0.980	0.984	0.004
13	169	0.983	0.987	0.003
14	142	0.993	0.990	0.002
15	110	0.991	0.993	0.000
16	83	1.000	0.994	0.003
17	42	0.977	0.996	0.015
18	28	1.000	0.997	0.000
19	24	0.960	0.998	0.036
20	12	0.923	0.998	0.073
21	1	1.000	0.998	0.000
22	1	1.000	0.999	0.000
Total	5131	—	—	0.343

calculate survival rates for a reasonable range of d.b.h., maximum diameter, and site index. Resulting values are plotted in figure 2. Annual survival probabilities range from 0.9 to nearly 1.0 and are most strongly affected by the subject tree's d.b.h. and the stand's site index. The survival probability for trees with d.b.h. ≥ 10 inches approaches 1.0 regardless of the site index. By contrast, the survival probability of trees with d.b.h. < 10 inches is considerably greater on poor sites (about 0.98 where d.b.h. = 4 inches) than on good sites (about 0.90 where d.b.h. = 4 inches).

The negative influence of the site index variable probably reflects the more intense competition that occurs on better sites—fewer trees will survive intensive competition. The relationship between site index and competition has also been noted in previous analyses of this study. Murphy and Shelton (1994) found that mortality in stand basal area increased with site index, while ingrowth in basal area decreased. This probably results from the greater intraspecific competition on the better sites in addition to more aggressive competition from nonpine vegetation. In addition, Shelton and Murphy (1994) noted the powerful effect of site quality on stand regeneration; pine seedlings and saplings tended to have much lower densities on the better sites as a result of aggressive competition from vines and other understory vegetation.

The survival function can be used to assess the first-year survival of trees under different uneven-aged stand structures. One hundred stand tables were randomly generated from the doubly truncated exponential distribution (Murphy and Farrar 1982) for each combination of the following attributes: a residual basal area of 60 ft^2/acre ; a q value of 1.2 (for 1-inch d.b.h. classes); maximum d.b.h.'s of 14, 16, 18, and 20 inches; and site indexes of 70, 85, and 100 ft at 50 years. These d.b.h. distributions are typical for uneven-aged loblolly pine stands with a 5-year cutting cycle, and the maximum d.b.h.'s are probably the operational range of diameters in loblolly pine management.

Table 3 shows the mortality rates as numbers of trees for the different combinations of maximum d.b.h. and site

index. Mortality rates range from 0.62 to 4.64 percent. Maximum d.b.h. affects mortality rate less than does site index; rates are six times greater on the best sites than on the poorest. These rates seem reasonable for the combinations of site index and maximum d.b.h. used in these simulations. This simulation supports the generally accepted tenet that uneven-aged loblolly pine stands are more difficult to create and maintain on better sites (Baker and others 1996).

The survival model described here is relatively simple but appears to give reliable estimates for stands managed by single-tree selection, as evidenced by the goodness-of-fit test and the simulations. Its full potential can be realized by incorporating it into an individual-tree growth and yield model.

ACKNOWLEDGMENT

Work was done in cooperation with the School of Forest Resources and the Arkansas Agricultural Experiment Station, University of Arkansas at Monticello. The authors thank the Arkansas Forestry Commission; Deltic Farm and

Table 3—Estimated first-year mortality rates for uneven-aged loblolly pine stand initial basal area of 60 ft^2/ac , a q value of 1.2, and specified maximum d.b.h. (based upon 100 simulations for each maximum d.b.h.-site index combination)

Maximum d.b.h.	Site index (ft)		
	70	85	100
-----Inches-----	-----	-----	-----
Percent	Percent	Percent	Percent
14	0.71	1.76	4.34
16	0.62	1.82	4.52
18	0.69	1.97	4.53
20	0.83	1.67	4.64

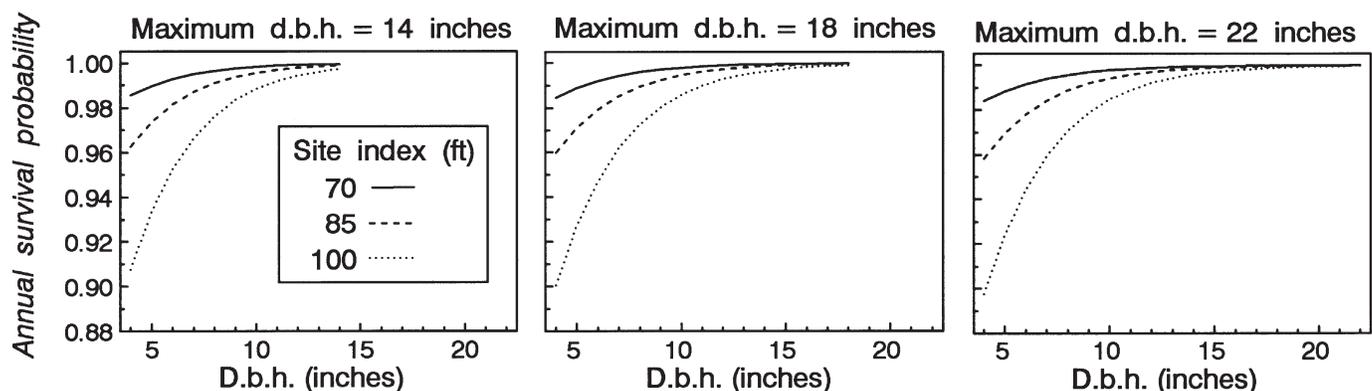


Figure 2—Calculated survival probabilities from equation (2) assuming reverse-J d.b.h.-class distributions with a q value of 1.2 and maximum d.b.h. of 14, 18, and 22 inches, which yields quadratic mean d.b.h. of 7.8, 8.8, and 9.2 inches, respectively.

Timber Company, Inc.; Georgia-Pacific Corporation; and Potlatch Corporation for their assistance in this study.

LITERATURE CITED

- Baker, James B.; Cain, Michael D.; Guldin, James M. [and others].** 1996. Uneven-aged silviculture for the loblolly-shortleaf pine forest types. Gen. Tech. Rep. SO-118. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 65 p.
- Breder, E.V.** 1973. Silviculture of loblolly pine in the Georgia Piedmont. Rep. 33. Macon, GA: Georgia Forest Research Council. 9 p.
- Buchman, R.G.; Pederson, S.P.; Walters, N.R.** 1983. A tree survival model with application to species of the Great Lakes region. Canadian Journal of Forest Research. 13: 601-608.
- Farrar, R.M., Jr.** 1973. Southern pine site index equations. Journal of Forestry. 71: 696-697.
- Hamilton, David A.; Edwards, Bruce M.** 1976. Modeling the probability of individual tree mortality. Res. Pap. INT-185. Ogden, UT: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station. 22 p.
- Hann, David W.** 1980. Development and evaluation of an even- and uneven-aged ponderosa pine/Arizona fescue stand simulator. Res. Pap. INT-267. Ogden, UT: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station. 95 p.
- Lynch, Thomas B.; Moser, John W., Jr.** 1986. A growth model for mixed species stands. Forest Science. 32: 697-706.
- McTague, John P.; Stansfield, William F.** 1995. Stand, species, and tree dynamics of an uneven-aged, mixed conifer forest type. Canadian Journal of Forest Research. 25: 803-812.
- Monserud, Robert A.** 1976. Simulation of forest tree mortality. Forest Science. 22: 438-444.
- Moser, J.W., Jr.** 1974. A system of equations for the components of forest growth. In: Fries, J., ed. Growth models for tree and stand simulation. Res. Note 30. Stockholm, Sweden: Royal College of Forestry: 260-287.
- Moser, J.W., Jr.** 1976. Specification of density for the inverse J-shaped diameter distribution. Forest Science. 10: 39-50.
- Murphy, P.A.; Farrar, R.M., Jr.** 1982. Calculation of theoretical uneven-aged stand structures with the exponential distribution. Forest Science. 28: 105-109.
- Murphy, Paul A.; Shelton, Michael G.** 1994. Growth of loblolly pine stands after the first five years of uneven-aged silviculture using single-tree selection. Southern Journal of Applied Forestry. 18: 128-132.
- Reynolds, R.R.** 1959. Eighteen years of selection timber management on the Crossett Experimental Forest. U.S. Department of Agriculture Tech. Bull. 1206. Washington, DC: U.S. Department of Agriculture. 68 p.
- Reynolds, R.R.** 1969. Twenty-nine years of selection timber management on the Crossett Experimental Forest. Res. Pap. SO-40. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 19 p.
- Reynolds, R.R.; Baker, J.B.; Ku, T.T.** 1984. Four decades of selection management on the Crossett Farm Forestry Forties. Bull. 872. Fayetteville, AR: Arkansas Agricultural Experiment Station. 43 p.
- SAS Institute, Inc.** 1989. SAS/STAT user's guide, version 6, 4th, ed., vol. 2. Cary, NC: SAS Institute, Inc. 846 p.
- Shelton, Michael G.; Murphy, Paul A.** 1994. Loblolly pine regeneration and competing vegetation 5 years after implementing uneven-aged silviculture. Canadian Journal of Forest Research 24: 2448-2458.
- Vanclay, J.K.** 1991. Mortality functions for north Queensland rainforests. Journal of Tropical Forest Science. 4: 15-36.
- Vanclay, Jerome K.** 1994. Modelling forest growth and yield: applications in mixed tropical forests. Wallingford, Oxon OX10 8DE, U.K.: CAB International. 312 p.

THE SELF-THINNING RULE AND EXTRAPOLATED RESULTS FROM GROWTH AND YIELD MODELS FOR UNTHINNED LOBLOLLY PINE PLANTATIONS

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Abstract—Growth and yield models are often used for short-term projection (5 to 10 years). Nevertheless, knowledge of how well these models behave in long-term simulations can give users more confidence in using the models. In this study, long-term projection from age 10 through age 100 was made using recent growth and yield models for unthinned loblolly pine plantations. Three levels of site index (50, 60, and 70 feet) and initial density at age 10 (500, 1000, and 1500 trees per acre) were included in this simulation. The objective of the study was to investigate the log(Q)-log(N) relationship, where N is surviving trees per acre and Q is quadratic mean diameter.

Except for two models, the growth and yield models did extrapolate well. The stands seem to approach a self-thinning curve, rather than a self-thinning line. Initial stand densities did not affect the shape of the self-thinning curves, whereas site index seemed to affect the level (or intercept) of these curves, but not the slope. It was concluded that most of the existing growth and yield models for unthinned loblolly pine plantations behave reasonably well in long-term projections.

INTRODUCTION

Growth and yield models have played a very important role in managing our forests. Foresters use these models to predict future yields based on current stand conditions, such as stand age, density, and site quality. The forecasts are usually short-term, from 5 to 10 years. Nevertheless, knowledge of how well the growth and yield models behave in long-term projections can give users more confidence in using the models.

In this study, we simulated stands from age 10 to age 100 using recent growth and yield models for unthinned loblolly pine plantations. The objective of the study was to investigate the relationship between tree size and stand density, and to observe if it follows the self-thinning rule.

THE SELF-THINNING RULE

Maximum average plant size for a given density was characterized by Yoda and others (1963) as following a self-thinning line. This line was expressed mathematically as

$$\log (W) = a + b \log (N)$$

where

W = maximum average plant size,

N = stand density,

a = the intercept,

b = the slope, and

log (x) = logarithm base 10 of x.

If the average plant size in the above equation is expressed as Q, the quadratic mean diameter of the stand, a graph of log (Q) versus log (N) shows the reciprocal relationship between quadratic mean diameter and stand density in an even-aged stand (fig. 1). The curve depicts a stand starting at an early age where there is little competition (A), then approaching the self-thinning line as it becomes older (B).

Reineke (1933) found the slope of this self-thinning line to be -1.605 for loblolly pine, when log (N) was plotted against log (Q). If the y-axis is log (Q) and the x-axis is log (N) as

presented in figure 1, the slope of his self-thinning line was $1/(-1.605)$ or -0.62. Subsequent authors reported slopes varying from -0.59 to -0.66.

SIMULATION

Long-term projections from age 10 to age 100 at 5-year intervals were made for three levels of site index (50, 60, and 70 feet, base age 25 years) and three levels of initial density at age 10 (500, 1,000, and 1,500 trees per acre).

Ten growth-and-yield models for unthinned loblolly pine plantations used in this study are listed in table 1. Also listed is a description of the data used in developing these models.

RESULTS AND DISCUSSION

Figure 2 shows the stand trajectories from different growth and yield models. There were nine curves for each model,

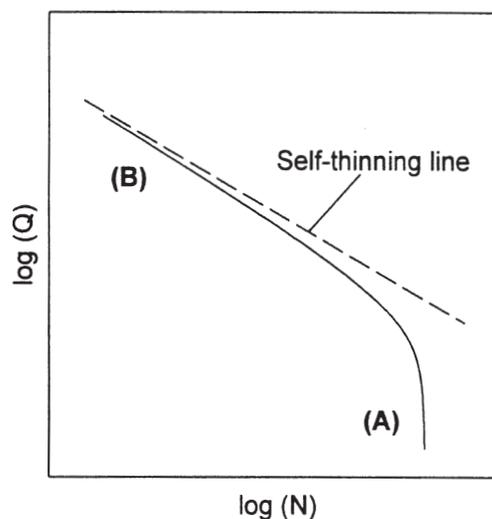
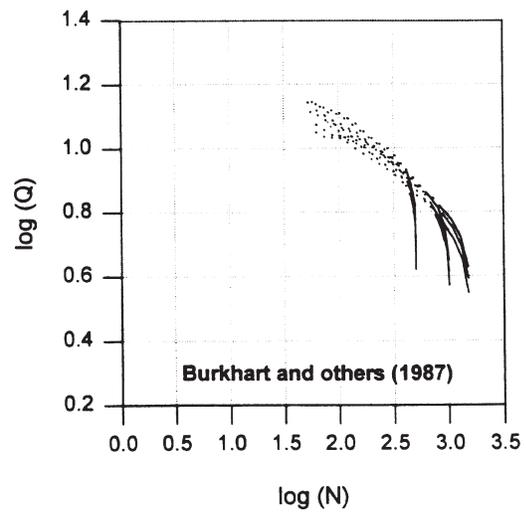
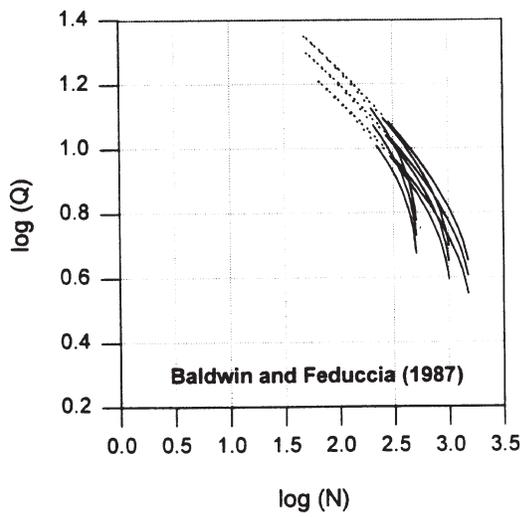
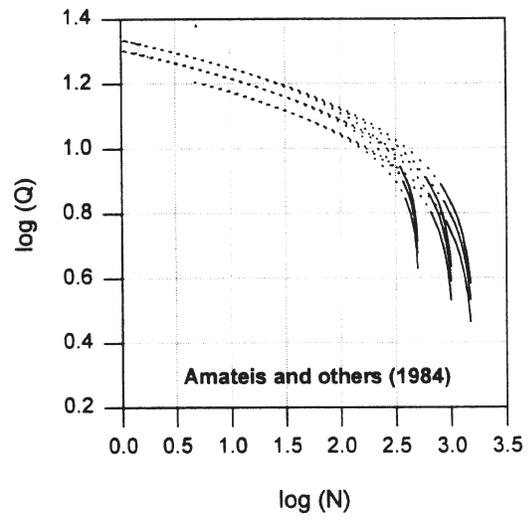
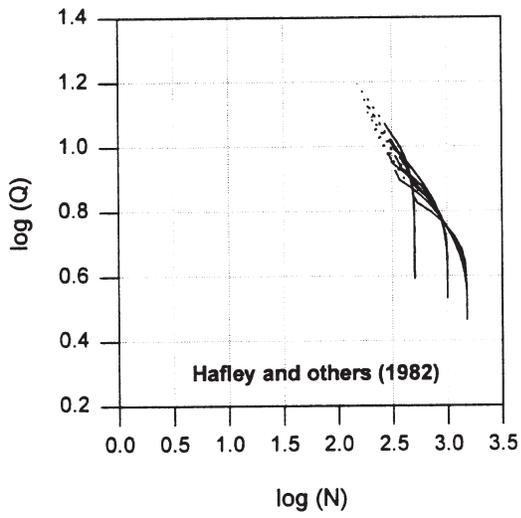
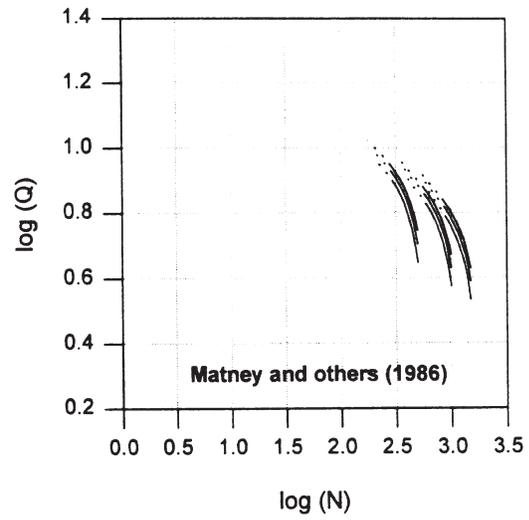
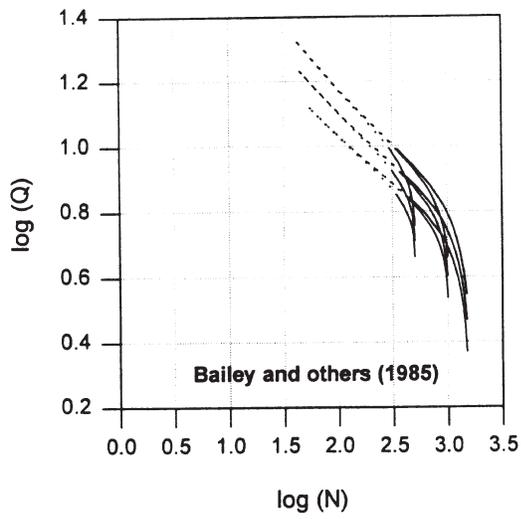


Figure 1—Size-density trajectory of an even-aged stand undergoing self-thinning.

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Table 1—Description of data used in developing loblolly pine plantation models included in the evaluation

Model	Location	Number of plots	Plot size -----Acres-----	Age Years	Site index feet @ 25 yrs	Trees per acre
Whole stand model						
Coile and Schumacher (1964)	AL, FL, GA, LA, MS NC, SC, TX	398 (Half of the plots were thinned)	0.10 (6-10 yrs) 0.20 (over 10 yrs)	5-35	35-80	—
Weibull diameter distribution models						
Smalley and Bailey (1974)	AL, GA, TN	267 (to fit models) 32 (to validate)	0.05	10-31	31-89	202-2240
Feduccia and others (1979)	AR, LA, MS, TX	409	varied, >0.10	3-45	22-78	250-1500
Amateis and others (1984)	AL, AR, GA, LA, MD, MS, NC, OK, SC, TN, TX, VA	186	0.50	8-25	33-97	275-950
Clutter and others (1984)	FL, GA, NC, SC	226	0.10	10-30	40-80	300-900
Bailey and others (1985)	AL, GA, SC	284	64 planting spaces	12-26	39-71	299-1500
Matney and others (1986)	AL, AR, LA, MS	230	0.25 (214 plots) 0.10 (16 plots)	1-26	51-71	101-801
Baldwin and Feduccia (1987)	LA, MS, TX	85 (unthinned)	—	5-45	40-79	100-2700 (planting)
SB diameter distribution model						
Hafley and others (1982)	LA, NC, SC, IL	—	—	5-44	48-93	—
Individual tree simulation model						
Burkhardt and others (1987)	AL, AR, GA, LA, MD, MS, NC, OK, SC, TN, TX, VA	186	0.50	8-25	83-90	270-1000



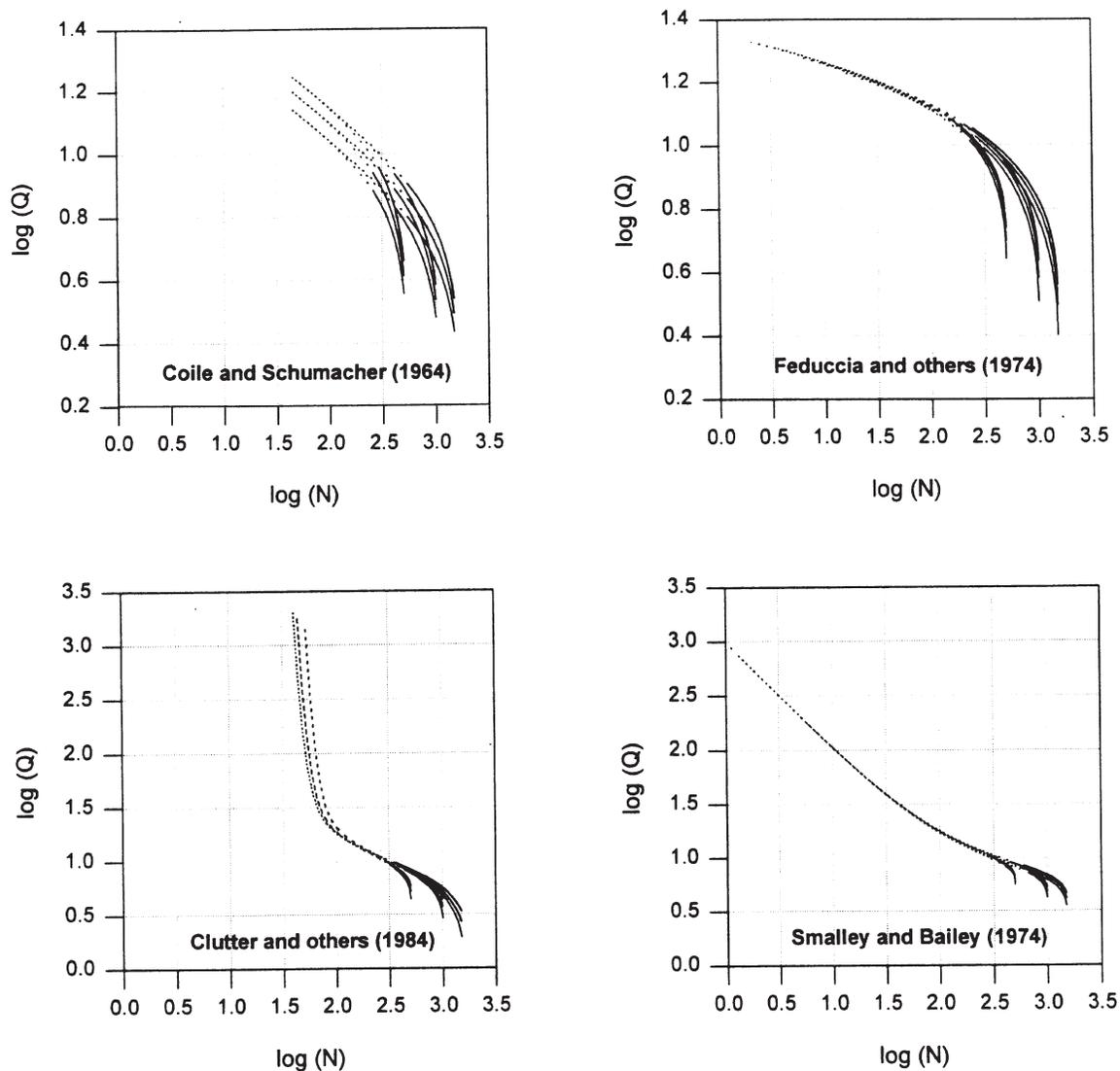


Figure 2—Size-density trajectories of unthinned loblolly pine plantations, simulated from 10 growth-and-yield models. The projections from age 10 to age 100 were made for three levels of site index (50, 60, and 70 feet) and stand density at age 10 (500, 1,000, and 1,500 trees per acre). Solid and dotted lines depict trajectories within and beyond the age range for each model, respectively.

each curve corresponding to a combination of site index (three levels) and initial stand density at age 10 (three levels). The solid lines depict the size-density trajectory within the age range, whereas the dotted lines represent extrapolation beyond the maximum age of the data used to construct that particular model. For example, since the maximum age of Baldwin and Feduccia's (1987) data was 45, solid lines were used for this model from age 10 to 45, and dotted lines from age 45 to 100.

The Self-Thinning Line

For some growth-and-yield models, the stand trajectories were close to a straight line at older ages. The slopes of the self-thinning lines varied between -0.3 to -0.4 (Bailey and others 1985), and between -0.35 to -0.6 (Matney and others 1986). The curves from Hafley and others' (1982) model were not smooth, but they tended to converge to a line having a slope close to -0.5.

For a majority of the growth-and-yield models evaluated here, stands converged at older ages to a curve, rather than a line, as expected from the self-thinning rule. Models exhibiting this characteristic included those from Amateis and others (1984), Baldwin and Feduccia (1987), Burkhart and others (1987), Coile and Schumacher (1964), and Feduccia and others (1974). For example, the slope of the self-thinning "curve" from Coile and Schumacher (1964) decreased (in absolute value) from -0.5 at age 35 to -0.3 at age 100. The curves for Burkhart and others' (1987) model were not smooth, possibly due to the random components in the model. The stand trajectories for this model represented averages of three stands for each combination of site index and initial stand density.

For two growth-and-yield models (Clutter and others 1984, Smalley and Bailey 1974), the stand trajectories behaved properly up to about 10 years beyond the maximum age of

the data, then the curves turned upward, resulting in unreasonably high values of quadratic mean diameter at older ages. Both models used the Weibull function to characterize diameter distributions of stands. The Weibull parameters in the models were predicted using regression equations; therefore no constraints were placed on the stand structure. This might cause the inconsistency in the long-term extrapolation attempted here.

Effect of Initial Stand Density

For the same site index, trajectories of stands of different initial stand densities converged to the same curve, as shown by most models evaluated here (Amateis and others 1984, Bailey and others 1985, Baldwin and Feduccia 1987, Clutter and others 1984, Coile and Schumacher 1964, Feduccia and others 1974, Hafley and others 1982, and Smalley and Bailey 1974).

Effect of Site Index

Curves for stands of different site indexes separated into three levels; a higher site index resulted in larger quadratic mean diameters. This trend was observed in models by Amateis and others (1984), Bailey and others (1985), Baldwin and Feduccia (1987), and Coile and Schumacher (1964). For other models, site index did not affect the stand trajectories (Clutter and others 1984, Feduccia and others 1974, and Smalley and Bailey 1974).

CONCLUSIONS

Since a number of existing growth-and-yield models, especially those developed many years ago, were based on empirical data rather than on biological principles, some foresters might be curious to see if the models behave properly when extrapolated beyond the age range of the data.

Results were encouraging. Except for two models, the growth-and-yield models did extrapolate well. The stands seem to approach a self-thinning curve, rather than a self-thinning line. Initial stand densities did not affect the shape of the self-thinning curves, whereas site index seemed to affect the level (or intercept) of these curves, but not the slope.

I hope that this paper gives foresters some assurance that most of the existing growth-and-yield models for unthinned loblolly pine plantations do a good job of projecting a stand through time, and the new crop of models that incorporate biological principles should perform even better.

LITERATURE CITED

- Amateis, R.L.; Burkhardt, H.E.; Knoebel, B.R.; Sprinz, P.T.** 1984. Yields and size class distributions for unthinned loblolly pine plantations on cutover site-prepared lands. Pub. No. FWS-2-84. Blacksburg, VA: School of Forest Wildlife Research, Virginia Polytechnic Institute and State University. 69 p.
- Bailey, R.L.; Grider, G.E.; Rheney, J.W.; Pienaar, L.V.** 1985. Stand structure and yields of site-prepared loblolly pine plantations in the Piedmont and upper coastal plain of Alabama, Georgia, and South Carolina. Res. Bull. No. 328. University of Georgia Agricultural Experiment Station.
- Baldwin, V.C., Jr.; Feduccia, D.P.** 1987. Loblolly pine growth and yield prediction for managed West Gulf plantations. Res. Pap. SO- 236. New Orleans, U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 27 p.
- Burkhardt, H.E.; Farrar, K.D.; Amateis, R.L.; Daniels, R.F.** 1987. Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas. Pub. No. FWS-1-87. Blacksburg, VA: School of Forest Wildlife Research, Virginia Polytechnic Institute and State University. 47 p.
- Clutter, J.L.; Harms, W.R.; Brister, G.H.; Rheney, J.W.** 1984. Stand structure and yields of site-prepared loblolly pine plantations in the lower coastal plain of the Carolinas, Georgia, and north Florida. Gen. Tech. Rep. SE-27. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 173 p.
- Coile, T.S.; Schumacher, F.X.** 1964. Soil-site relations, stand structure, and yields of slash and loblolly pine plantations in the Southern United States. T.S. Coile, Inc., 296 p.
- Feduccia, D.P.; Dell, T.R.; Mann, W.F., Jr. [and others].** 1979. Yields of unthinned loblolly pine plantations on cutover sites in the West Gulf region. Res. Pap. SO-148. New Orleans, U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 88 p.
- Hafley, W.L.; Smith, W.D.; Buford, M.A.** 1982. A new prediction model for unthinned loblolly pine plantations. Tech. Rep. No. 1. Raleigh, NC: Southern Forest Research Center, School of Forest Research, North Carolina State University. 65 p.
- Matney, T.G.; Sullivan, A.D.; Ledbetter, J.R.** 1986. Diameter distributions and merchantable volumes for planted loblolly pine on cutover site-prepared land in the West Gulf coastal plain. Tech. Bull. 132. Mississippi State University Agricultural and Forest Experiment Station. 12 p.
- Reineke, L.H.** 1933. Perfecting a stand-density index for even-aged forests. Journal of Agricultural Research. 46: 627-638.
- Smalley, G.W.; Bailey, R.L.** 1974. Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama, and Georgia Highlands. Res. Pap. SO-96. New Orleans, U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 81 p.
- Yoda, K.; Kira, T.; Ogawa, H.; Hozumi, K.** 1963. Self-thinning in overcrowded pure stands under cultivated and natural conditions. Series D. Journal of Biology, Osaka City University. 14: 107-129.

STABILITY OF PARAMETERS THAT PREDICT GROWTH AND YIELD OF NATURAL STANDS OF LONGLEAF PINE (*Pinus palustris* Mill.)

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Abstract—Data from remeasured longleaf pine growth-and-yield plots were modeled at the stand level as well as the individual tree level to evaluate the stability of model parameters over time. Parameter stability analyses were performed using subsets of the database (with nonoverlapping remeasurement periods) where the effects of stand dynamics have been accounted for or isolated as much as possible. Likelihood Ratio tests determined that the model parameters changed significantly from one time period to the next. Further tests also identified the parameters in need of modification to account for the observed changes. The selection of a particular model or model form did not change the fact that the model parameters have changed significantly over the last 25 years. Results were consistent across several dependent variables and several model forms. Results indicate that long-term prediction from growth-and-yield models for longleaf pine will be biased and inefficient without appropriately modifying some of the model parameters.

INTRODUCTION

Linear and nonlinear regression are the primary statistical techniques widely used to quantify biological relationships. Various empirical fits are available to represent biological relationships (Clutter 1963). Somers and Farrar (1991) use biomathematical models that try to explain the biological relationship to a reasonable extent. When a biological relationship is represented by statistical techniques, it is essential to examine the stability of that relationship for a long period of time (Chow 1960). In this context, growth-and-yield models based on periodic remeasurements may be useful to project yield for a certain period of time. The parameters of these growth-and-yield models might be questionable if the projection is for a long period of time. The question of instability of parameters of growth-and-yield models can be answered statistically by testing the parameters over different time periods. This investigation has importance in light of the present concern of growth changes due to climate change (Zahner and others 1989).

DATA

The growth data used in this investigation were part of the Regional Longleaf Pine Growth Study (RLGS) which was initiated by the USDA Forest Service in the mid-1960's (Kush and others 1987). The RLGS data set contains observations from periodically remeasured, naturally regenerated, even-aged longleaf pine (*Pinus palustris* Mill.) stands. Longleaf pine ecosystems once covered vast acreages in the Southeast. These ecosystems provide valuable wood products, unique multiple-use benefits, maintain biological diversity, and supply the necessary habitat for certain rare and endangered wildlife species (Boyer 1991). Plots in the RLGS come from a broad array of physiographic provinces covering a wide range of soil, site, and climatic conditions.

To examine possible productivity changes with respect to time, a series of plots has been established every 10 years in young stands (9-15 years old) on the Escambia Experimental Forest (EEF) in Brewton, AL. These plots are

termed the "time replication plots" or "timerep" plots. The plots are purposely located on similar sites and cover similar densities. There are 21 timerep 1 plots (established in the mid-1960's); 15 timerep 2 plots (established in the mid-1970's); and 29 timerep 3 plots (established in the mid-1980's). The plots have been periodically remeasured about every 5 years.

The purpose of establishing the timerep plots was to examine the differences in growth due to differences in the environmental factors after reducing the differences in initial stand characteristics (age, site, and trees/ac) as much as possible. The controlled nature of the timerep plots already isolates most of the stand characteristics included in growth models. For instance, the close proximity of plots to each other and similarity of soil types have isolated most of the effects due to site quality. To help minimize differences between timereps, a subset of nonoverlapping time remeasurements termed as "band" that are similar in site (SI=65 ft), trees/ac (<3000/ac), and ages (15-25 years) were selected from the overall timerep plot data.

In order to check the applicability of the findings from the timerep band data to older age classes, pseudo timerep data sets were created and tested in a similar manner. RLGS observations from nonoverlapping remeasurement years and older age classes were divided into seven arbitrary age classes (e.g., 25-34, 35-44, etc.). These data sets, however, lacked the controlled ranges of site index, trees/ac, and geographical location that were built into the timerep band data.

METHODS

Determining the stability of the parameters in the growth model was a two-stage process. First, differences in the parameters estimated for each of the three timereps were examined using various growth-and-yield models. Tests were conducted to determine whether any or all of the parameters were stable across all the timereps. If differences existed, then the parameters would be

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considered unstable and a specification error was suspected. The second stage attempted to identify which parameter(s) in the models were most unstable.

The models selected to conduct the tests included stand level basal area and volume projection models, stand level increment projection models, and an individual tree basal area increment model.

Thinning Considerations

The effect of thinning was an important factor to be considered prior to the parameter stability analysis. During establishment, plots were assigned a target basal area class of 30, 60, 90, 120, or 150 ft²/ac. They were left unthinned to grow into that class if they were initially below the target basal area. In subsequent remeasurements, if the basal area of the plots was found to be more than 7.5 ft²/ac above the target BA the plots were thinned back to the previously assigned class. The thinning was generally of low intensity and from below. Somers and Farrar (1991) found that the thinnings did not significantly impact the functional form of their prediction equations. Similarly, Quicke and others (1994) found that there were no consistent patterns of under- or overprediction in plots of residuals against actual basal area removed in the process. Both of the above studies were based on portions of the same RLGS data.

To test for a difference in growth due to thinning, plots were considered thinned if more than 5 percent of the trees were removed. A likelihood ratio (LR) test was used to test the null hypothesis that a single set of parameters in the projection model was sufficient versus the alternative that separate parameters are needed depending upon thinning history. If the LR test was significant, then an extra parameter would be included in the model to reflect changes in the functional form by thinning.

Stand Level Models

Empirical models were chosen to provide a maximum fit to the data. Dependent variables chosen were: basal area (BA) (ft²/ac), basal area increment (BAI) (ft²/ac/year), diameter²*height (D²H) as a surrogate for volume (SV), and D²H increment (SVI) as a surrogate for volume growth. SV and SVI were chosen since their use eliminates the need to select a specific volume equation and because volume is often a linear function of D²H.

Stand level projection models—A number of basal area projection models exist that could have been used in this study. One requirement of this study was to select a model flexible enough to fit the timerep band data. After an evaluation of several alternative models, the nonlogarithmic form of the two parameter basal area projection models suggested by Clutter and Jones (1980) was selected:

$$BA_2 = BA_1 \left(\frac{A_1}{A_2} \right)^{\beta_0} e^{\beta_1 \left[1 - \left(\frac{A_1}{A_2} \right)^{\beta_0} \right]} \quad (1)$$

where

BA₁ = initial stand basal area (ft²/ac) at initial age A₁,
 BA₂ = projected stand basal area (ft²/ac) at projected age A₂,
 β₀, β₁ = parameters to be estimated.

The analysis based on model (1) was repeated using a second dependent variable SV₂ with the following model:

$$SV_2 = SV_1 \left(\frac{A_1}{A_2} \right)^{\beta_0} e^{\beta_1 \left[1 - \left(\frac{A_1}{A_2} \right)^{\beta_0} \right]} \quad (2)$$

Stand level increment models—To further investigate possible changes over time, a linear basal area increment equation including stand characteristics was developed. Stepwise and Maximum R-square procedures were employed when developing the candidate models. Any nonsignificant parameters were dropped from the model. Best candidate models with 2 and 3 parameters were selected.

Individual Tree Models

A distance-independent individual tree basal area increment model developed for thinned, even-aged stands of naturally regenerated longleaf pine was also selected as a candidate model (for further detail see Quicke and others 1994). This model has the following form:

$$bai = a_0 e^{a_1 BA^{0.5}} e^{b_0 BAL} e^{c_1 (1 - e^{c_2 DBH}) - c_0} A \quad (3)$$

where

bai = annual basal area increment per tree(in²/year),
 BA = stand basal area (ft²/ac),
 BAL = basal area of all trees >the subject tree(ft²/ac),
 A = mean age of dominant and codominant trees (years),
 DBH = tree diameter outside-bark at breast height (in).

Parameter Stability Analyses

Parameters for each of the selected models were tested for stability over different time periods by a series of LR tests. The first series of tests was conducted on the timerep band data set and then repeated on the pseudo timerep data sets. Each series consisted of a general hypothesis that all the parameters were the same for each time period. Rejection of this hypothesis indicated a difference in parameters over the time periods and led to a second series of tests to determine whether each time period required different parameters.

In general, let β₀, β₁, and β₂ be the parameters in a projection or increment model that is fitted from the stand characteristics in the band data. The timereps (1, 2, and 3) are represented by the decomposition of the above parameters β₀, β₁, and β₂ such that

$$\begin{aligned}\beta_0 &\rightarrow \beta_{01}, \beta_{02}, \beta_{03} \\ \beta_1 &\rightarrow \beta_{11}, \beta_{12}, \beta_{13} \\ \beta_2 &\rightarrow \beta_{21}, \beta_{22}, \beta_{23}\end{aligned}$$

e.g., β_{01} is the β_0 parameter representing timerep 1, and so on.

In order to perform an LR test for an overall difference among the timereps, the following null and alternative hypotheses were used:

$$(1) \quad H_0: \begin{aligned} \beta_{01} &= \beta_{02} = \beta_{03} = \beta_0 \\ \beta_{11} &= \beta_{12} = \beta_{13} = \beta_1 \\ \beta_{21} &= \beta_{22} = \beta_{23} = \beta_2 \end{aligned} \quad H_A: \begin{aligned} \beta_{01} &\neq \beta_{02} \neq \beta_{03} \neq \beta_0 \\ \beta_{11} &\neq \beta_{12} \neq \beta_{13} \neq \beta_1 \\ \beta_{21} &\neq \beta_{22} \neq \beta_{23} \neq \beta_2 \end{aligned}$$

The full model (H_A) and a reduced model (H_0) for the above hypotheses consisted of the parameters respectively: $\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{21}, \beta_{22}, \beta_{23}$, and $\beta_0, \beta_1, \beta_2$. LR test statistics were used to test the above hypotheses. If $F > F_{(\alpha, p, v-k)}$ then H_0 is rejected, where α is the level of significance and $r, n-k$ is the numerator and denominator degrees of freedom for the likelihood F-statistic, respectively. If the above H_0 was rejected (indicating a significant difference among parameters) then the following hypotheses were tested to isolate and identify which individual parameters were candidates for further modification.

$$(2-\beta_0) \quad H_0: \begin{aligned} \beta_{01} &= \beta_{02} = \beta_{03} = \beta_0 \\ \beta_{11} &\neq \beta_{12} \neq \beta_{13} \neq \beta_1 \\ \beta_{21} &\neq \beta_{22} \neq \beta_{23} \neq \beta_2 \end{aligned} \quad H_A: \text{for all } H_0\text{'s is:} \\ H_A: \begin{aligned} \beta_{01} &\neq \beta_{02} \neq \beta_{03} \neq \beta_0 \\ \beta_{11} &\neq \beta_{12} \neq \beta_{13} \neq \beta_1 \\ \beta_{21} &\neq \beta_{22} \neq \beta_{23} \neq \beta_2 \end{aligned}$$

$$(2-\beta_1) \quad H_0: \begin{aligned} \beta_{01} &\neq \beta_{02} \neq \beta_{03} \neq \beta_0 \\ \beta_{11} &= \beta_{12} = \beta_{13} = \beta_1 \\ \beta_{21} &\neq \beta_{22} \neq \beta_{23} \neq \beta_2 \end{aligned}$$

$$(2-\beta_2) \quad H_0: \begin{aligned} \beta_{01} &\neq \beta_{02} \neq \beta_{03} \neq \beta_0 \\ \beta_{11} &\neq \beta_{12} \neq \beta_{13} \neq \beta_1 \\ \beta_{21} &= \beta_{22} = \beta_{23} = \beta_2 \end{aligned}$$

Generalization to Pseudo Timerep Plots

To check the consistency of the time-replication analyses, it was desirable to generalize the study to older stands. To accomplish this, the pseudo timerep data sets were used and similar LR tests were performed.

RESULTS AND DISCUSSION

The results presented in this paper deal primarily with the parameter stability analysis, which was performed to determine whether there were any significant changes in the parameters of the selected models during a projection period of 25 years. Candidate models for each dependent variable BA_2, SV_2, BAI, SVI , and bai were developed and tested for the stability of parameters.

Thinning Considerations

LR tests for the thinning component obtained, using the projection models [equations (1) and (2)], consistently found that there were no significant effects of thinning in the timerep band data. This result was not unexpected

since, as already mentioned, thinning was light to moderate and was from below, and previous studies had failed to identify any significant impacts.

Stand Level Models

Stand level projection models—Parameters in projection models [equations (1) and (2)] were estimated using standard nonlinear estimation techniques. The results of the hypothesis tests ($1-2-\beta_0$, and $2-\beta_1$) are given in table 1.

Stand level increment models—The following candidates for the increment models were developed using Stepwise and Maximum R-square procedures:

$$BAI = \beta_0 + \beta_1 \left(\frac{N_1}{A_1} \right) + \beta_2 \left(\frac{SI}{A_1} \right) \quad (4)$$

$$SVI = \beta_0 + \beta_1 \left(\frac{N_1}{A_1} \right) + \beta_2 \left(\frac{SV_1}{A_1} \right) \quad (5)$$

Results of the LR tests for equations (4) and (5) are given in table 2.

Individual Tree Growth Model

A distance-independent individual tree growth model [equation (3)] was fitted using a nonlinear estimation technique. This model fitted well with the timerep band data set ($N=23190$). The individual tree model [equation (3)] has six parameters and, therefore, decomposing all of its parameters simultaneously to represent three timereps would result in a full model with 18 parameters in the LR tests (as expected, these models failed to converge due to over-parameterization). To overcome this, an LR test was performed by decomposing one parameter at a time to represent different timereps while keeping all the other parameters constant. Table 3 gives the parameters and respective likelihood F-ratios, and their corresponding level of significance.

Parameter Stability Analysis

These analyses used the timerep band data, where effects due to thinning and other stand dynamics are already accounted for or minimized. The projection models [equations (1) and (2)], increment models [equations (4) and (5)] utilizing stand level values were used in the analysis. Table 1 shows the results of the hypotheses tests for all parameters for the above mentioned projection models. The LR test based on hypothesis 1 (table 1) indicated an overall significant difference among the parameters across all three timereps. The overall significant difference was consistent for the surrogate volume projection model [equation 2] (table 1). Results of hypothesis tests $2-\beta_0$ and $2-\beta_1$ in table 1 further show that the parameter β_0 is the more suitable candidate to represent differences in the three timereps and the parameters β_{11}, β_{12} , and β_{13} can be collapsed to a single parameter β_1 .

Analyses performed using the increment models [equations (4) and (5)] (table 2) also indicated an overall significant

Table 1—Results of LR tests on three timereps using projection models [equation (1) and (2)]

Hypothesis	BA ₂	SV ₂	Parameter tested
	F-Value/ Prob > F	F-Value/ Prob > F	
1. Full model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}$) Reduced model (β_0, β_1) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ H _A : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$	8.6002/ 0.0001	4.1480/ 0.0036	both β_0, β_1
2- β_0 . Full model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}$) Reduced model ($\beta_0, \beta_{11}, \beta_{12}, \beta_{13}$) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$ H _A : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$	3.5873/ 0.0308	3.6636/ 0.0287	β_0 only
2- β_1 . Full model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}$) Reduced model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_1$) H ₀ : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ H _A : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$	2.9188/ 0.0580	2.2640/ 0.1085	β_1 only

$$BA_2 = BA_1 \left(\frac{A_1}{A_2} \right)^{1.44324} e^{5.51592 \left[1 - \left(\frac{A_1}{A_2} \right)^{1.4432} \right]}$$

and

$$SV_2 = SV_1 \left(\frac{A_1}{A_2} \right)^{1.64644} e^{10.00578 \left[1 - \left(\frac{A_1}{A_2} \right)^{1.64644} \right]}$$

difference among the parameters across all three timereps. Parameter β_1 was the most suitable candidate for modification (table 2) for the increment models.

The results of the individual tree growth model LR tests (table 3) verify the results found using stand level models. Furthermore, they suggest that any parameter is a suitable candidate for modification.

The timerep analyses clearly indicated that the parameters of the models change across all timereps. This strongly indicates that the parameters of the projection and increment models do not remain stable over the 25-year period. Furthermore, the analysis also indicates that unless the difference is explained by some means, these models may be biased and inefficient when predicting growth for a longer period of time.

Generalization to Pseudo Timerep Plots

Generalization of the timerep analysis was conducted using pseudo timerep data. Hypothesis test 1 was performed using only the basal area increment model [equation (4)] (table 4). In general, the results identified significant

differences for all age classes except above 85 years. The pseudo timerep results coincide with the findings obtained in the previous parameter stability analyses and support the generalization of significant growth changes occurring over time within the overall RLGS data set.

CONCLUSIONS

Significant changes in the parameters among three time periods were found in all of the models tested, using a number of model forms and dependent variables. The instability of the parameters might be due to omission of some relevant variable that is attributable for that change. The omitted variable might be identified by correlating several important variables with the residuals from the mis-specified models (for details see Rayamajhi 1996). In the timerep band, most important variables included in a growth-and-yield model were either isolated or accounted for in the model form. The only important variable, which was left out is climate. Inclusion of climate might help stabilize the parameters of these models. The other way of avoiding the instability would be to only project growth-and-yield for short periods of time.

Table 2—Results of LR tests (together and separately for each coefficient) on three timereps using increment models [equation (4) and (5)]

Hypothesis	BAI	SVI	Parameters tested
	F-Value/ Prob > F	F-Value/ Prob > F	
1[a]. Full model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{21}, \beta_{22}, \beta_{23}$) Reduced model ($\beta_0, \beta_1, \beta_2$) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$ H _A : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$ $\beta_{21} \neq \beta_{22} \neq \beta_{23} \neq \beta_2$	2.7131/ 0.0572	2.7379/ 0.0161	all β_0, β_1 & β_2
[b]. Full model ($\beta_{01}, \beta_{02}, \beta_{03}, \beta_1, \beta_2$) Reduced model ($\beta_0, \beta_1, \beta_2$) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$ H _A : $\beta_{01} \neq \beta_{02} \neq \beta_{03} \neq \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$	0.1913/ 0.9413	1.5081/ 0.2045	only β_0
[c]. Full model ($\beta_0, \beta_{11}, \beta_{12}, \beta_{13}, \beta_2$) Reduced model ($\beta_0, \beta_1, \beta_2$) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$ H _A : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} \neq \beta_{12} \neq \beta_{13} \neq \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$	2.9313/ 0.0238	3.7545/ 0.0066	only β_1
[d]. Full model ($\beta_0, \beta_1, \beta_{21}, \beta_{22}, \beta_{23}$) Reduced model ($\beta_0, \beta_1, \beta_2$) H ₀ : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_2$ H _A : $\beta_{01} = \beta_{02} = \beta_{03} = \beta_0$ $\beta_{11} = \beta_{12} = \beta_{13} = \beta_1$ $\beta_{21} \neq \beta_{22} \neq \beta_{23} \neq \beta_2$	0.5067/ 0.7309	3.1693 0.0164	only β_2
$BAI = 1.44608 + .03009 \left(\frac{NI}{AI} \right) + .59807 \left(\frac{SI}{AI} \right) \text{ and}$			
$SVI = 230.02059 + 1.31047 \left(\frac{NI}{AI} \right) + 1.13628 \left(\frac{SVI}{AI} \right)$			

Table 3—Likelihood F-ratio and probability levels for testing the sensitivity of individual parameters in the model to changes over time while all other parameters are unchanged for the band data set

Parameter	F-ratio	P > F
a_0	76.5699	0.0001
a_1	96.9675	0.0001
b_0	72.5176	0.0001
c_0	86.9414	0.0001
c_1	89.0613	0.0001
c_2	85.3636	0.0001

Table 4—Generalization of timerep analysis using pseudo timerep data sets and the increment model [equation (4)]

Age range	N	F-value	Prob >F	Conclusion
25-34	142	2.0894	0.0585	Significant at 10% level
35-44	138	2.2212	0.0450	Significant difference
45-54	123	1.8736	0.0913	Significant at 10% level
55-64	143	3.3978	0.0038	Significant difference
65-74	116	1.9939	0.0728	Significant at 10% level
75-84	79	1.8961	0.0936	Significant at 10% level
85-94	55	0.8620	0.5921	No difference

ACKNOWLEDGMENT

The following organizations and their responsible individuals deserve our sincere thanks for the use of their land and for their assistance in installing, treating, and maintaining the RLGS: T.R. Miller Mill Company; USDA Forest Service, Southern Research Station; USDA Forest Service—National Forests System; Champion International Corporation; Cyrene Turpentine Company; Eglin Air Force Base; Florida Forest Service; Gulf States Paper Corporation; John Hancock Companies; International Paper Company; Kaul Trustees; Kimberly-Clark Corporation;

Mobile County (Alabama) School Board; Resource Management Service; Wefel Trust; and AmSouth Bank (Birmingham, AL). This study was funded by the U.S. Department of Agriculture, Forest Service, Southern Global Change Program.

LITERATURE CITED

- Boyer, William D.** 1991. Longleaf pine. In: Burns, R.M.; Honkala, B.H. tech. coords. *Silvics of North America*, Vol. I, Conifers. Agric. Handb. 654. U.S. Department of Agriculture, Forest Service, Washington, DC: 405-412.
- Chow, G.C.** 1960. Tests of equality between sets of coefficients in two linear regressions. *Econometrica*. 28(3): 591-605.
- Clutter, Jerome L.** 1963. Compatible growth and yield models for loblolly pine. *Forest Science*. 9(3): 354-370.
- Clutter, Jerome L.; Jones, Earle P., Jr.** 1980. Prediction of growth after thinning of old-field slash pine plantations. Res. Pap. SE-217. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station: 14 p.
- Kush, John S.; Meldahl, Ralph S.; Dwyer, Stephen P.; Farrar, Robert M., Jr.** 1987. Naturally regenerated longleaf pine growth and yield research. In: Phillips, Douglas R., comp. *Proceedings of the fourth biennial southern silvicultural research conference*. 1986 November 4-6; Atlanta, GA: Gen. Tech. Rep. SE-42. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station: 343-344.
- Quicke, Harold E.; Meldahl, Ralph S.; Kush, John S.** 1994. Basal area growth of individual trees: A model derived from a regional longleaf pine growth study. *Forest Science*. 40(3): 528-542.
- Rayamajhi, Jyoti N.** 1996. Productivity of natural stands of longleaf pine in relation to climatic factors. Auburn University, AL: Auburn University. 192 p. Dissertation.
- Somers, Greg L.; Farrar, Robert M., Jr.** 1991. Biomathematical growth equations for natural longleaf pine stands. *Forest Science*. 37(1): 227-244.
- Zahner, Robert.; Saucier, Joseph R.; Myers, Rick K.** 1989. Tree-ring model interprets growth decline in natural stands of loblolly pine in southeastern United States. *Canadian Journal of Forest Research*. 19: 612-621.

SIMULATING THE UNEVEN-AGED MANAGEMENT OF SOUTHERN LOBLOLLY PINE: FEATURES OF THE SOUTHPRO COMPUTER PROGRAM

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Abstract—The SouthPro software simulates the growth of uneven-aged stands of mixed pines and hardwoods in the Southern United States. It is based on a density-dependent matrix growth model calibrated from permanent plots of the SO-FIA database. The model was calibrated with out-of-sample plots and predictions of undisturbed growth. The model keeps track of the number of trees in 13 diameter classes for pines and other softwoods, soft hardwoods, and hard hardwoods. Regeneration, growth, and mortality are affected by stand state and by the interaction between trees of different species and size. Stand state also affects tree form and, thus, the standing volumes of saw logs and pulpwood. SouthPro also monitors the Shannon diversity of stands by tree species and size and the income obtained from harvests. The SouthPro software is an Excel add-in, with features to facilitate data input, error checking, and output interpretation. The output of SouthPro is tabular and graphical. The effects of different residual diameter distributions on diversity and income in loblolly pine stands, given different initial conditions, is discussed.

INTRODUCTION

The 50 million acres of loblolly-shortleaf pine forests in the Southeastern United States contain over two-thirds of the region's standing merchantable volume (Powell and others 1993). These forests are generally managed with even-aged silvicultural techniques; yet there is growing interest in uneven-aged management, particularly among the private nonindustrial owners who control over two-thirds of this acreage (Murphy and Farrar 1983), and within government land management agencies.

Uneven-aged silviculture of this forest type has much to recommend it. It has been used successfully to manage individual stands for more than 50 years (Reynolds and others 1984, Baker 1986). It produces butt logs of comparable volume and form class and of higher grade than logs from even-aged plantations (Guldin and Fitzpatrick 1991), and sawtimber board-foot volumes are at least as high as those from even-aged stands (Guldin and Baker 1988). Williston (1978) notes that uneven-aged management may be particularly appealing to landowners who can afford little capital investment, have few acres to manage, wish to maintain a continuous forest cover, or who desire a more frequent income. He also points out that it is better adapted to steep slopes, fragile soils, very dry sites, and high water tables; maintains genetic variability; and provides the structural diversity favored by many game and nongame species. In addition, it can be used to rehabilitate understocked stands (Baker 1986, Baker 1989, McLemore 1983) or to approximate advanced successional stages (Guldin 1996). Moreover, the present net value of returns from uneven-aged silviculture can be higher than that from even-aged silviculture when interest rates are high (Chang 1981, Redmond and Greenhalgh 1990) or when the initial stand is not valued as a cost (Guldin and Guldin 1990). Finally, selection harvests disturb a smaller percentage of the stand than clearcuts (Kluender and Stokes 1994) and,

for many people, are likely to be less esthetically displeasing.

Uneven-aged management of these forests is not a panacea, however. It will likely require greater technical expertise than even-aged management (Hotvedt and Ward 1990). It is more difficult to regulate harvests on an even-flow basis (Williston 1978). And uneven-aged stands are not as efficient at producing pulpwood (Baker and others 1991, Guldin and Baker 1988, Williston 1978). Nevertheless, growing public sentiment against even-aged silviculture and the increasing recognition of the importance of ecosystem management principles suggest that the acreage of loblolly pine managed with uneven-aged techniques is likely to increase.

The SouthPro program was developed to help forest owners and managers evaluate the economic and ecological consequences of alternative uneven-aged management regimes for loblolly pines. This paper discusses the growth model used by SouthPro, describes the SouthPro software, and provides examples illustrating some of its features.

THE GROWTH MODEL

The growth model of SouthPro is an extension of the matrix model of Lu and Buongiorno (1993). The growth matrix describes the effects of standing trees on the ingrowth of different species and on the probabilities that trees of a given size-class and species will stay in their present size-class, grow into the next larger size-class, or die during a 1-year interval. To express the effect of stand density on tree growth and mortality, the growth matrix is a function of the residual stand basal area (Solomon and others 1986, Buongiorno and others 1995, Lin and others 1996). The growth matrix also varies with site productivity (Lin and others, in press).

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Data

Data from the Southern Forest Inventory and Analysis (SO-FIA) database were used to estimate equations for ingrowth, upgrowth, and mortality. The complete SO-FIA database consists of approximately 18,000 permanent point plots. Plots with more than one age class in the major or dominant species are classified as "mixed age." Site productivity ranges from 1 to 7. A rating of site 7 is used for a potential yield of less than 20 cubic feet/acre/year, site 6 for 20 to 49, site 5 for 50 to 84, site 4 for 85 to 119, site 3 for 120 to 164, site 2 for 165 to 224, and site 1 for more than 225 cubic feet/acre/year.

Calibration plots—A subset of 991 permanent plots was selected for use in calibrating the growth model. It consisted of all the plots that: (1) had been remeasured, (2) were classified as being of the loblolly pine forest type in the previous inventory, (3) were classified as "mixed age" in the previous inventory, and (4) showed no evidence of having been regenerated artificially. Each plot had been measured twice between 1978 and 1994, at an average interval of 7.3 years.

Species groups and size-classes—To estimate the model, trees were grouped as in the SO-FIA species group classifications: pines and other softwoods, soft hardwoods, and hard hardwoods. Within each species group, trees were also classified into 13 diameter at breast height (d.b.h.) size-classes. Size-classes ranged from 2 to 26+ inches at 2-inch intervals. Size-class 2 (the smallest) included trees with diameters ranging from 1 inch to less than 3 inches. Size-class 26+ (the largest) included all trees 25 inches in diameter and larger.

Model Estimation

Each plot gave one observation on ingrowth, upgrowth, and mortality. Ingrowth was defined as the number of trees per acre per year that became larger than 1 inch d.b.h. The transition probabilities and ingrowth were also expressed per year. The parameters of the equations were obtained by multiple regression, across all plots. The upgrowth probability is a function of tree species, residual stand basal area, tree diameter, and site productivity class. The mortality probability is a function of tree species, residual stand basal area, and tree diameter. For a given species, the annual ingrowth is a linear function of the stand basal area and the number of trees of that species (Lin and others, in press).

Model Validation

The growth model was developed from 80 percent of the plots selected randomly from the 991 available. To test its accuracy, the model was then used to predict the state of the remaining plots at the time of their second measurement, given their state at the first inventory and possible harvest. A series of t-tests showed that the predicted mean number of trees in each size-class category was not significantly different from the observed mean, at the 5 percent significance level.

To test the long-term behavior of the growth model, it was used to predict the growth of stands, without harvest, over

three centuries. Simulations were performed for a high productivity site and a low productivity site. In both cases, the model predictions were consistent with ecological studies of climax forests of this type (Lin and others, in press).

Tree Volumes and Values

To calculate economic returns, SouthPro multiplies the pulpwood and sawtimber volumes of the average tree in each size-class and species group by the appropriate price per unit of volume and then sums the resulting individual tree values over all trees. For saplings (trees with d.b.h. less than 5 inches), tree values are set to zero. For trees with d.b.h. equal to or greater than 5 inches, the individual tree volumes of sawtimber and pulpwood depended on tree and stand conditions. SouthPro includes equations for calculating tree heights, saw log lengths, pulpwood volumes, and saw log volumes (Lin and others, in press).

The total height of a tree is a function of species, d.b.h., residual stand basal area, and site productivity. The equations were estimated from more than 18,000 trees on the 991 plots used to develop the growth model. Approximately 12,000 trees of saw log length were used to estimate the equations for saw log length, a function of species, d.b.h., and total tree height. SouthPro recognizes two potential sources of pulpwood: pulpwood trees and the tops of sawtimber trees. The volume of pulpwood trees is a function of species, d.b.h., total tree height, and saw log length, based on Clark and Souter's data (1994). The volume of saw logs in cubic feet is a function of species, d.b.h., total tree height, and saw log length, also based on the data of Clark and Souter (1994). SouthPro used Koch's conversion table (1972, p. 1593) to convert sawtimber cubic-foot volumes to board-foot volumes.

THE SOUTHPRO SOFTWARE

SouthPro was written in the Visual Basic programming language and compiled as an add-in program for Microsoft Excel. Once installed and activated, it can be accessed whenever Excel is running, giving the user simultaneous access to both its features and those of Excel. It is available in a Windows and a Macintosh version, each version featuring menu-driven commands and dialog-box interfaces typical of its respective environment. The SouthPro users manual describes the program's features and includes a step-by-step tutorial designed to help new users learn the program (Schulte and others, in press).

Input Data

SouthPro provides an Input Data worksheet for entering the data for a simulation. Required input data include the initial stand state, target stand state(s), cutting cycle parameters, loblolly pine site index, stumpage prices, interest rate, and fixed costs of administration and hardwood control.

The initial stand state or distribution is defined as the number of live trees per acre in each species group and size-class at the start of a simulation. Similarly, a target stand state is the desired number of live trees per acre in each species-size category after a harvest. All trees in excess of the target number in any species-size category

are cut at each harvest. To accommodate the simulation of silvicultural systems with an intermediate harvest, SouthPro allows users to enter two target states. In addition, to assist users who practice the BDq method of stand regulation (see for example Baker and others 1996, Farrar 1996), SouthPro includes a BDq Distribution Calculator. It can be used to calculate BDq distributions and copy them to the Input Data worksheet as either initial or target stand states for each species group. Before performing a simulation, SouthPro checks the input data for errors.

Output Worksheets

SouthPro writes the output from each simulation to two worksheets. The Stand Development output worksheet shows, for each year, the size distribution and total basal area of each species group and the Shannon indices of species and size diversity based on the basal area of trees at the beginning of each year and after each harvest.

The Products output worksheet shows, for each harvest year: the basal area cut, the volume of pulpwood and sawtimbers removed by species group, the gross income generated, and the net present value of the harvest. It also reports the total net present value of the stand and its mean annual production in terms of basal area cut and volumes harvested.

Output Charts

SouthPro can generate six different types of charts. Each chart type has a separate dialog box for selecting the years and data series to be plotted. Data on a Stand Development worksheet can be used to create Diversity Indices charts, Size Distribution charts, or Stand Basal Area charts, for pre- and postharvest stand conditions. Diversity Indices charts show changes in the Shannon index of species and/or size diversity. Size Distribution charts show the number of live softwood, soft hardwood, and/or hard hardwood trees in each size-class. Stand Basal Area charts show the per-acre basal area of softwoods, soft hardwoods, hard hardwoods and/or the entire stand.

Data on a Products worksheet can be used to create Basal Area Cut charts, Gross Income charts, or Volume Cut charts, for selected harvest years. Basal Area Cut charts show the total basal area cut, in square feet per acre. Gross Income charts show the gross income generated, in dollars per acre. Volume Cut charts show the cubic-foot volume of pulpwood and/or sawtimber removed from each of the three species groups.

Setup Files

SouthPro also provides dialog boxes for creating, deleting, and loading setup files. Setup Files are collections of related input data, stored together on a Setup File worksheet. Setup files may contain input data for initial stand states, target stand states, cutting cycle parameters, stumpage prices, and fixed costs. Once sets of input data have been stored as setup files, they can be loaded in various combinations to either the Input Data worksheet to run a single simulation or to SouthPro's Batch File

worksheet to run a batch of simulations. SouthPro allows batches of up to 500 simulations to be run sequentially.

Stock-and-Cut Tables and Marking Guides

SouthPro can generate stock-and-cut tables from any user-selected preharvest and target distribution. SouthPro's stock-and-cut tables show, for the selected preharvest and target distributions, the cut and residual stand distributions, by species and size. The tables also list the basal area, cubic-foot volume, and board-foot volume of the trees in each species-size category. Additionally, they show the annual cubic-foot growth of stands with the preharvest, target, and residual distributions. Lastly, for each stock-and-cut table, SouthPro calculates the corresponding marking guide: the number of trees to cut, for each species group, in each of four product classes: pulpwood, small sawtimbers, medium sawtimbers, and large sawtimbers.

EXAMPLE APPLICATION OF SOUTHPRO

SouthPro can be used to simulate a wide range of management regimes, from doing nothing to cutting at different intensities and timings. As an example, we summarize the effects of three management regimes, defined by a cutting cycle and target distribution, on stand characteristics and revenues over 120 years. The simulations were performed for a high (site class 3) and a low (site class 5) productivity site. The initial stand state for each simulation was the average species-size distribution at the second inventory of calibration plots with the same site class (table 1), and each simulation used a cutting cycle of 6 years.

Table 1—Average number of trees per acre on site 3 and site 5 calibration plots at the second inventory, by species group and size-class

D.b.h.	Site 3			Site 5			
	SW	SH	HH	D.b.h.	SW	SH	HH
<i>in</i>				<i>in</i>			
2	69.4	147.2	144.6	2	157.8	145.3	290.3
4	35.4	41.7	35.2	4	67.6	23.3	45.8
6	21.2	15.0	11.2	6	40.9	8.8	9.8
8	16.1	6.6	5.9	8	25.9	4.7	6.0
10	11.4	3.0	3.0	10	9.1	1.8	1.4
12	10.2	1.5	1.7	12	6.8	0.7	1.6
14	7.9	0.6	1.3	14	3.9	0.3	1.2
16	5.2	0.3	0.9	16	1.4	0.1	0.0
18	3.3	0.2	0.5	18	0.7	0.1	0.7
20	1.6	0.1	0.3	20	0.2	0.0	0.3
22	0.8	0.0	0.2	22	0.1	0.0	0.2
24	0.4	0.0	0.1	24	0.1	0.0	0.2
26+	0.3	0.0	0.1	26+	0.1	0.0	0.1

D.b.h. = diameter at breast height.

SW = pines and other softwoods, SH = soft hardwoods, HH = hard hardwoods.

Target States

The target distribution for the “diversity” regime (table 2) had a merchantable (5 inches d.b.h. and larger) basal area of 55 square feet per acre. The basal area was distributed evenly among each size-class and species group, giving it the largest possible values of Shannon’s diversity index by species (1.10) and size (2.56).

The “income” regime used as a target for softwoods a BDq distribution with a merchantable basal area of 55 square feet per acre, a maximum diameter of 15 inches, and a q-ratio of 1.3. It assumed that all hardwoods were cut at each harvest and that no submerchantable softwoods were cut (table 2).

The “compromise” regime used BDq distributions for all three species groups. Each distribution had a maximum diameter of 15 inches and a q-ratio of 1.3. The merchantable basal areas for the softwoods, soft hardwoods, and hard hardwoods were 47.5, 2.5, and 5 square feet per acre, respectively. No submerchantable tree was cut (table 2).

Simulation Results

Table 3 shows the present value of gross income, timber productivity, and diversity values, by site and management regime. A real interest rate of 3 percent per year and 1996 average stumpage prices for Southeastern States (Timber Mart-South, 1st Quarter 1997) were used to calculate the present value of gross income. Gross, rather than net, revenues were used because costs were not known. However, sensitivity analysis showed that, in each case, the net present value would decrease by \$6 for each

Table 2—Target number of trees per acre after a cut, by species group and size-class

	Diversity guide			Income guide			Compromise guide			
	D.b.h.	SW	SH	HH	SW	SH	HH	SW	SH	HH
<i>in</i>										
2	9.0	9.0	9.0	*	0.0	0.0	*	*	*	
4	6.9	6.9	6.9	*	0.0	0.0	*	*	*	
6	5.3	5.3	5.3	36.2	0.0	0.0	31.3	1.6	3.3	
8	4.1	4.1	4.1	27.8	0.0	0.0	24.0	1.3	2.5	
10	3.1	3.1	3.1	21.4	0.0	0.0	18.5	1.0	1.9	
12	2.4	2.4	2.4	16.5	0.0	0.0	14.2	0.7	1.5	
14	1.9	1.9	1.9	12.7	0.0	0.0	10.9	0.6	1.2	
16	1.4	1.4	1.4	0.0	0.0	0.0	0.0	0.0	0.0	
18	1.1	1.1	1.1	0.0	0.0	0.0	0.0	0.0	0.0	
20	0.8	0.8	0.8	0.0	0.0	0.0	0.0	0.0	0.0	
22	0.7	0.7	0.7	0.0	0.0	0.0	0.0	0.0	0.0	
24	0.5	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	
26+	0.4	0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	

D.b.h. = diameter at breast height.

* = Size-classes without a cut, SW = pines and other softwoods, SH = soft hardwoods, HH = hard hardwoods.

Table 3— Income, timber productivity, and Shannon diversity of stands managed under three different harvest regimes, over 120 years

Site class:	Diversity guide		Income guide		Compromise guide	
	3	5	3	5	3	5
Present value of gross income (\$/acre):	3556	1149	6230	3796	5947	3457
Basal area cut (ft ² /acre/year):	1.9	1.6	3.0	2.8	3.5	3.0
Annual productivity (ft ³ /acre):						
Softwood sawtimber	22.3	12.1	78.2	62.6	61.6	47.7
Softwood pulpwood	3.7	3.7	2.2	3.5	1.7	2.9
Soft hardwood sawtimber	1.7	0.8	0.4	0.1	2.0	1.7
Soft hardwood pulpwood	1.1	0.3	1.1	0.5	10.1	3.7
Hard hardwood sawtimber	5.0	2.4	0.8	0.3	3.5	2.0
Hard hardwood pulpwood	2.7	1.3	1.3	0.7	7.6	5.7
Average diversity:						
Species	1.09	1.08	0.09	0.10	0.96	0.96
Size	2.50	2.49	2.01	2.00	2.04	2.01

additional \$1 in fixed costs. The present value produced by the income guide was 1.7 times larger than that produced by the diversity guide for good sites and over 3 times larger for poor sites; whereas the present value produced by the compromise guide was only 5 percent less than that produced by the income guide for good sites and 9 percent less for poor sites. Regardless of regime, the present values were about \$2400 per acre larger on good sites than on poor sites.

For both sites, the income and compromise guides cut similar amounts of basal area and volumes, but the composition of the harvest was quite different, with the compromise guide producing a much higher proportion of hardwoods. The diversity guide also produced a high proportion of hardwoods, but cut considerably less basal area and volume than the other guides.

The diversity guide produced the largest average diversity of tree size and species over 120 years. The species diversity obtained by the income guide was very low, dropping to zero for post-harvest stand states. By leaving some trees in each species at all times, the compromise guide produced stands with species diversity approaching that produced by the diversity guide. The income and compromise guides each resulted in stands with moderate size diversity.

Summary Charts

Figures 1 and 2 illustrate two of SouthPro's charting options. Figure 1 shows the basal area growth of a stand managed with the compromise regime on a poor site. The

basal area of softwoods increased over the first 30 years, then declined thereafter. The basal area of the soft hardwoods and hard hardwoods increased gradually over the 120-year period, with hard hardwoods remaining dominant to soft hardwoods throughout.

Stand Basal Area - Compromise Regime on a Poor Site

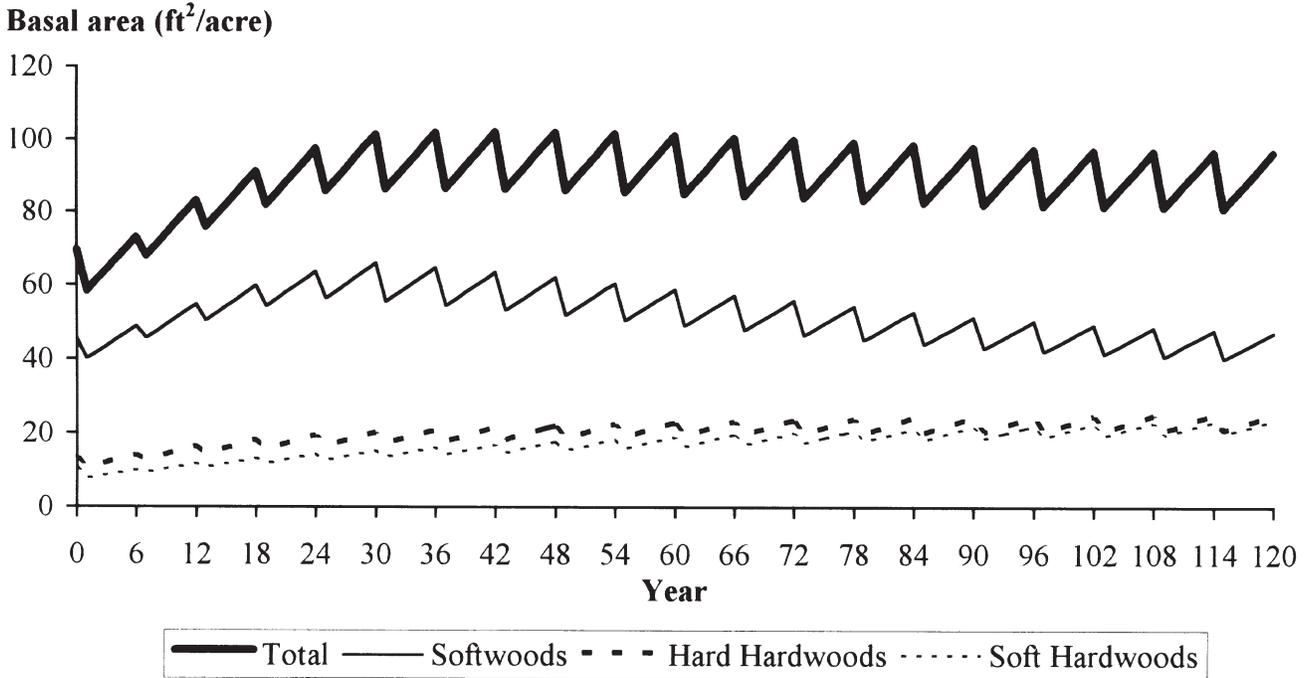


Figure 1—Basal area growth of a loblolly pine stand managed with the compromise regime on a poor site.

Softwood Size Distributions - Compromise Regime on a Poor Site

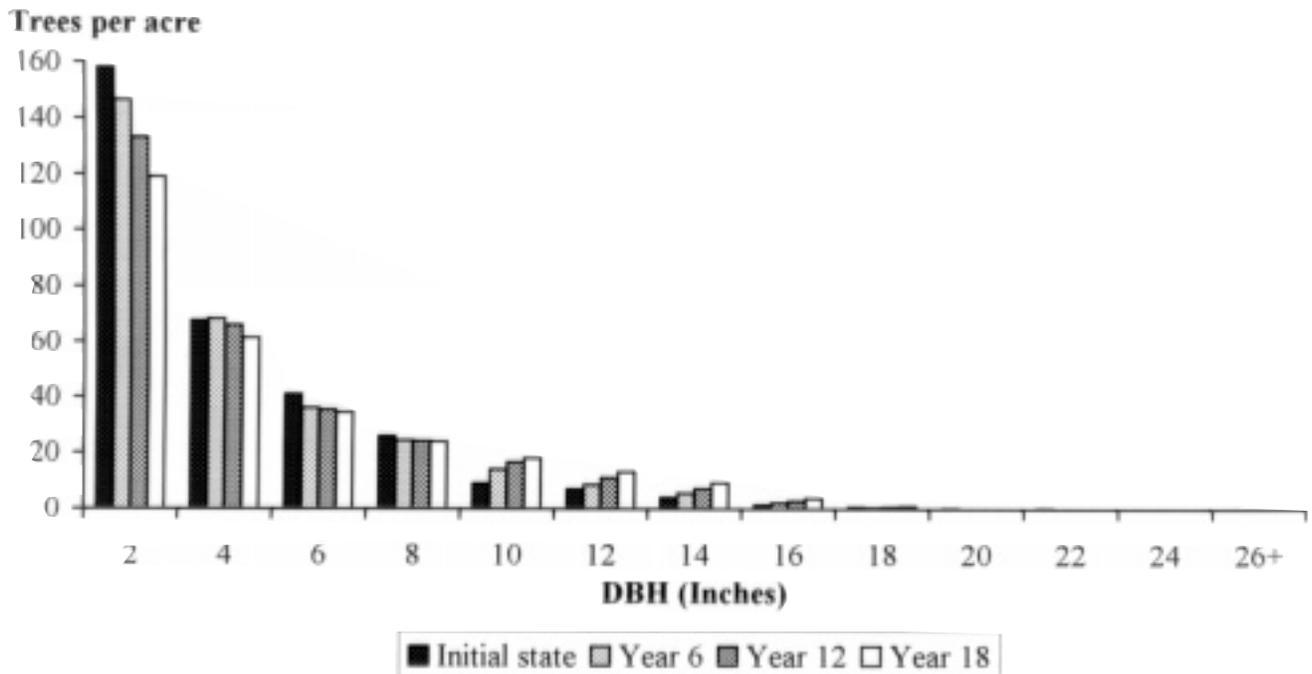


Figure 2—Preharvest size distributions of softwoods for a stand managed with the compromise regime on a poor site.

Figure 2 shows, for the same stand, the number of softwood trees per acre in each size-class for the first four pre-harvest states. The number of softwoods in the four smallest size-classes decreased over this interval, whereas the number of softwoods in the remaining size-classes increased. SouthPro can produce similar charts for the hardwood species.

ACKNOWLEDGMENT

This work has been supported in part by the U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Wood Utilization for Ecosystem Management Project, and by the School of Natural Resources, University of Wisconsin, Madison. We thank Larry Westrick for his assistance with the FIA database and the personnel of the U.S. Department of Agriculture, Forest Service, Southern Region, especially Dave Meriwether and Alex Clark, for their comments and collaboration.

LITERATURE CITED

- Baker, James B.** 1986. The Crossett Farm forestry forties after 41 years of selection management. *Southern Journal of Applied Forestry*. 10(4): 233-237.
- Baker, James B.** 1989. Recovery and development of understocked loblolly-shortleaf pine stands. *Southern Journal of Applied Forestry*. 13(3): 132-139.
- Baker, James B.; Cain, Michael D.; Guldin, James M. [and others].** 1996. Uneven-aged silviculture for the loblolly and shortleaf pine forest cover types. Gen. Tech. Rep. SO-118. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 65 p.
- Baker, James B.; Guldin, James M.; Guldin, Richard W.** 1991. Natural regeneration methods for loblolly and shortleaf pines. *Forest Farmer*. 50(3): 59-63.
- Buongiorno, Joseph; Reyron, Jean-Luc; Houllier, Francois; Brucamacchie, Max.** 1995. Growth and management of mixed-species, uneven-aged forests in the French Jura: Implications economic returns and tree diversity. *Forest Science*. 41(3): 397-429.
- Chang, Sun Joseph.** 1981. Determination of the optimal growing stock and cutting cycle for an uneven-aged stand. *Forest Science*. 27(4): 739-744.
- Farrar, Robert M., Jr.** 1996. Fundamentals of uneven-aged management in southern pine. Moser, W. Keith; Brennan, Leonard A., eds. Misc. Pub. No. 9. Tallahassee, FL: Tall Timbers Research Station. 68 p.
- Guldin, James M.** 1996. The role of uneven-aged silviculture in the context of ecosystem management. *Western Journal of Applied Forestry*. 11(1): 4-12.
- Guldin, James M.; Baker, James B.** 1988. Yield comparisons from even-aged and uneven-aged loblolly-shortleaf pine stands. *Southern Journal of Applied Forestry*. 12(2): 107-114
- Guldin, James M.; Fitzpatrick, Michael W.** 1991. Comparison of log quality from even-aged and uneven-aged loblolly pine stands in South Arkansas. *Southern Journal of Applied Forestry*. 15(1): 10-17.
- Guldin, James M.; Guldin, Richard W.** 1990. Economic assessments of even-aged and uneven-aged loblolly-shortleaf pine stands. In: Hickman, Clifford A., comp. *Proceedings of the southern forest economics workshop on evaluating even- and all-aged timber management options for southern forest lands*; 1990 March 29-30; Monroe, LA. Gen. Tech. Rep. SO-79. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station: 55-64.
- Hotvedt, James E.; Ward, Keith B.** 1990. A dynamic programming optimization model for uneven-aged loblolly-shortleaf pine stands in the mid-South. In: Hickman, Clifford A., comp. *Proceedings of the southern forest economics workshop on evaluating even- and all-aged timber management options for southern forest lands*; 1990 March 29-30; Monroe, LA. Gen. Tech. Rep. SO-79. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station: 35-52
- Kluender, Richard A.; Stokes, Bryce J.** 1994. Productivity and costs of three harvesting methods. *Southern Journal of Applied Forestry*. 18(4):168-174.
- Lin, Ching-Rong; Buongiorno, Joseph; Prestemon, Jeff; Skog, Kenneth.** [In press]. A growth model for uneven-aged loblolly pine stands, with simulations and management implications.
- Lin, Ching-Rong; Buongiorno, Joseph; Vasievich, Mike.** 1996. A multi-species, density-dependent matrix growth model to predict tree diversity and income in northern hardwood stands. *Ecological Modeling*. 91: 193-211.
- Lu, Hsien-Chih; Buongiorno, Joseph.** 1993. Long- and short-term effects of alternative cutting regimes on economic rotations and ecological diversity in mixed-species forests. *Forest Ecology and Management*. 58: 173-192.
- McLemore, B.F.** 1983. Recovery of understocked, uneven-aged pine stands and suppressed trees. In: Jones, Earle P., Jr. ed. *Proceedings of the second biennial southern silvicultural research conference*; 1982 Nov. 4-5; Atlanta, GA. Gen. Tech. Rep. SE-24. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station: 226-229.
- Murphy, Paul A.; Farrar, Robert M., Jr.** 1983. Sawtimber volume predictions for uneven-aged loblolly-shortleaf pine stands on average sites. *Southern Journal of Applied Forestry*. 7: 45-50.
- Powell, D.S.; Faulkner, J.L.; Darr, D.R. [and others].** 1993. Forest resources of the United States, 1992. Gen. Tech. Rep. RM-234. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Forest and Range Experiment Station. 132 p.
- Redmond, Claire, H.; Greenhalgh, Richard.** 1990. An economic analysis of even-aged versus uneven-aged management on nonindustrial private land in southern Arkansas. In: Hickman, Clifford A., comp. *Proceedings of the southern forest economics workshop on evaluating even- and all-aged timber management options for southern forest lands*; 1990 March 29-30; Monroe, LA. Gen. Tech. Rep. SO-79. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station: 69-79.
- Reynolds, R.R.; Baker, James B.; Ku, Timothy T.** 1984. Four decades of selection management on the Crossett Farm Forestry Forties. *Bull.* 872. Fayetteville, AR:

University of Arkansas, Division of Agriculture,
Agricultural Experiment Station. 43 p.

Schulte, Benedict; Buongiorno, Joseph; Lin, Ching-Rong; Skog, Kenneth. [In press]. SouthPro: A computer program for uneven-aged management of loblolly pine stands.

Solomon, Dale S.; Hosmer, Richard A.; Hayslett, Homer T., Jr. 1986. A two-stage matrix model for predicting

growth of forest stands in the Northeast. *Canadian Journal of Forest Research*. 16(3): 521-528.

Timber Mart-South. 1st Quarter 1997. Highlands, NC: Timber Marts, Inc.

Williston, Hamlin L. 1978. Uneven-aged management in the loblolly-shortleaf pine type. *Southern Journal of Applied Forestry*. 2(3): 78- 82.

AN INDIVIDUAL TREE BASAL AREA GROWTH MODEL FOR UNEVEN-AGED STANDS OF SHORLEAF PINE (*PINUS ECHINATA* Mill.) IN THE OUACHITA MOUNTAINS IN ARKANSAS AND OKLAHOMA

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Abstract—A distance-independent individual tree basal area growth model was developed for natural, uneven-aged stands of shortleaf pine in the Ouachita Mountains of Oklahoma and Arkansas; the data came from permanent inventory plots located on private industrial forests of the Deltic Farm & Timber Company, Inc. There were 452 plots established in 1965-66; in 1983 and 1988 there were, respectively, 132 and 236 additional plots established in order to cover all forest conditions found in the region. Only plots that have basal area between 30 and 90 ft²/acre were used for this study. This resulted in 3,663 observations which were divided into two subsets: one for calibration (2,692 observations) and the second for validation (971 observations). A potential-modifier type model was chosen as the best individual tree basal area growth model for uneven-aged stands. The model had the lowest and most uniform deviations when predicted values were compared to observed values. The model explained 44.01 percent of the total variations and had a mean square error value of 0.000069. The model tended to overestimate basal area growth of trees with d.b.h. equal to or larger than 16 inches. The overestimation might have been due to having few sample trees in the larger d.b.h. classes. This model can be incorporated into a system including ingrowth and mortality functions in order to assess global stand growth dynamics in uneven-aged stand conditions. The model should be limited to conditions similar to the area of study.

INTRODUCTION

Modeling growth and yield of shortleaf pine (*Pinus echinata* Mill.) in the Southern United States has concentrated on even-aged stands. However, recently there are more concerns in managing forest stands by adopting an uneven-aged system in order to respond to an ever increasing desire of noncommodity products from public and private forests. Baker and others (1996) developed silvicultural guidelines for uneven-aged stands of loblolly and shortleaf pine stands to respond to new trends. In order to facilitate planning, a model for predicting future yields in such stands needs to be used. A literature survey showed little has been done in terms of individual tree modeling for uneven-aged stands of shortleaf in the Southern United States. However, a recent publication presents an individual-tree basal area growth model for loblolly pine (*P. taeda* L.) stands in Arkansas and Louisiana (Murphy and Shelton 1996).

Basal area of individual trees is used in mathematical models to estimate the volume of standing trees by measuring diameter at breast height (d.b.h.) and total or merchantable height. The aggregation of tree basal areas on a given unit area gives an indication of the degree of stocking, a useful description of stands and their development over time. The availability of sophisticated data analysis procedures has made the development of individual tree basal area models feasible.

The objectives of the study were (1) to develop a distance-independent individual-tree basal area growth model for natural uneven-aged stands of shortleaf pine in the interior highlands of Arkansas and Oklahoma, (2) to validate the model with data from an independent data set, and (3) to

make recommendations regarding use of the model. The model predicts future individual-tree basal area growth for trees with d.b.h. equal to or greater than 5 inches.

DATA

The data came from 0.2-acre continuous forest inventory (CFI) plots established by the Deltic Farm & Timber Company, Inc., on forest lands located in the Ouachita Mountains of Arkansas. A total of 452 permanent plots were established between 1965-66 and were remeasured every 5 years until 1993. There were 132 and 236 additional plots installed in 1983 and 1988, respectively, in order to get a more representative data sample of all stand conditions found in the study area. A total of 820 permanent plots were installed in the uneven-aged stands, from which the data for the current study were drawn.

Individual tree records were maintained for all trees with d.b.h. 5.0 inches and larger. Pertinent information collected included (1) d.b.h. to the nearest 0.1 inch, (2) total merchantable height, (3) saw log height, (4) merchantable height for sawtimber trees, (5) tree history, and (6) crown position. Site index at base age 50 was estimated for each plot for shortleaf pine. Six basal area classes and four site index classes were selected to cover all major conditions found in the region of study. Stand basal area was estimated by aggregating all individual shortleaf basal area estimates per plot by class, and then multiplying by an expansion factor of five to obtain values on a per-acre basis. Summary statistics are provided for site index class and merchantable basal area class combinations for the shortleaf pine data set (table 1).

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Table 1—Summary of treatment combinations comprising six classes of basal area per acre and four classes of site index (base age 50) for Deltic data set

Variable	Class	
	Range	Midpoint
Merchantable	<11	-
Basal area (ft ² /acre)	11 - 29	20
	30 - 49	40
	50 - 69	60
	70 - 89	80
	>89	-
Site Index (Base age 50)	<45	40
	46 - 55	50
	56 - 65	60
	>65	70

Table 1 illustrates a potential of 24 treatment combinations, similar to Murphy and others (1985), with slight variations. Sawtimber basal area as used by Murphy and others (1985) was not included when creating the combinations because the purpose of the study was to develop an individual-tree basal area growth model without regard to product classification. Plots retained for model development were in natural uneven-aged stands with relatively uniform spacing of trees. All plots had at least 70 percent of basal area in shortleaf pine and less than 10 percent mortality of initial plot basal area. There had been no harvesting or thinning activities during the entire growth period.

Elimination of all the plots that did not meet this selection criteria left 319 plots. Many of these plots had basal area less than 30 or more than 90 ft² per acre. However, very few plots were present in merchantable basal area classes less than 11 ft² per acre and classes greater than 90 ft² per acre. The same is true for all combinations of site index less than 45 and greater than 65. Baker and others (1996) suggested that uneven-aged stands having less than 30 ft² of basal area per acre are understocked while those with more than 90 ft² are overstocked. Therefore, the data were balanced by further eliminating plots having less than 30 or more than 90 ft² basal area per acre of shortleaf pine at the beginning of the growth period.

The remaining plots were randomly assigned identification numbers. An unbalanced condition of data was expected since the data were collected from ordinary forest inventory operations, not from controlled permanent research plots. When establishing research plots, all forest conditions are represented with equal frequency in each condition, consequently requiring no further data balancing. For each merchantable basal area class, the maximum number of plots allowed was restricted to 25. Because the data were mainly concentrated in a few site index and basal area classes, a restricted set of observations was randomly

chosen to obtain a more uniform sample across the range of data. This procedure was also adopted by Murphy and Farrar (1985). The final selection reduced the total plots to 157, from which a total 3,663 observations (individual trees) were available for model development. Seventy-five percent of the total observations (2,692 observations) were randomly selected for calibrating the model and the remaining 25 percent (971 observations) were used for validation. Summary statistics are presented for variables used in the individual-tree basal area growth model development for the combined, calibration, and validation data sets (tables 2, 3, and 4, respectively).

The purpose of dividing the original data set into two subsets is to allow the development of estimates of the regression parameters using the calibration data set. Once the final model had been selected, the validation data set could then be used to determine the robustness of the model. It is important that two data sets contain variables that have common statistical properties, while remaining independent and mutually exclusive of each other. Tests of hypothesis about the mean for each variable concluded there were no significant differences (0.05 level) between corresponding

Table 2—Summary of descriptive statistics for seven variables included in the complete data set (3,663 observations) used for individual tree basal area growth model development

Variable at midpoint	Minimum	Maximum	Mean	Std Dev	Cv
	<i>Percent</i>				
Site index (feet at base age 50)	35.00	73.5	56.34	7.35	13.06
Stand basal area (ft ² /acre)	37.08	115.23	72.71	17.07	23.48
Diameter at breast height for shortleaf pine (inches)	5.20	19.8	9.14	2.50	27.38
Quadratic mean diameter (inches)	6.35	12.95	8.93	1.22	13.66
Individual tree basal area (ft ²)	0.1475	2.1409	0.4911	0.2788	56.77
Average annual individual tree basal area growth (ft ²)	-0.00422	0.06849	0.0164	0.0111	67.24
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.99141	0.5866	0.2816	48.02

Table 3—Summary of descriptive statistics for seven variables included in the calibration data set (2,692 observations) used for individual tree basal area growth model development

Variable at midpoint	Minimum	Maximum	Mean	Std Dev	Cv
<i>Percent</i>					
Site index (feet at base age 50)	35.00	73.5	56.39	7.34	13.03
Stand basal area (ft ² /acre)	37.08	115.23	72.27	17.07	23.61
Diameter at breast height for shortleaf pine (inches)	5.25	19.8	9.11	2.50	27.49
Quadratic mean diameter (inches)	6.35	12.95	8.92	1.22	13.71
Individual tree basal area (ft ²)	0.1504	2.1409	0.4884	0.2779	56.91
Average annual individual tree basal area growth (ft ²)	-0.00419	0.06849	0.0164	0.0110	67.31
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.99141	0.5888	0.2813	47.78

variables, therefore, allowing the use of one subset for model development and the other for model validation.

MODEL DEVELOPMENT

Most individual-tree basal area growth models that have been developed are for even-aged stands and can be classified into two categories: direct and indirect models. An example of a simple direct basal area growth model is fully described by Wykoff (1986) in PROGNOSIS. Several variants of direct and indirect models were tried to establish the best to fit the data (Bitoki 1996). The basic structure for the individual-tree basal area growth model adopted was of the potential*modifier form. Theoretically, the potential growth model sets an upper limit size that a given tree cannot exceed (Hann and Leary 1979).

A potential*modifier function model is created in two stages. In the first step, a potential function is developed based on growth theories. Secondly, a modifier function is developed to adjust the potential growth to the actual growth achieved. The modifier function reflects environmental stress on the tree. These two steps produce a complex model having the following form:

Table 4—Summary of descriptive statistics for seven variables included in the validation data set (971 observations) used for individual tree basal area growth model development

Variable at midpoint	Minimum	Maximum	Mean	Std Dev	Cv
<i>Percent</i>					
Site index (feet at base age 50)	35.00	73.5	56.51	7.38	13.14
Stand basal area (ft ² /acre)	37.08	115.23	73.91	17.03	23.05
Diameter at breast height for shortleaf pine (inches)	5.20	19.2	9.21	2.45	27.06
Quadratic mean diameter (inches)	6.35	12.95	8.97	1.21	13.52
Individual tree basal area (ft ²)	0.1475	2.0115	0.4987	0.2812	56.40
Average annual individual tree basal area growth (ft ²)	-0.00422	0.05970	0.0167	0.0112	67.08
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.98690	0.5804	0.2826	48.68

$$\text{Individual tree growth} = (\text{potential}) * (\text{modifier}).$$

Several researchers have used this technique to produce an individual-tree growth model (Shifley 1987, Fairweather 1988, Belcher and others 1982, Hitch 1994). The potential function used was described by Shifley (1987) and is a version of the Chapman-Richards growth function.

$$\text{Pot} = b_1(\text{BA})^{b_2} - b_1/M^{(1-b_1)} * \text{BA} \quad (1)$$

where

pot = potential basal area growth,

BA = individual tree basal area (ft²),

M = parameter, maximum tree basal area, and

b₁ = parameters to be estimated.

This Chapman-Richards potential growth function ordinarily has three parameters, but only two need to be estimated with this version because the specification of maximum tree size fixes one of the parameters (Shifley and Brand 1984). This fixed parameter (M) represents the maximum size a

tree can achieve and, therefore, is mathematically the maximum asymptote, where growth equals zero. The value of the parameter M equals 7.068384 (adapted from Hitch 1994).

The modifier function adopted was previously described by Murphy and Shelton (1993, 1996). This type uses individual tree competitive status in the stand, expressed by the total basal area of all trees per acre as large or larger than the subject tree (BAL) to adjust the potential growth to the growth actually achieved. The following general form was adopted:

$$\text{Modifier function} = \frac{1}{1 + \exp(b_1 \text{BAL} + b_3 \text{BA} + \dots \text{add'l vars})} \quad (2)$$

For a tree growing without competition, its actual growth equals the potential growth. Consequently, the modifier function has a maximum value of one, which is approached at an asymptotic rate, when the exponential term approaches zero. Alternatively, if the tree receives extensive competition, the modifier function approaches zero; this occurs when the exponential term grows to infinity. Parameters of the function are estimated by linear or nonlinear regression. The use of linear regression was accomplished by rearranging the equation as:

$$\text{BAG} = \frac{\text{POTBAG}}{1 + \exp[b_1 \text{BAL} + b_2 \text{BA} + \dots \text{add'l vars}]} \quad (3)$$

and,

$$\ln\left[\frac{\text{POTBAG}}{\text{BAG}} - 1\right] = (b_1 \text{BAL} + b_2 \text{BA} + \dots \text{add'l vars}) \quad (4)$$

where

POTABAG = potential basal area growth (ft²/yr),

BAG = individual tree basal area growth (ft²/yr),

BA = individual tree basal area (ft²), and

BAL = proportion of basal area per acre of all trees as large or larger than the subject tree.

Equation (4) is in a form where multiple linear regression can be applied. It is flexible so that additional variables that explain basal area growth can also be included; those not significant may be removed through stepwise techniques. The final implicit form obtained was:

$$\text{AABAG} = \frac{b_1 \text{BA}^{b_2} - b_1 / M^{(1-b_2)} \cdot \text{BA}}{1 + \exp(b_3 + b_4 \text{SBA} + b_5 \text{BAL} + b_6 \text{SI} + b_7 \text{d.b.h.})} \quad (5)$$

where

aabag = annual average basal area growth (ft²),

BA = individual tree basal area (ft²),

M = individual tree maximum basal area,

SBA = stand basal area (ft²/acre),

BAL = proportion of basal area of all trees per acre of all trees as large or larger than the subject tree,

SI = site index (base age 50), and

d.b.h. = diameter at breast height (in).

The fit index for this model was calculated as following:

$$\text{Fit Index} = 1 - \left\{ \frac{\text{Error Sum of Squares}}{\text{Corrected Total Sum of Squares}} \right\} \quad (6)$$

The model had a fit index value of 0.43129 when fitted to the calibration data set and was consistent in terms of parameters when checked with the validation data set.

Although this fit index may be low compared to other studies, it is reasonable because the data came from normal inventory plots, not from research plots where variation is controlled. This statistic measures the proportion of variation of the dependent variable accounted for by the model. Other statistics used to test the model were mean square error and average deviations. When fitted to the entire data set, the following explicit model (with a fit index value of 0.44016 when used with the entire data set) was obtained:

$$\text{aabag} = \frac{0.179337 \text{BA}^{0.861911} - 0.1793377 \cdot 0.68384^{0.138089} \cdot \text{BA}}{1 + \exp(-1.161198 + 0.015529 \text{SBA} - 0.012761 \text{SI} + 1.026556 \text{BAL} - 0.013322 \text{dbh})} \quad (7)$$

Residual plots for the model by d.b.h. class were examined to detect any model bias and the model performed well throughout all d.b.h. classes (fig. 1). However, it must be noted that, due to having few trees represented in diameter classes 16 inches and greater, these trees were combined into one class to assess the model performance. The minimum tree diameter observed was 5.0 inches and the maximum tree diameter observed in the data of study was in the 20-inch d.b.h. class. Therefore, this model should not be used to predict growth of trees having sizes not represented in this study.

Similar plots were also analyzed by site index class and, again, the model performed well for all classes (fig. 2). Finally, the residuals were examined to detect model bias for stand basal area class and the model performed reasonably well across all classes (fig. 3). It can also be observed that average basal area growth deviations decrease as the stand basal area increases. The model shows that individual tree basal area growth decreases as the stand basal area increases, which is in agreement with forest growth theories (fig. 3).

RECOMMENDATIONS

Forest managers concerned with uneven-aged stands of shortleaf pine in the Ouachita Mountains of Oklahoma and Arkansas are the primary potential users of this model. This is the first individual-tree basal area growth model for uneven-aged shortleaf stands in the region, and should be a useful tool to predict future growth to provide needed

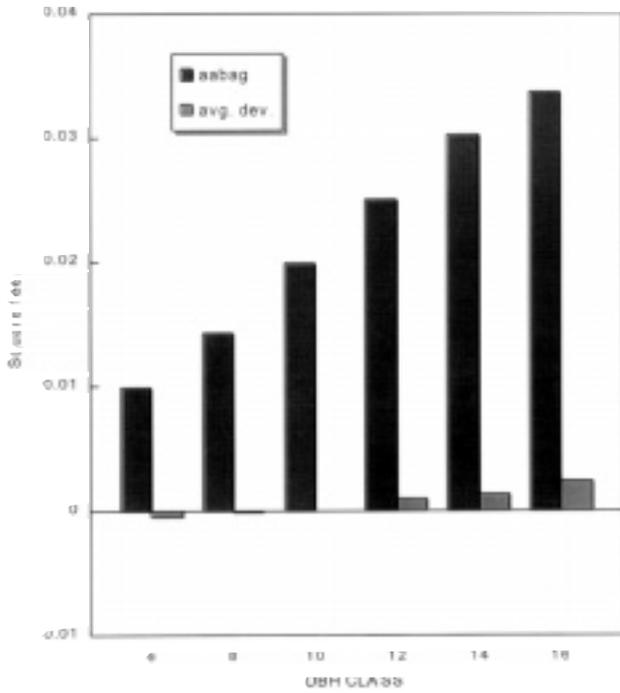


Figure 1—Average deviations and mean annual basal area growth (aabag) by d.b.h. class (2-inch classes) for the model when fitted to the entire data set.

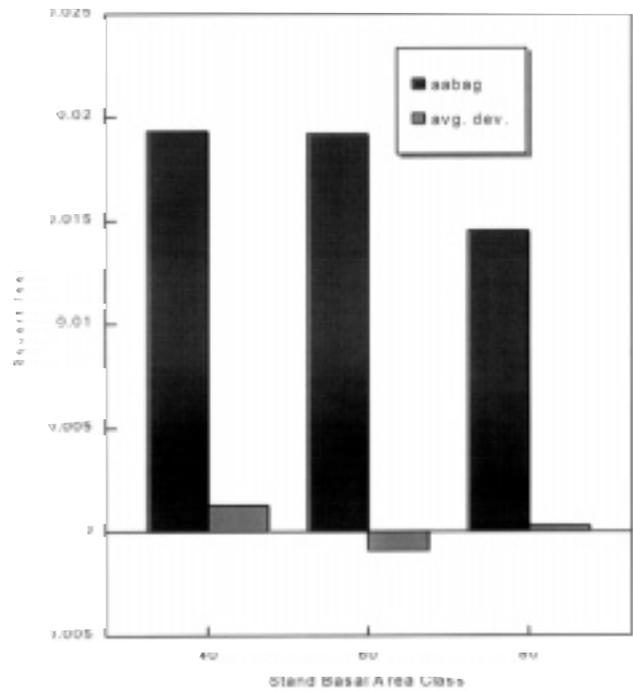


Figure 3—Average deviations and mean annual basal area growth (aabag) by stand basal area class (ft²/ac) for the model when fitted to the entire data set.

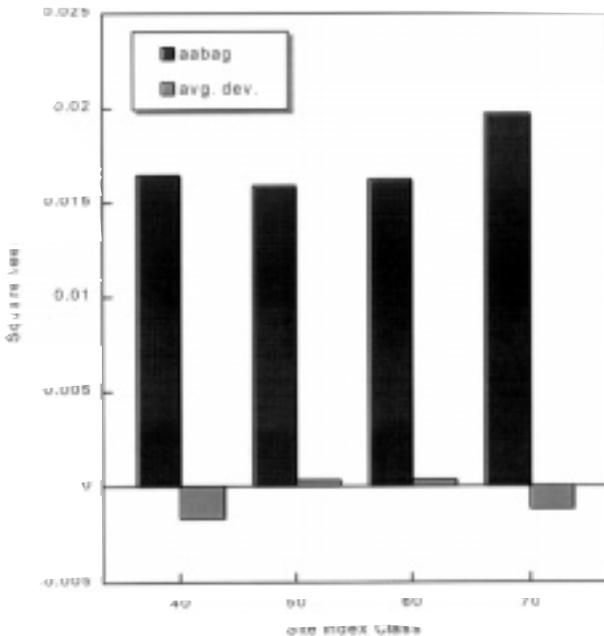


Figure 2—Average deviations and mean annual basal area growth (aabag) by site index class (base age 50 years) for the model when fitted to the entire data set.

information for management decisions. This model can be incorporated with other functions such as survival and ingrowth models to predict uneven-aged shortleaf pine growth dynamics in the Ouachita Mountains. The model should be restricted to the region represented by the data used for the model development. Therefore, users are

encouraged to check and compare their stand characteristics to the data described in this study in order to discern whether their conditions are similar to the study area.

The data used for this study were from ordinal inventory permanent plots, not research plots. The data, therefore, may not have equally represented all possible stand conditions. Trees with a large d.b.h. (>16 inches) were not well represented in the data set; therefore, this model may not perform correctly for other stands that do have large trees. Also, site index classes 40 and 70 were not adequately represented. The user of this model should take into account these shortcomings. The equation presented here could also be improved in the future by collecting more data on sites with site index of less than 45 and by refitting the model to well-balanced data, especially from research plots.

LITERATURE CITED

- Baker, J.B.; Cain, M.D.; Guldin, J.M.; Murphy, P.A.; Shelton, M.G.** 1996. Uneven-aged silviculture for loblolly and shortleaf pine forest cover types. Gen. Tech. Rep. SO-118. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station.
- Belcher, D.W.; Holdaway, M.R.; Brand, G.J.** 1982. A description of STEMS. Gen. Tech. Rep. NC-79. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 18 p.
- Bitoki, O.** 1996. A distance-independent individual-tree basal area growth model for natural uneven-aged stands of shortleaf pine (*Pinus echinata* Mill.) in the Ouachita Mountains. Oklahoma State University. 74 p. M.S. thesis.

- Fairweather, E. Stephen.** 1987. Development of an individual tree growth model for Pennsylvania. In: Forest growth modeling and prediction. Vol.1. Proc. of the IUFRO conference; 1987 August 23-27. Minneapolis, MN.
- Hann, J.T.; Leary, R.S.** 1979. Potential diameter growth functions. In: A generalized forest growth projection system applied to the Lake States. Gen. Tech. Rep NC-49. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station: 22-26.
- Hitch, K.L.** 1994. A distance -independent individual tree basal area growth model for natural even-aged stands for shortleaf pine in eastern Oklahoma and eastern Arkansas. Oklahoma State University. 62 p. M.S. thesis.
- Murphy, P.A.; Farrar, R.M., Jr.** 1985. Growth and yield of uneven-aged shortleaf pine stands in the Interior Highlands. Res. Pap. SO-218. New Orleans: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. 11 p.
- Murphy, P.A.; Shelton, M.G.** 1993. Progress report: Effects of density, site quality and maximum diameter on growth and yield of uneven-aged loblolly pine stands. FS-SO-4106-36 (Problem 4). Monticello, AR.
- Murphy, P.A.; Shelton, M.G.** 1996. An individual-tree basal area growth model for loblolly pine stands. Canadian Journal of Forest Research. 26: 327-331.
- Shifley, S.R.** 1987. A generalized system of models forecasting central states tree growth. Res. Pap. NC-279. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 10 p.
- Shifley, S.R.; Brand, G.B.** 1984. Chapman-Richards growth function constrained for maximum tree size. Forest Science. 30(4): 1066-1070.
- Wykoff, W.R.** 1986. Supplement to the user's guide for the stand Prognosis model. Version 5.0. Gen. Tech. Rep. INT-218. U.S. Department of Agriculture, Forest Service. 36 p.

DETECTING RESPONSES OF LOBLOLLY PINE STAND DEVELOPMENT TO SITE-PREPARATION INTENSITY: A MODELING APPROACH

Mingguang Xu, Timothy B. Harrington, M. Boyd Edwards¹

Abstract—Data from an existing site preparation experiment in the Georgia Piedmont were subjected to a modeling approach to analyze effects of site preparation intensity on stand development of loblolly pine (*Pinus taeda* L.) 5 to 12 years since treatment. An average stand height model that incorporated indicator variables for treatment provided an accurate description of responses to site-preparation intensity, with steepness of the height trajectory increasing with site-preparation intensity. Stand basal area and volume varied similarly among treatments as found for height, with their development increasing with treatment intensity.

INTRODUCTION

It has been widely accepted that development of stand height follows a certain pattern that can be described by the following model (Clutter and others 1983):

$$\ln(H)=a+b/AGE \quad (1)$$

where

H is stand height,

AGE is stand age, and

a and b are estimated parameters.

Similarly, growth of stand basal area (BA) and stand volume (V) follow a similar pattern as:

$$\ln(Y)=a+b/AGE \quad (2)$$

where

Y is stand basal area or stand volume, and

a, b, and AGE are as defined previously.

The growth pattern of stand height, basal area, and volume can vary with site quality, species, and silvicultural treatments, such as site preparation. For a given species and site, effects of treatments can be detected by incorporating their effects into this model:

$$\ln(Y)=a+b/AGE+cTRMT \quad (3)$$

where

TRMT represents treatment effects,

c is an estimated parameter, and

a, b, and AGE are as defined previously.

In this study, an indicator variable was specified for each treatment to detect effects of site preparation intensity on stand development.

DATA

The data were from an existing site preparation study of loblolly pine (*Pinus taeda* L.) initiated in 1980 in the Georgia Piedmont. After clearcutting mature loblolly pine, the six site-preparation intensities were applied, ranging from absence of site preparation to combinations of mechanical, herbicide, and fertilizer treatments. The treatments are listed as follows in order of increasing intensity:

- (1) Clearcut only.
- (2) Chainsaw. All residual trees greater than 2.5 centimeters diameter at breast height (d.b.h.) were felled with a chainsaw in August 1981.
- (3) Shearing of residual trees with a KG blade mounted on a D7 tractor in September 1981 and chopping of woody debris with a single pass of a rotary-drum chopper in November 1981.
- (4) Shear, chop, and herbicide. Treatment 3 plus application of 0.5 cubic centimeters Velpar (TM) Gridball pellets (hexazinone at 10 percent active ingredient) in a 0.6-meter x 0.6-meter grid pattern at a rate of 2.8 kilograms per hectare in March 1982.
- (5) Shear, rootrake, burn, and disk. Residual trees were sheared and rootraked into windrows in September 1981 and burned in October 1981. The remaining debris and ash were scattered with a dozer blade and the plots were disked with an offset harrow to a depth of 15-20 centimeters in October 1981.
- (6) Shear, rootrake, burn, disk, fertilize, and herbicide. Treatment 5 plus a broadcast application of ammonium- nitrate fertilizer at 114 kilograms N per hectare and a 1.2-meter band application of Oust (TM) (sulfometuron) at 0.42 kilograms active ingredient per hectare in March and April 1983.

Each treatment was replicated five times in a randomized complete-block design. In January and February 1982, seedlings of loblolly pine were hand-planted at a spacing of 1.8 meters x 3 meters. After establishment of the stands, measurements of d.b.h. (centimeters) and height (meters)

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of each planted pine were taken 5, 8, 10, and 12 growing seasons after treatment, from which stand average height (H), basal area (BA) (square meters per hectare), and volume (V) (cubic meters per hectare) were calculated. All models were fitted with linear regression using a 95 percent significance level.

RESULTS AND DISCUSSION

Stand Average Height

Fitting stand average height to model (3) resulted in the following equation:

$$\ln(HT)=3.26-9.84/AGE-0.415T1-0.256T2-0.126T3-0.316T4-0.0991T5 \quad (4)$$

where

T1, T2, T3, T4, and T5 are indicator variables that represent treatments 1 to 5, respectively.

For example, T1=1 if treatment is 1, otherwise T1=0. Equation (4) indicates that development of average height differed significantly among treatments, with rate increasing with site-preparation intensity. Average height of untreated stands was significantly less than that of treated stands ($P \leq 0.05$).

Numerous studies have reported increases in stand height in response to site preparation (Glover and Zutter 1993, Harrington and Edwards 1996, Pienaar and Rheney 1995, Thomson and McMinn 1989). Since stand height is relatively similar for a wide range of stand densities, increases in the rate of height development probably are more attributable to improvement in site quality due to site preparation. Measurements of soil properties on this site demonstrated that the treatments improved growing conditions for pine by decreasing bulk density and increasing pore space (Miller and Edwards 1985).

Stand Basal Area

The following equation resulted from fitting stand basal area to model [3]:

$$\ln(BA)=4.75-9.19/AGE-1.52T1-1.04T2-0.308T3-0.477T4 \quad (5)$$

The fitted equation (5) indicates that development of stand basal area also increased with treatment intensity, with the clearcut only treatment having the slowest rate of basal area development.

Stand Volume

Model [3] also was used to test responses of stand volume to treatment:

$$\ln(V)=6.28+24.4/AGE-1.68T1-1.09T2-0.326T3-0.617T4 \quad (6)$$

Stand volume differed significantly among site-preparation intensities, with the clearcut-only treatment having the slowest rate of volume development (fig. 1). Development of

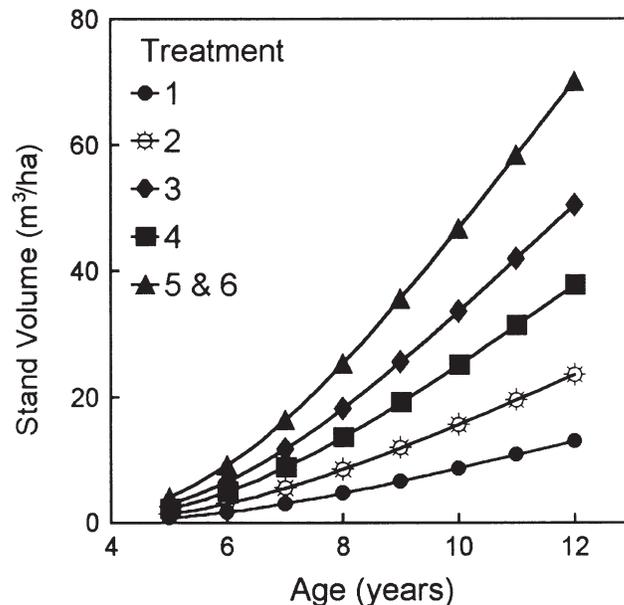


Figure 1—Effects of increasing site-preparation intensity on development of stand volume of loblolly pine.

stand volume did not differ significantly ($P \geq 0.05$) between treatments 5 and 6—the two treatments having the greatest rate of stand development.

CONCLUSIONS

Effects of site preparation on stand development were modeled as adjustments to a growth model for stand average height, basal area, and volume. This approach provided an accurate description of stand responses to site-preparation intensity. Results suggest that the more intensive site preparation treatments lead to greater rates of stand development.

LITERATURE CITED

Clutter, Jerome L.; Fortson, James C.; Pienaar, Leon V. [and others]. 1983. Timber management: a quantitative approach. Malabar, FL: Krieger Publishing Company. 333 p.

Glover, Glenn R.; Zutter, Bruce R. 1993. Loblolly pine and mixed hardwood stand dynamics for 27 years following chemical, mechanical, and manual site preparation. *Canadian Journal of Forest Research*. 23: 2126-2132.

Harrington, Timothy B.; Edwards, M. Boyd. 1996. Structure of mixed pine and hardwood stands 12 years after various methods and intensities of site preparation in the Georgia Piedmont. *Canadian Journal of Forest Research*. 26: 1490-1500.

Miller, James H.; Edwards, M. Boyd. 1985. Impacts of various intensities of site preparation on Piedmont soils after 2 years. In: Shoulders, Eugene, ed. Proceedings of the third biennial southern silvicultural research conference; 1984 November 7-8; Atlanta, GA. Gen. Tech. Rep. SO-54. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station: 65-73.

Pienaar, Leon V.; Rhenev, John W. 1995. Modeling stand level growth and yield response to silvicultural treatments. *Forest Science*. 41: 629-638.

Thomson, Alan J.; McMinn, Robert G. 1989. Effects of stock type and site preparation on growth to crown closure of white spruce and lodgepole pine. *Canadian Journal of Forest Research*. 19: 262-269.

TESTING FOR PREDICTION BIAS WHEN PROJECTING GROWTH FROM INCREMENT CORES

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Abstract—Predictors of periodic basal area growth are derived from each of two data sources, one using breast-high diameters, and increment cores measured at the same time, and another using breast high diameters measured at two points in time. The increment cores are used to estimate diameters of merchantable-sized (d.b.h. > 4.5 inches) survivor trees at a point in the past (in this case, 5 years before the age at measurement). Although this first approach requires more work at the initial measurement, it has the advantage of not requiring a revisit to the site at some future time, and yields immediate information on the periodic growth of survivor trees. We investigated whether regression prediction equations yielded similar parameter estimates when fitted to the two data sets using the same mathematical formulation of initial basal area stocking and initial stand age. Comparisons between the two were not statistically significant when compared on a parameter-by-parameter basis, which supported the conclusion that using increment-core growth this way is an efficient, unbiased way to develop a periodic basal area growth predictor. Although the individual parameters were not statistically different, the fitted models were statistically different when predicted basal area growth was compared at a common initial basal area stocking and initial stand age. Specifically, the model using the 1991-96 data over predicted the model fit to the 1986-91 data by 0.79 ft² per 1/5-acre (or approximately 4 ft² per acre) for the 5-year period. Adjusting for mean monthly growing season rainfall reduced the difference to 0.19 ft² per 1/5-acre (or approximately 1 ft² per acre). This adjusted difference was not statistically significant, suggesting that a simple adjustment procedure using growing season rainfall works.

INTRODUCTION

We often want to use temporary inventory plots to develop predictions of periodic growth, or when installing permanent growth plots, we might want to analyze growth without having to wait for the first remeasurement. One objective of this research is to investigate the suitability of using increment-core derived periodic basal area growth in the development of a periodic basal area growth predictor. For example, Lloyd and Waldrop (1993) used such an approach in their analysis of the relative growth dynamics of the pine and hardwood components in mixed pine and hardwood stands in the Piedmont physiographic region. A second objective is to test the accuracy of using increment-core derived basal area growth from the period prior to plot measurement to project survivor growth for the period immediately following plot establishment. Our hypothesis is that a linear expression of initial basal area stocking and initial stand age fit to increment-derived growth is the same model derived from remeasured diameters, that is, has the same parameter estimates. This analysis uses a subset of the data from a more comprehensive study of growth dynamics in natural, mixed pine and hardwood stands (Lloyd 1991).

METHODS

The permanent plots were installed in naturally regenerated pine-hardwood stands located on the Piedmont Ranger District of the Sumter National Forest and on the Clemson University Experimental Forest. The study design consists of 39 circular, 1/5-acre plots located in even-aged mixed pine-hardwood stands ranging from 25 to 61 years of age. A stand was deemed to be a pine-hardwood mixture when the pine component made up between 10 percent and 70 percent of the merchantable

basal area. The 10 percent lower limit was selected on the subjective criterion that this is not enough pine to meaningfully affect stand dynamics. The 70 percent upper limit was used as a criterion to assure that at least a portion of the generally shorter hardwood stand component received direct light from above. In other words, that the pine was not so densely stocked that it formed a closed overstory canopy.

Attention was paid to choosing sample stands that had no identifiable evidence of natural or man-imposed disturbance. Plots would be dropped from further analysis if mortality appeared to be caused by forces other than self-thinning due to competition effects. No such rejections were necessary for this 10-year growth cycle. All merchantable-sized trees were identified by species, marked by paint with a number, and located by azimuth and distance from the plot center. At plot establishment in 1991, increment cores were extracted from all merchantable-sized trees, and 5- and 10-year radial increment was accurately measured on a digital ring measurement device. Tree diameter accuracy was maintained by measuring all trees (merchantable and nonmerchantable) at a painted mark at breast height. Height growth patterns were examined using stem analyses of a dominant oak and pine selected near each permanent plot, but only the ring count from the stump of the stem-analyzed pine was used in this analysis as the stand age. Within the plot, total tree height was obtained on a subsample by choosing every fifth tree within each 1-inch diameter class within the pine and hardwood species categories. One sample tree height was always taken in any diameter classes with less than five trees. None of the height data were used in this investigation.

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All tree attributes (except increment cores) were remeasured during the 1995 dormant season, and the tree identification numbers of merchantable-sized trees were repainted. Total tree height was remeasured on the sample trees established at plot installation, except in instances of sample height tree mortality. When a sample tree would die, another tree of the same species (if possible) or species genus (for sure) and approximate diameter was substituted by measuring its total height. Any ingrowth trees between 1991 and 1996 were assigned a number and measured the same way as other merchantable trees.

The data used in this analysis are the 1991 diameters measured at plot establishment in 1991, the 1996 remeasured diameters, and the 5-year radial growth from the increment cores that were extracted in 1991. The increment core measurements for all trees alive at the 1996 remeasurement were multiplied by 2 and subtracted from the 1991 diameters to estimate 1986 diameters. The three diameter measures for 1986, 1991, and 1996 were used to compute periodic plot basal area growth for the 1986-91 and 1991-96 periods.

Five-year periodic basal area growth was fit by least squares estimation to the model form:

$$b_i = a_{i0} + a_{i1}t + a_{i2}B \quad (1)$$

where

t = stand age at the beginning of the growth period,

B = basal area at the beginning of the growth period of all trees alive at the end of the growth period, and

a_{i0} , a_{i1} , and a_{i2} are parameters, which are estimated using periodic growth data from the i th period (that is, 1986-91 or 1991-96.) The simple linear form for the stand age variable (t) was used in equation (1) because curvilinear expressions (such as the reciprocal of stand age) proved unstable due to a lack of data in the middle of the observed stand age range. This design defect was an artifact of the areas in which we initially searched for stands. The plan is to improve the study design by installing additional plots in the deficient age classes. The models to be compared are:

$$b_1 = a_{10} + a_{11}t + a_{12}B \quad (2)$$

for the 1986-91 period, and

$$b_2 = a_{20} + a_{21}t + a_{22}B \quad (3)$$

for the 1991-96 period.

Our hypothesis asserts that the models in equations (2) and (3) have the same parameter values. Therefore, the first analytical step was to test the three null hypotheses:

$$H_0: a_{10} = a_{20}, H_0: a_{11} = a_{21}, H_0: a_{12} = a_{22}. \quad (4)$$

The statistical tests for the hypotheses expressed in equations (4) were performed by least-square analyses

using the combined data from both periods. The parameters in this combined model [equation (5) below] are presented in terms of the parameters in the separate models in equations (2) and (3). The purpose of doing this is to make it clear how the combined model is working. Periodic basal area growth for the combined data set is expressed as:

$$b = a_{10} + a_{11}t + a_{12}B + (a_{20} - a_{10})D + (a_{21} - a_{11})Dt + (a_{22} - a_{12})DB, \quad (5)$$

where all variables are as previously defined, except the new variable D , which is an indicator variable (Gujarati 1995) that takes a value of 0 for the 1986-91 period and a value of 1 for the 1991-96 period. Of course, the $(a_{2j} - a_{1j})$ parameters are estimated as single values of the respective differences.

RESULTS

The results of fitting equation (5) to the combined data set are found in table 1. The three null hypotheses in equations (4) for the individual parameters are tested with the t -statistics for the parameter differences identified in the parameter identifier column of the table. The test statistics of 1.216, -1.352, and 0.303 associated with these differences are not statistically significant, so the assertion of no difference in parameters is not rejected; in other words, our hypothesis is supported. No effort was devoted here to evaluating the power of these statistical tests because our measurement methods were as good as can be obtained from established technique, and thus our ability to detect differences cannot be improved. Table 2 is included to aid interpretation of the statistical tests resulting from the least-squares analysis of equation (5). It contains the parameter estimates for equations (2) and (3) fitted separately to the data sets for periods 1986-91 and 1991-96. Comparisons between tables 1 and 2 verify that the differences in table 1 are in fact the ones we wanted to analyze.

Although the null hypotheses in equations (4) were not rejected, this does not mean that other linear functions of these parameters might not be deemed statistically significant. One such linear function expressed as the null hypothesis is:

$$H_0: (a_{20} - a_{10}) + (a_{21} - a_{11})t_c + (a_{22} - a_{12})B_c = 0 \quad (6)$$

which is the difference in predicted basal area growth between the two fitted models at the common initial stand age t_c and initial basal area stocking B_c . The value used for t_c is the overall mean age for both periods (34.3 years) and the corresponding 1/5-acre value for B_c is the average stocking across both periods (21.4 ft² or 107 ft² on an acre basis). These values and the parameter estimates from table 1 are substituted into the linear function in equation (6) to produce the estimated difference of:

$$(4.318 - 3.371) + (-0.07889 + 0.06786)(34.3) + (0.1094 - 0.09882)(21.4) = 0.79 \text{ ft}^2. \quad (7)$$

Table 1—Regression statistics using both periods to fit the combined model

Parameters	Parameter estimates	t-statistics	Probability > t
a_{10}	3.371414	6.637	0.000
$(a_{20} - a_{10})$	0.946511	1.216	0.228
a_{11}	-0.67869	-11.618	0.000
$(a_{21} - a_{11})$	-0.011031	-1.352	0.181
a_{12}	0.098822	3.827	0.000
$(a_{22} - a_{12})$	0.010540	0.303	0.763

Table 2—Regression statistics for models fitted separately to the period data

1986-91 period		1991-96 period	
Parameter	Estimate	Parameter	Estimate
a_{10}	3.371414	a_{20}	4.317925
a_{11}	-0.067860	a_{21}	-0.078891
a_{12}	0.098822	a_{22}	0.109362

The value of the test statistic for this linear combination of parameters is 42.85, which is highly significant. In other words, pairwise comparisons of parameters between periods were not significantly different, but a difference in predicted periodic basal area growth between fitted versions of equations (2) and (3) evaluated at t_c and B_c was significantly different from 0.

The cause of these differences might be a drought in the Piedmont physiographic region during the 1986-91 period. We asked whether making a simple adjustment for rainfall differences between 1986-91 and 1991-96 might account for the observed difference. The approach was to compute an index (I) defined as the cumulative mean monthly rainfall (in inches) for the months of May, June, July, and August, where the “mean” was from monthly values over the 5-year period. Our plots clustered near two permanent weather stations, so the index took on one of four values, depending on the growth period from which the data came and the location of the plot. The indexes were appended to the combined data set through use of a variable called I. The indexes were incorporated into the intercept term of the combined model found in equation (5). The reasoning behind only modifying the intercept term was based on the apparent equality of the parameter estimates for the individual variables suggested by the failure to reject the hypotheses in equations (4). This meant that we expected the significant effect to be due to a difference in elevation between the two parallel plains representing the growth

relationships between periods. This assumption yielded the new model:

$$b = i_{20}I(D) + i_{10}I(1-D) + a_{11}t + a_{12}B + (a_{21} - a_{11})Dt + (a_{22} - a_{12})DB, \quad (8)$$

where all variables are as previously defined, and the coefficients i_{10} and i_{20} are for the modified intercept terms. The associated null hypothesis is:

$$H_0: (i_{20} - i_{10})I_c + (a_{21} - a_{11})t_c + (a_{22} - a_{12})B_c = 0, \quad (9)$$

where I_c is the mean rainfall index across both periods. The least-squares estimated parameters for the modified model in equation (8) are listed in table 3. As with the first combined model [equation (5)], the separate fitted models are presented in table 4 to aid interpretation of table 3. Note in table 3 that the differences $(a_{21} - a_{11})$ and $(a_{22} - a_{12})$ are still not significant, as was the case in the first combined model in table 1. The adjusted (for rainfall) basal area growth between periods is:

$$(0.2658 - 0.2562)(16.905) + (-0.09234 + 0.08742)(34.3) + (0.1123 - 0.1031)(21.4) = 0.19 \text{ ft}^2, \quad (10)$$

where 16.905 is the average rainfall index across both periods (the variable I_c in [equation (9)]). The value of the test statistic for this linear function is 2.67, which is not statistically significant.

Table 3—Regression statistics for the combined model with modified intercepts

Parameters	Parameter estimates	t-statistics	Probability > t
i_{10}	0.256190	6.241	0.000
i_{20}	0.265770	6.971	0.000
a_{11}	-0.087421	-12.617	0.000
$(a_{21} - a_{11})$	-0.004919	-0.494	0.623
a_{12}	0.103144	3.863	0.000
$(a_{22} - a_{12})$	0.009201	0.256	0.799

Table 4—Regression statistics for modified intercept models fitted separately to periods

1986-91 period		1991-96 period	
Parameter	Estimate	Parameter	Estimate
i_{10}	0.256190	i_{20}	0.265771
a_{11}	-0.087421	a_{21}	-0.092340
a_{12}	0.103144	a_{22}	0.112345

CONCLUSION

This research examined two hypotheses. The first one is that increment cores can function in an unbiased way in estimating future periodic basal area growth of survivor trees. Stating this first hypothesis differently, the null assertion is that the same regression estimates result whether periodic basal area growth of survivor trees is measured from radial growth on increment cores, or obtained from diameter remeasurements made after waiting a time equivalent to the length of the growth period. The consistent non-significance of the parameter differences $a_{21} - a_{11}$ and $a_{22} - a_{12}$, combined with the fact that we used the best measurement technique, is strong support for using the increment core data this way. A second hypothesis was supported which asserts that simple expressions of rainfall differences can be used to adjust the fitted growth projections when the projection

model is fit to growth from one time period and used to estimate growth for the time period in the immediate future.

LITERATURE CITED

- Gujarti, D.N.** 1995. Basic econometrics. 2d ed. New York: McGraw-Hill Book Co.: 446-448.3
- Lloyd, F.T.** 1991. Forecasting growth of pine-hardwood mixture from their ecological land class. In: Ecological land classification: Applications to Identify the productive potential of southern forests. Proceedings. Charlotte, NC.
- Lloyd, F.T.; Waldrop, T.A.** 1993. Relative growth of oaks and pines in natural mixtures on intermediate to xeric Piedmont sites. In: Proceedings, oak regeneration: serious problems, practical recommendation. Gen. Tech. Rep. SE-84. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station.