PREDICTION OF ULTRASONIC PROPERTIES FROM GRAIN ANGLE

KABIR, M. F.
Dept. of Wood Science and Forest Products, Virginia Tech. Brooks Forest Products Center. Blacksburg, USA

ABSTRACT

The ultrasonic properties of rubber wood were evaluated in three main symmetry axes – longitudinal (L), radial (R) and tangential direction and also at an angle rotating from the symmetry axes at different moisture content. The ultrasonic velocity were determined with a commercial ultrasonic tester of 45 kHz pulsed longitudinal waves. The experimental results were compared with the predicted value using some empirical formula such as Hankinson, Osgood and Jacoby equation. For LR and LT rotations, the predicted ultrasonic velocity using Hankinson and Osgood equations are in close agreement with the measured value for almost all moisture content. Jacoby equation predicted well only at an angle greater than 50°. In RT rotation, all these equations can be used for the prediction of ultrasonic velocity of rubber wood. In RT rotation, the value of the exponent for Hankinson and Osgood are higher compared to LR and LT rotations. Whereas in the Jacoby equation it was found to be lower.

KEYWORDS:

Ultrasonic Properties, Wood Anisotropy, Grain Angle, Elastic Stiffness Constant, Hankinson Formulae.

INTRODUCTION

Wood as an anisotropic orthotropic solid, obeys Hooke's law

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \quad (1)$$

and

$$\varepsilon_{ij} = S_{ijkl}\,\sigma_{kl} \quad (2)$$

where C_{ijkl} are the elastic stiffness constant and S_{ijkl} the elastic compliance. The physical significance of the compliance is that $S_{ijkl} = 1/E_i$ i.e., the Young's modulus for the material along the i axis. The relationship between the stiffness constant and ultrasonic velocity of the orthotropic material are given by Christofel equation (Musgrave, 1970)

$$[\Gamma_{ij} - \rho V^2 \delta_{ik}] = 0 \quad (3)$$

Where Γ_{ij} is the Christofell stiffness, which is a function of stiffness matrix Cij and the components of, unit wave-normal vector n_j; ρ is the density of the material and δ_{ik} is the Kronecker delta-tensor of rank two.

The six diagonal terms of the stiffness matrix can be written in general form as:

$$C_{ij} = V^2_{ij}\,\rho \qquad I = 1, 2, 3 \quad (4)$$

In wood, these six diagonal stiffness constant may be expressed as C_{LL}, C_{RR}, C_{TT}, C_{LR}, C_{LT}, and C_{RT} where L, R, T stands for three anisotropic directions - longitudinal, radial and tangential respectively. Similar notation can be used for ultrasonic velocity V, such as V_{LL}, V_{RR}, V_{TT}, V_{LR}, V_{LT} and V_{RT}.

Ultrasonic properties, such as ultrasonic velocity and elastic stiffness constant are greatly affected by the grain directions and grain angles (Suzuki & Sasaki 1990, Mishiro 1996, Bucur 1988). Usually, the ultrasonic velocity and elastic stiffness constant are greater in the L direction compare to the R and the T directions (Kabir *et al.* 1997, Kamioka 1988, Bucur & Feeney 1992).

The first empirical equation, known as Hankinson's formula (Anon, 1987) developed by the U.S. Army in 1921 for predicting strength properties of wood from grain angle is as follows:

$$N = \frac{P \times Q}{P \sin^n \theta + Q \cos^n \theta} \quad (5)$$

Where

N = the strength property at an angle θ.
P = the strength property parallel to grain.
Q = strength property perpendicular to gram.
n = an empirically determined constant.
θ = grain angle.

This Hankinson's formula has been used widely for various mechanical properties, such as modulus of elasticity, compressive strength, bending strength, etc. from the grain angle. It may also be suitable for the estimation of ultrasonic velocity and elastic stiffness constant (Armstrong *et al.*, 1991). The value of 'n' in Hankinson's formula may vary from 1.5 to 2.5 for these mechanical properties.

Another less-well known formula termed as Osgood equation (Kim, 1985) can be expressed by

$$N = \frac{P \times Q}{Q + (P - Q)(\mathrm{Sin}^n\theta + a\cos^n\theta)\sin^n\theta} \quad (6)$$

where N, P, and Q are defined as in Hankinson's formula, and "a" is the coefficients depending on the species, for which Osgood gives the value 0.35 for southern yellow pine.

The relationship between ultrasonic velocity and grain angles have been given by the Jacoby equation (Mishiro, 1996) which can be written as:

$$V_\theta = V_0\cos^n\theta + V_{90}\sin^n\theta \quad (7)$$

Where V_θ is the ultrasonic velocity at angle θ from the grain direction, V_0 is the ultrasonic velocity parallel to grain, V_{90} is the ultrasonic velocity perpendicular to grain, and n is an empirically determined exponent. The value of 'n' may vary from 1.9 to 2.5 for some Japanese wood species.

An attempt has also been made for estimating the ultrasonic properties from grain angle using statistical regression analysis. For doing so, second order parabolic and hyperbolic equations were used. The parabolic equation takes the form:

$$V, E = A + B\theta + C\theta^2 \quad (8)$$

and the Hyperbolic equation is of the form

$$V, E = A + B/\theta + C/\theta^2 \quad (9)$$

where V is the ultrasonic velocity, E is the elastic stiffness constant, θ is the angle of the grain in degrees, and A, B, and C are the constants of the regression.

MATERIALS AND METHODS

The experiment was carried out by using a commercial ultrasonic tester (BP V- Steinkamp, Germany) of 45 kHz pulsed longitudinal waves. Two exponential piezoelectric conical transducers were used for transmitting and receiving the pulses. The transmission time of the pulses through the specimen was digitally displayed and recorded manually. Transmitting time was measured to an accuracy of ± 0.1 µs. The ultrasonic velocity was calculated by dividing the specimen length by the transmitting time.

For measuring the ultrasonic velocity in the rotation of LT, LR and TR directions, specimens of rubber wood *(Hevea brasiliensis)* were prepared semi-circular in shape. Each surface of the specimen was abraded using a belt sander so that there was a good contact between the sample and transducers. The transducers were placed perpendicularly on the faces of each direction. The average thickness of the specimen was about 1.5 cm and 12 cm to 15 cm in diameter. The transmitting transducer was placed at the center of the main orthotropic axis and the receiving transducer was rolling on the circular faces. The measurements were taken from 0° to 90° at an interval of 10° from L to R and T, and R to T directions. Before taking measurement, the instrument was calibrated with a standard block.

To measure the ultrasonic velocity at different M.C. the specimen was fully soaked in water for a sufficiently long period of time, weighed and measurement was taken. Then, it was dried in air to reduce moisture. This cycle of measuring and weighing was repeated several times until the specimen showed no change in weight by drying. Finally, the oven-dry weight of the specimen was taken by drying in an electric oven at 100°C ± 3°C for 24 hours. The mean density of the rubber wood was determined based on the oven-dry weight and the values were 578 kg/m^3, 595 kg/m^3 and 620 kg/m^3 for green, air-dry and oven-dry condition respectively.

RESULTS AND DISCUSIONS

The ultrasonic velocity and elastic stiffness constants were predicted from grain angle using Hankinson's formula (5) Osgood (6) and Jacoby equation (7). It is generally assumed that since ultrasonic wave velocity is a function of dynamic modulus of elasticity (MOE), the relationship between velocity and grain angle would conform to Hankinson's as well as Osgood formula.

The predicted ultrasonic velocity using different equations and regression analysis along with experimental data at different MC in LR rotation are presented in Figure 1.

Similar results for LT and RT rotations are shown in Figure 2 and Figure 3 respectively.

Figure 4, Figure 5 and Figure 6 showed the elastic stiffness constant for LR, LT and RT rotations respectively. The exponent n in each equation of Hankinson, Osgood and Jacoby were determined empirically for the best fitted data and the values are presented in Table 1.

The results from the regression analysis for second order parabolic and hyperbolic equations are shown in Table 2 with the r^2 value. The usefulness of the regression equations for predicting ultrasonic properties from grain angle are determined by r^2 values. On the other hand, Hankinson, Osgood and Jacoby equations are not statistically derived and its relative accuracy cannot be determined by r^2 values. Therefore, Average Absolute Errors (AAE) were used to determine the best equation for predicting ultrasonic properties from the grain angle following the method of Armstrong *et al.* (199 1) with slight modifications. The calculated AAE percentage in LR, LT and RT rotations are presented in Table 3 and Table 4 for ultrasonic velocity and elastic stiffness constant respectively. The negative sign in these tables mean that the average experimental value is greater than the predicted value.

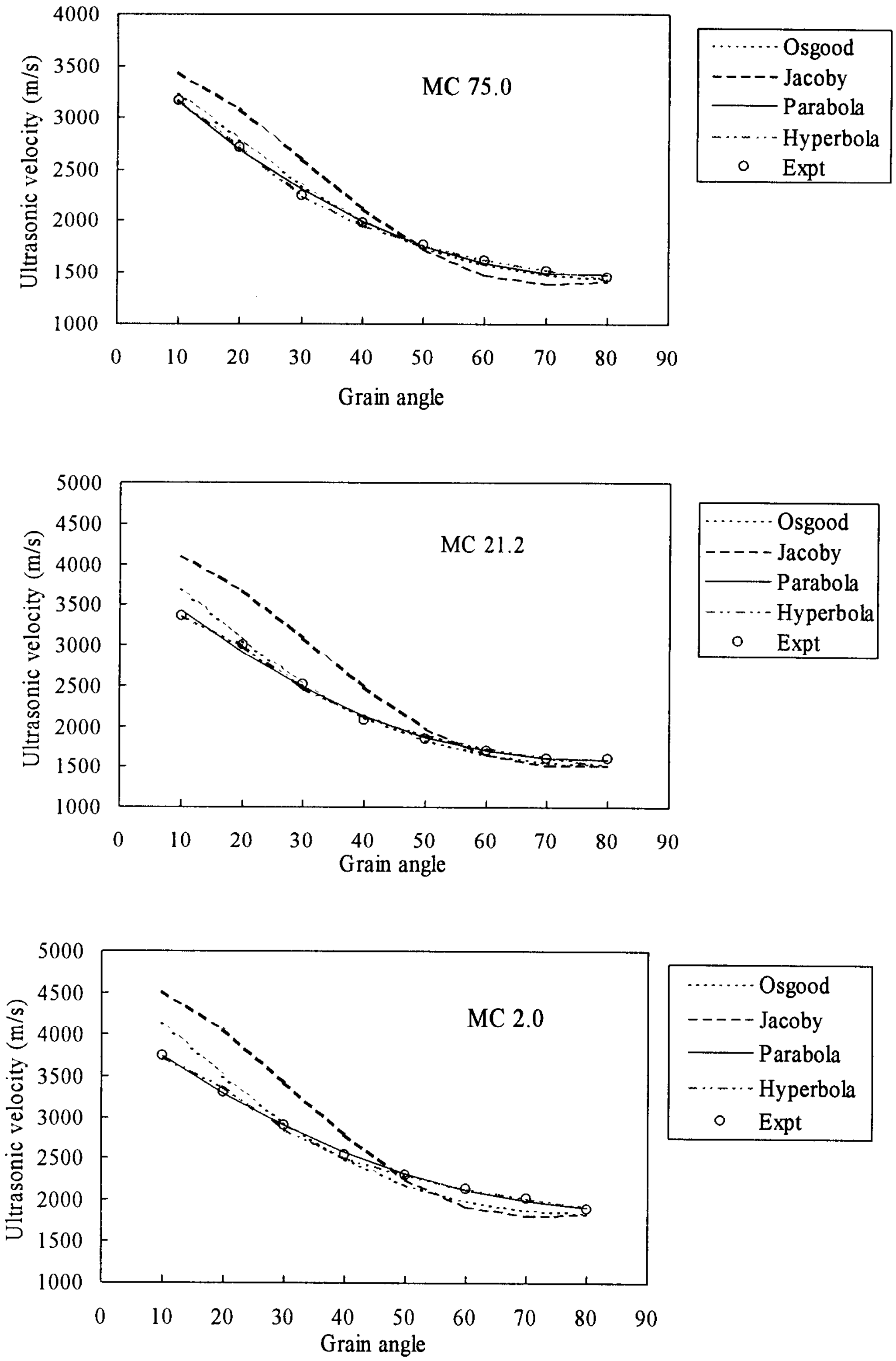

Figure 1. Experimental and predicted ultrasonic velocity using different equations in LR rotation.

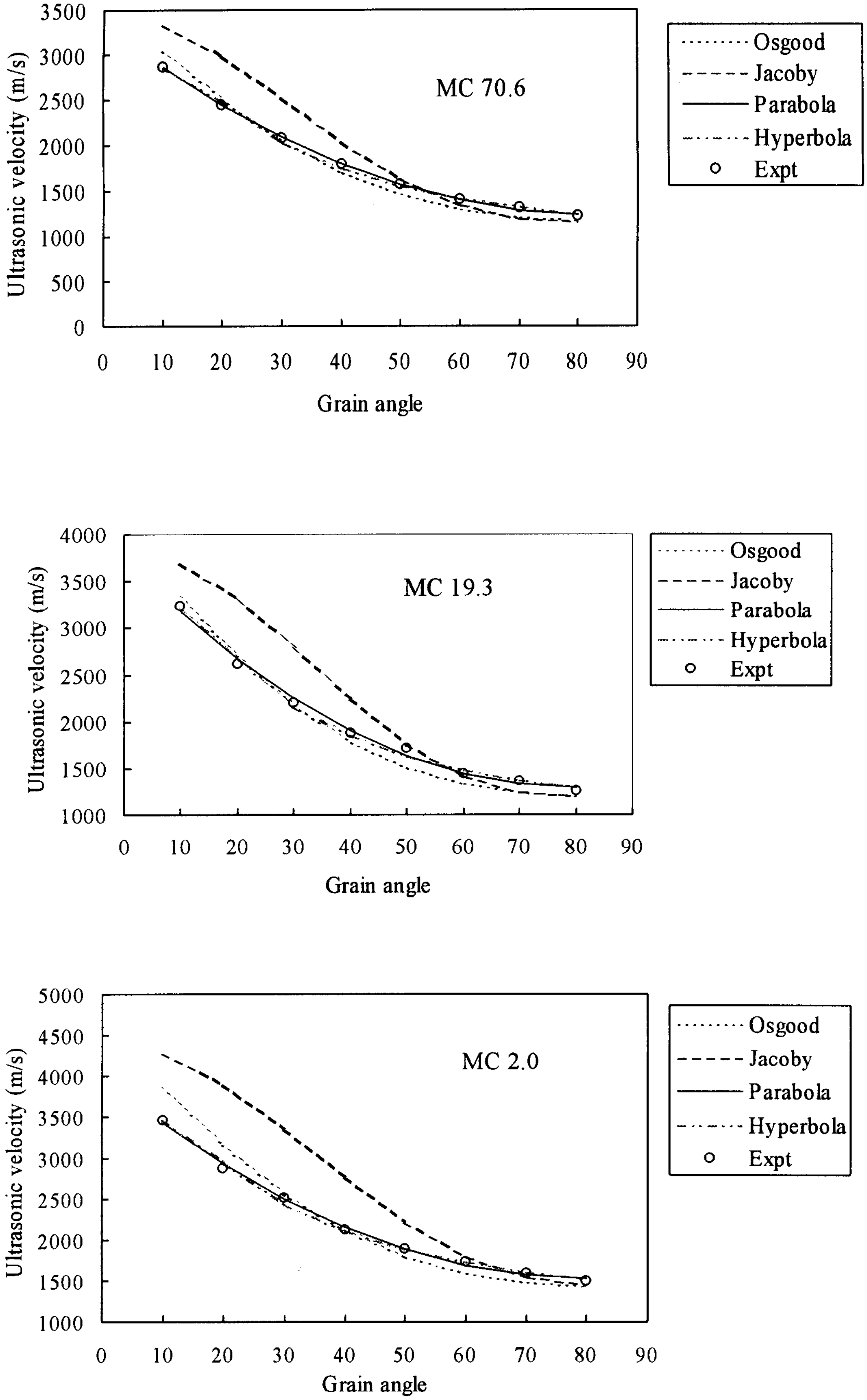

Figure 2. Experimental and predicted ultrasonic velocity using different equations in LT rotation.

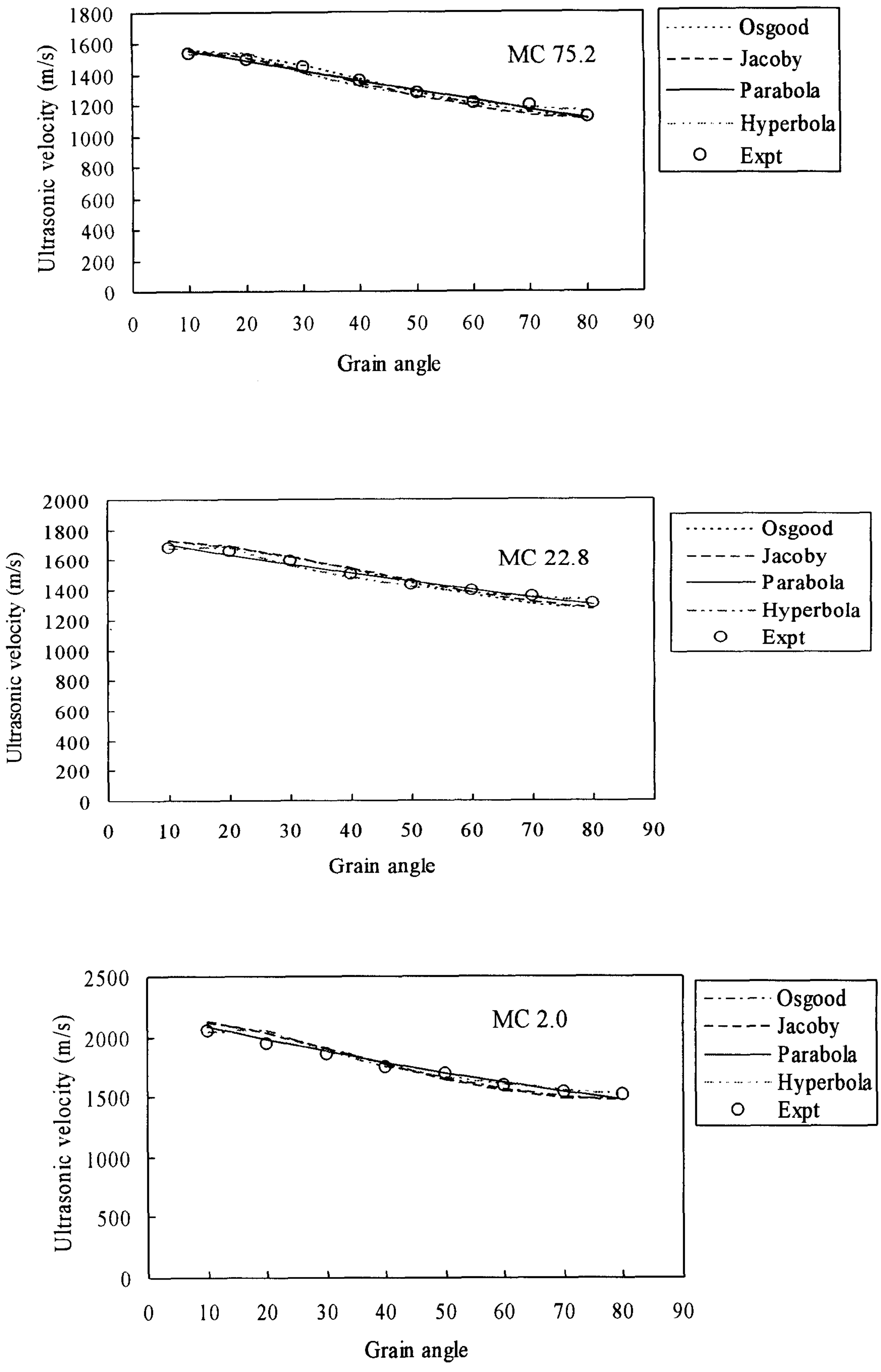

Figure 3. Experimental and predicted ultrasonic velocity using different equations in RT rotation.

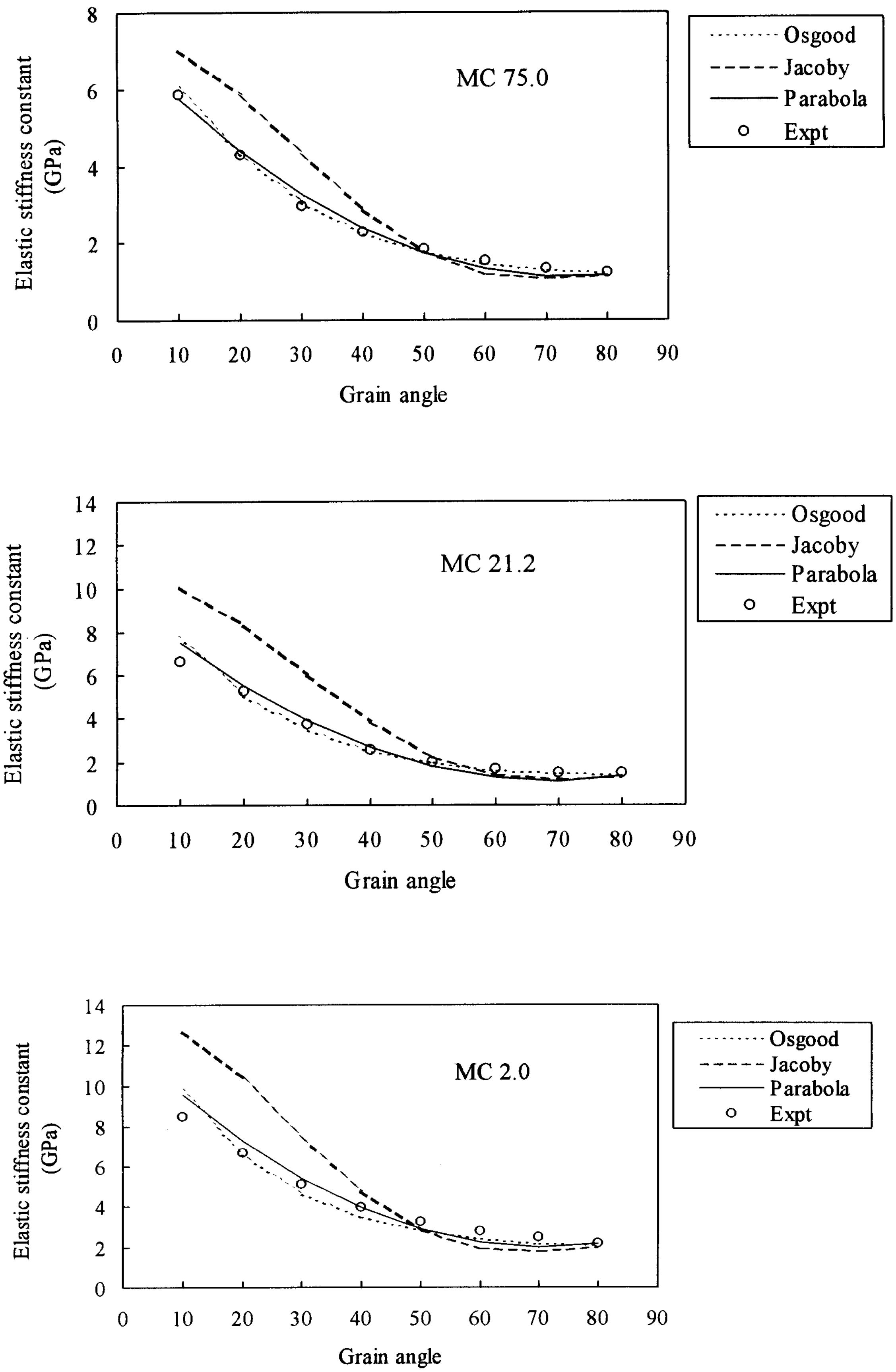

Figure 4. Experimental and predicted elastic stiffness constant, using different equations in LR rotation.

ISSN 0020-3203

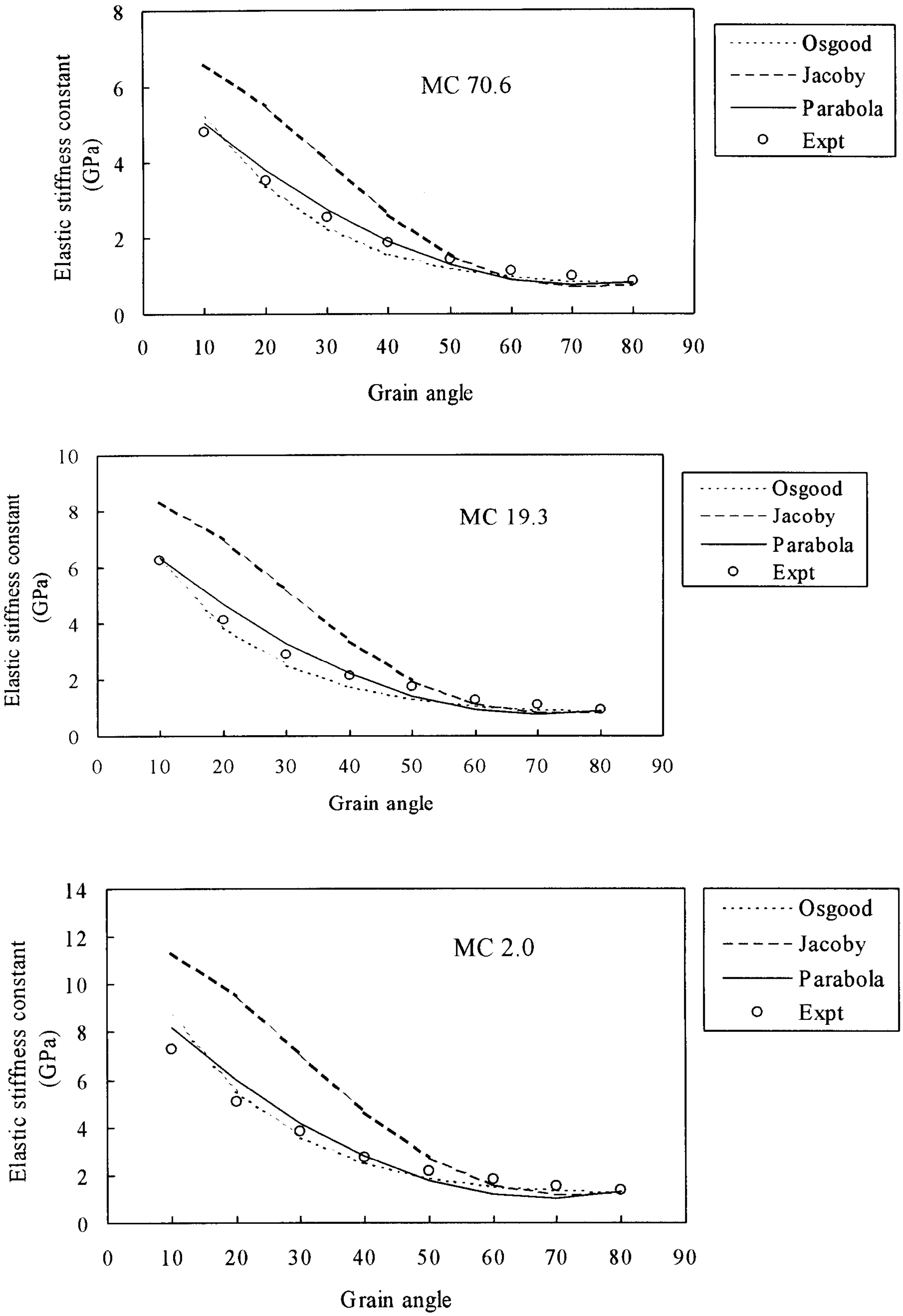

Figure 5. Experimental and predicted elastic stiffness constant, using different equations in LT rotation.

ISSN 0020-3203

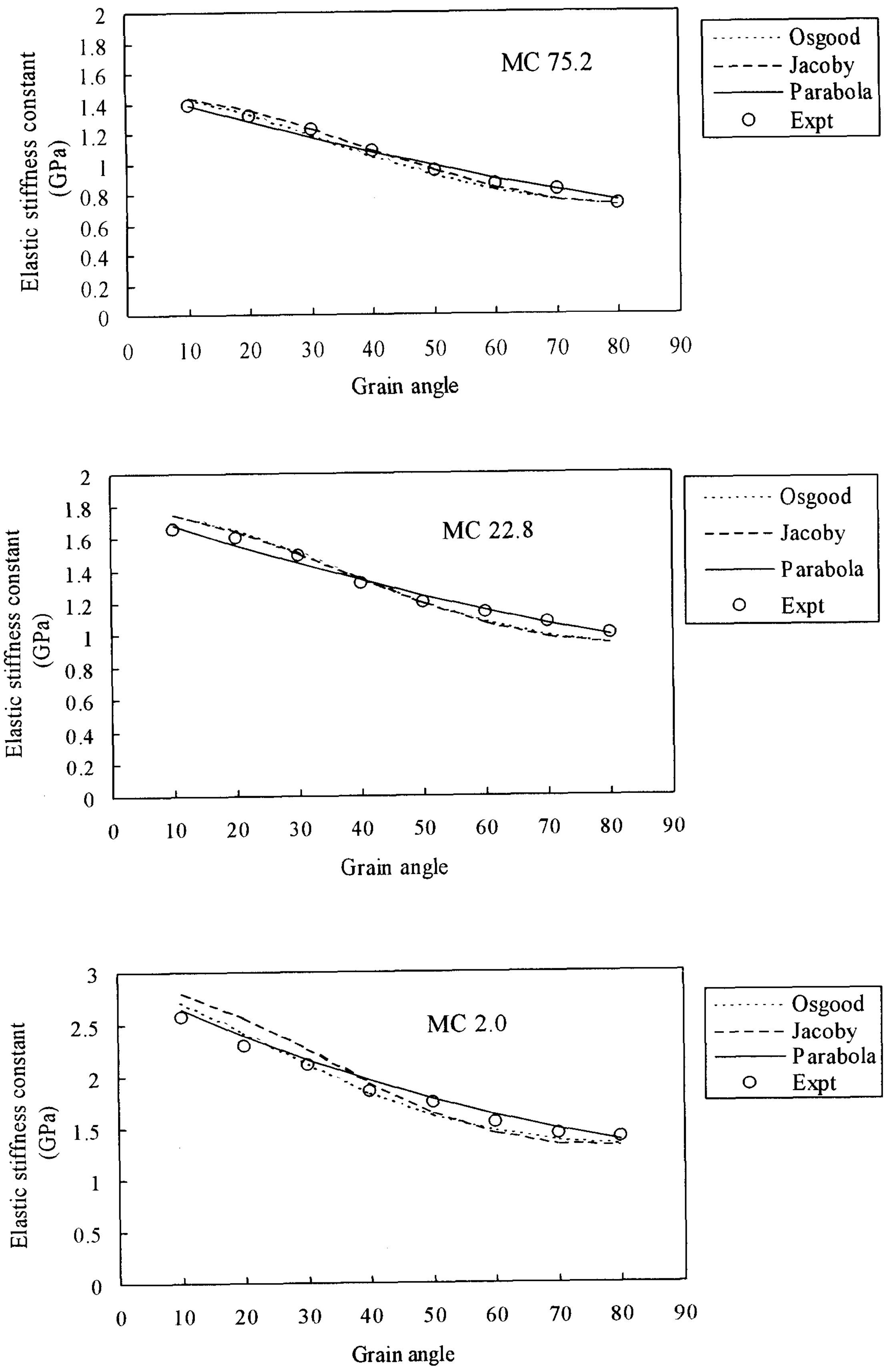

Figure 6. Experimental and predicted elastic stiffness constant, using different equations in RT rotation.

It is observed from Figure 1 that Hankinson and Osgood equations fitted very well for ultrasonic velocity in LR rotations. The AAE values are found below 10% when the grain angle increases from 20° to 80° (Table 3). At lower MC with a 10° grain angle, the AAE values are slightly higher. In LT rotation, the predicted values are also close to the experimental value, although in some cases, the AAE values are higher than 10% (Table 3 and Figure 2). By neglecting the sign and regardless of the moisture content and the grain angle, the AAE values vary from 0.11 to 12.08 for Hankinson and from 0.76 to 11.8 for Osgood equations for LR and LT rotations. The predicted ultrasonic velocities using Hankinson and Osgood equations in RT rotation are in agreement with experimental results (Figure 3). This is clearly seen from Table 1 in which the AAE ranges from 0.11 to 4.09 for Hankinson and from 0.01 to 4.93 for Osgood.

Elastic stiffness constants also fitted well using Hankinson and Osgood equations in LR rotation (Figure 4 and Table 2). In the case of Hankinson's formula, AAE values lie between 0.18 to 12.7. The Osgood equation showed very good agreement with experimental data as seen from AAE values, which vary from 0.18 to 12.8. In the LT rotation, the prediction of elastic stiffness constants by both of these equations are found close to the measured value (Figures 5). At the intermediate angle of about 50°, the AAE values are comparatively high. The predicted values from these two equations are in close agreement with those of experimental data in RT rotation (Figure 6). The AAE values were found to vary from 0.33 to 7.23 for Hankinson and 0.58 to 8.29 for Osgood (Table 2).

Table 1. Values of n for different type of equations.

		Ultrasonic Velocity			Elastic Stiffness Constant		
Rotation	MC(%)	Hankinson	Osgood	Jacoby	Hankinson	Osgood	Jacoby
LR	75.0	1.6	1.1	2.8	1.8	1.3	2.5
	21.2	1.6	1.0	2.8	1.6	1.2	2.1
	2.0	1.6	1.0	2.8	1.6	1.1	2.1
LT	70.6	1.6	1.1	2.5	1.9	1.3	2.9
	19.3	1.6	1.1	2.8	1.9	1.3	2.8
	2.0	1.6	1.1	2.5	1.6	1.3	2.5
RT	75.2	2.0	1.6	2.1	2.1	1.5	2.2
	22.8	2.0	2.0	1.6	2.1	1.9	2.2
	2.0	1.9	1.4	2.2	1.8	2.5	1.2

Table 2. Parabolic and hyperbolic equations for ultrasonic velocity.

MC(%)		Parabolic Equation	r^2	Hyperbolic Equation	r^2
LR	75.0	$3680.26 - 57.05\theta + 0.3686\theta^2$	0.98	$844.87 + 51459.8/\theta - 283026/\theta^2$	0.99
	21.2	$4021.59 - 64.19\theta + 0.4224\theta^2$	0.98	$799.37 + 61932.6/\theta - 362989/\theta^2$	0.97
	2.0	$4254.04 - 54.95\theta + 0.3214\theta^2$	0.98	$1226.19 + 60310.3/\theta - 353340/\theta^2$	0.98
LT	70.6	$3338.02 - 50.79\theta + 0.3072\theta^2$	0.98	$639.70 + 52026.40/\theta - 297799/\theta^2$	0.98
	19.3	$3754.01 - 61.90\theta + 0.3900\theta^2$	0.98	$648.31 + 55289.60/\theta - 295229/\theta^2$	0.97
	2.0	$4001.19 - 61.61\theta + 0.3843\theta^2$	0.96	$856.12 - 57576.00/\theta - 315974/\theta^2$	0.98
RT	75.2	$1628.05 - 7.16\theta + 0.0096\theta^2$	0.95	$955.22 + 17750.30/\theta - 119487/\theta^2$	0.99
	22.8	$1776.28 - 7.35\theta + 0.0176\theta^2$	0.98	$1143.31 + 16427.30/\theta - 110593/\theta^2$	0.98
	2.0	$2204.21 - 11.65\theta + 0.0329\theta^2$	0.98	$1238.69 + 24863.50/\theta - 167411/\theta^2$	0.96

θ = grain angle

Table 3. Average Absolute Error (AAE) for the ultrasonic velocity.

Grain angle (degree)	LR Rotation			LT Rotation			RT Rotation		
	MC			MC			MC		
	75.0	21.2	2.0	76.0	19.3	2.0	75.2	22.8	2.0
Hankinson									
10	-0.33	-11.4	-11.7	-4.59	-1.81	-10.3	-1.79	-2.73	-3.61
20	2.06	-2.00	-4.87	1.10	0.45	-6.07	-0.90	-1.13	-4.09
30	1.08	0.23	-0.11	5.35	4.61	1.81	0.90	0.03	-1.60
40	3.49	-2.58	2.17	7.70	7.36	3.18	0.89	-0.54	-1.18
50	3.85	-0.92	4.01	7.69	12.08	5.75	0.46	0.15	2.22
60	3.10	-0.11	5.23	6.79	6.63	7.43	1.02	2.63	1.19
70	1.96	-0.21	4.84	6.58	6.93	5.16	3.47	3.44	1.24
80	0.11	3.03	1.05	2.62	2.93	3.17	-0.11	2.32	1.63
Osgood									
10	-2.33	-13.2	-11.0	-6.07	-3.01	-11.7	-2.04	-2.95	-3.69
20	-2.72	-2.68	-5.61	-2.94	-3.22	-10.1	-2.06	-2.17	-4.93
30	-4.41	0.30	-1.09	1.28	1.07	-2.06	-1.04	-1.74	-2.87
40	-0.76	-1.02	1.87	5.05	5.33	0.81	-1.25	-2.49	-2.32
50	1.42	2.50	4.68	6.79	11.8	5.23	-1.22	-1.37	1.66
60	2.41	4.15	6.75	7.38	7.76	8.34	0.14	1.83	1.25
70	2.51	5.20	6.80	8.01	8.74	6.86	3.27	3.26	1.69
80	1.07	3.05	2.80	4.02	4.52	4.69	-0.01	2.41	2.06
Jacoby									
10	-8.54	-22.3	-21.2	-16.1	-14.4	-23.8	-1.67	-3.05	-4.15
20	-13.6	-22.6	-22.4	-22.2	-26.5	-35.6	-0.80	-2.24	-5.29
30	-15.6	-22.5	-17.9	-21.5	-27.3	-33.9	-0.94	-1.97	-2.92
40	-6.73	-19.6	-9.16	-14.0	-19.3	-30.2	1.00	-3.14	-1.94
50	2.46	-6.96	1.91	-4.02	-2.73	-17.2	0.88	-2.43	2.44
60	8.03	2.59	-.9.69	4.69	2.30	-3.50	1.77	0.68	2.24
70	7.71	5.00	10.3	9.10	8.69	2.60	4.27	2.38	2.43
80	2.43	5.32	3.36	4.37	4.62	3.50	0.35	2.02	2.23

Table 4. Average absolute error (AAE) for the elastic stiffness constant.

Grain Angle (degree)	LR Rotation			LT Rotation			RT Rotation		
	MC			MC			MC		
	75.0	21.2	2.0	76.0	19.3	2.0	75.2	22.8	2.0
Hankinson									
10	-3.43	-12.7	-17.0	-13.9	-6.30	-6.29	-4.21	-6.20	-5.68
20	1.79	10.2	0.79	-0.13	1.72	7.97	-2.78	-3.50	-4.62
30	-0.46	12.5	8.24	9.06	10.6	19.2	1.01	-1.21	1.22
40	3.31	3.2	9.08	12.8	14.6	16.9	1.29	-2.13	1.72
50	3.01	1.85	9.13	11.9	21.7	16.6	0.66	-0.47	7.23
60	1.25	0.06	9.11	9.69	10.6	15.9	1.84	-4.64	4.04
70	-0.18	-1.80	7.47	9.73	11.1	9.59	6.64	6.39	3.12
80	-1.91	4.84	0.65	3.45	4.31	5.38	-0.33	4.42	3.34
Osgood									
10	-3.47	-17.8	-15.3	-8.24	-0.18	-18.7	-3.07	-5.63	-5.57
20	-1.47	2.57	0.57	3.06	5.28	-9.08	-1.30	-3.23	-5.90
30	-3.81	6.71	8.88	11.5	13.3	6.81	2.49	-1.43	-0.46
40	1.99	0.31	11.4	15.8	17.7	9.07	2.98	-2.36	0.60
50	3.78	2.10	12.8	15.6	25.1	13.2	2.66	-0.24	7.12
60	3.45	2.39	13.5	13.7	14.7	15.6	3.92	5.35	4.84
70	2.39	1.23	11.6	13.1	14.5	10.9	8.29	7.24	4.39
80	-0.18	6.87	3.49	5.49	6.39	6.74	0.58	4.95	4.37
Jacoby									
10	-20.2	-52.3	-49.2	-36.9	-33.3	-55.0	-3.91	-5.61	-9.06
20	-37.3	-58.5	-56.3	-57.1	-70.0	-88.6	-3.16	-2.89	-12.3
30	-47.2	-62.5	-46.7	-59.8	-79.7	-84.2	-0.66	-1.14	-7.49
40	-24.6	-52.1	-20.4	-39.4	-60.0	-67.9	-1.10	-2.3 1	-4.75
50	3.27	-11.2	12.7	-7.65	-12.4	-24.3	-1.15	-0.11	5.38
60	20.9	16.4	31.4	18.4	8.81	12.7	1.58	5.92	6.11
70	19.3	17.8	27.4	25.4	23.1	20.7	7.58	8.10	6.48
80	5.27	11.5	7.8	14.4	10.8	11.2	0.57	5.47	5.04

Hankinson and Osgood also provide a good fit for the experimental data of other mechanical properties, such as tensile stress parallel to grain as reported by Kim (1985).

The Jacoby equation is not suitable for predicting ultrasonic velocity and elastic stiffness constants when grain angles are less than about 50° for both the LR and the LT rotations. At a grain angle greater than 50°, these properties can be predicted well as shown in Figure 1, Figure 2, Figure 4 and Figure 5. In the RT rotation, both of these properties show very good fittings with the experimental data for all MC. The AAE value ranges from 0.35 to 5.29 for ultrasonic velocity and from 0.57 to 12.3 for elastic stiffness constant.

The value of the exponents in the RT rotation is higher than LR and LT for both Hankinson and Osgood for ultrasonic velocity. Regardless of the MC and the rotational direction, the exponent vary from 1.6 to 2.0 and 1.0 to 2.0 for Hankinson and Osgood respectively (Table 1). These values are within the range as mentioned in the Wood Handbook (Anon, 1987) for various mechanical properties of wood. The value of the exponents for elastic stiffness constant in LR rotation are slightly lower compare to LT and RT rotations for both the Hankinson and the Osgood equations. The exponents for ultrasonic velocity in the Osgood equation are also lower than elastic stiffness constant. The RT direction showed lower value of the exponent for the Jacoby equation. These results suggest that the rotational direction is the important factor for the value of the exponent. The Jacoby equation exhibited higher values of the exponent than those found for some Japanese wood species (Mishiro, 1996).

It is found from Table 2 that the second order parabolic and second order hyperbolic equations can be used for estimating ultrasonic velocity and elastic stiffness constant from the grain angles. The constants A, B and C were found to increase with MC for both the parabolic and the hyperbolic functions for all the rotational planes. The r^2 values, which vary from 0.95 to 0.99, showed that the ultrasonic velocity could be calculated from grain angles by using these equations. Armstrong *et al.* (1991) also reported similar equations for some other species.

CONCLUSIONS

The ultrasonic velocity and elastic stiffness constant at different grain angle can be predicted using some empirical equations and also with parabolic and hyperbolic regression equations. The predicted ultrasonic velocity and elastic stiffness constant lie close to measured value for both LR and LT rotation using the Hankinson and the Osgood equations. The Jacoby equation showed good agreement only at angles greater than 50° for these rotational directions. The high values the exponent in Hankinson and Osgood are used in RT direction for fitting the experimental data compare to LR and LT directions. The second order parabolic and hyperbolic equations can also be used for estimating ultrasonic properties from grain angle having high r^2 values.

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