EQUIVALENT CIRCUIT MODELING OF THE DIELECTRIC PROPERTIES OF RUBBER WOOD AT LOW FREQUENCY

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ABSTRACT

Dielectric properties of rubber wood were studied at various moisture contents and grain directions at low frequencies from 10^{-2} to 10^{5} Hz. Results showed that the moisture content of wood affected the dielectric properties considerably. Dielectric data at different anisotropic directions, i.e., longitudinal, radial, and tangential and at various moisture contents were modeled using universal capacitance. The equivalent circuits for different dielectric processes observed in wood were expressed by the combinations of diffusive, quasi-dc elements together with nondispersive capacitance and resistance. Three equivalent circuits for wood with moisture contents—a) above fiber saturation point, b) below fiber saturation point, and c) very low moisture content or oven-dried wood—are obtained for three grain directions. The experimental data are in close agreement with the value obtained from equivalent circuit.

Keywords: Dielectric constant, dielectric loss factor, moisture content, universal capacitance, susceptibility, diffusive, quasi-dc.

INTRODUCTION

Dielectric properties of wood have both theoretical and industrial applications. They also provide a better understanding of the molecular structure of wood and wood-water interactions. The behavior of water with the constituents of wood such as cellulose and lignin can be understood more clearly by studying dielectric properties. These properties can be used for determining the density and moisture content of wood by nondestructive electrical measurements. It is also reported that the detection of knots, drying defects, and spiral grain are possible by measuring dielectric properties of wood (Martin et al. 1987). The dielectric properties of rubber wood at different moisture contents (MC) were reported earlier (Kabir et al. 1998a). The dielectric properties above 25–30% MC showed almost a single mechanism. As the moisture content decreases from around 25%, the dielectric constant and dielectric loss factor decrease abruptly (Kabir et al. 1998b). The shape of the curves changes accordingly, indicating a different dielectric mechanism involved in the processes.

The interaction of wood with a low frequency electrical field has not yet been studied. It may provide a valuable basis for determining fiber saturation point (FSP) and defects in wood nondestructively, and also may help to understand wood-water interactions and hygroscopic characteristics of wood. This paper deals with the modeling of the dielectric prop-

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erties using an electrical circuit consisting of a capacitor and a resistor.

THEORETICAL CONSIDERATIONS

The classical expression relating the complex permittivity to the frequency is based on Debye (1945), in which the susceptibility function can be given by

$$\chi^*(\omega) \propto \frac{1}{1 + i\omega/\omega_p} \qquad (1)$$

where χ^* is the complex susceptibility, ω is the angular frequency, $\propto$ indicates proportionality.

The Debye response is suitable for presenting the dielectric properties of some liquids and gases. Since the behavior of most dielectric materials deviates in varying degree from the Debye responses Cole-Cole (1941) modified the Debye equations by introducing a parameter α. The Cole-Cole susceptibility function can be written as

$$\chi^*(\omega) \propto \frac{1}{1 + (i\omega/\omega_p)^{1-\alpha}} \qquad (2)$$

where α is an empirical parameter that lies between 0 and 1, still having a symmetrical loss peak, but broader than the Debye function. Davidson and Cole (1951) later modified this expression to fit the experimental data, which is given by

$$\chi^*(\omega) \propto \frac{1}{1 + (i\omega/\omega_p)^{\beta}} \qquad (3)$$

where β is a parameter in the range from 0 to 1. The parameters α and β used in these equations were determined empirically and have no physical significance. Most dielectric behavior cannot be explained by either of these two equations. Havrilak and Negami (1966) proposed a generalized form by combining these two equations, which yields

$$\chi^*(\omega) \propto \frac{1}{\{1 + (i\omega/\omega_p)^{1-\alpha}\}^{\beta}} \qquad (4)$$

The above-mentioned equations, such as Cole-Cole, Davidson-Cole, and Havrilak-Negami are usually associated with a distribution of relaxation times. The best result can be obtained with two parameter Havrilak-Negami functions for fitting of experimental data.

Jonscher (1983, 1990a) introduced the concepts of power law frequency response of dielectric relaxations that were found suitable for fitting experimental data for a wide range of dielectric materials. According to this universal law, the complex capacitance and corresponding susceptibility can be expressed by

$$\chi^*(\omega) \propto C^*(\omega) = B(i\omega)^{n-1}$$
$$= B\{\sin(n\pi/2) - i\cos(n\pi/2)\}\omega^{n} \qquad (5)$$

where C* is the complex capacitance, B is the proportional constant, and the exponent n defines the frequency dependence that lies between 0 and 1. Another important feature for this power law relation is that the real and imaginary components maintain a constant frequency-independent ratio (Jonscher and Pickup 1985; Jonscher and Robinson, 1988), i.e.,

$$\frac{C''(\omega)}{C'(\omega) - C_\infty} = \frac{\chi''(\omega)}{\chi(\omega)} = \cot(n\pi/2) \qquad (6)$$

Based on the concept of universal law, the experimentally observed behavior for different dielectric dispersions can be explained with dipolar, quasi-dc, and diffusive mechanisms (Hill and Pickup 1985; Zaidi and Jonscher 1987). For the bound dipolar case, the fractional power law behavior is given by

$$\chi^*(\omega) \propto \chi(0)(i\omega/\omega_p)^{n-1} \quad \text{for } \omega \gg \omega_p \qquad (7)$$

and

$$\chi(0) - \chi^*(\omega) \propto \chi(0)(i\omega/\omega_p)^{m} \quad \text{for } \omega \ll \omega_p \qquad (8)$$

where ω_P is the peak frequency, $\chi(0)$ is the susceptibility at very low frequency. We can write the alternative equations for bound dipolar case by using Eq. (6) as

$$\chi''(\omega) = \cot(n\pi/2)\chi'(\omega)\alpha\omega^{n-1} \quad \text{for } \omega \gg \omega_p \tag{9}$$

$$\chi''(\omega) = \tan(m\pi/2)[\chi(0) - \chi'(\omega)]\alpha\omega^{m} \quad \text{for } \omega \ll \omega_p \tag{10}$$

where χ' and χ'' are the real and imaginary parts of the susceptibility, respectively. According to Jonscher, this susceptibility function can be expressed as

$$\chi''(\omega) \propto \frac{1}{(\omega/\omega_p)^{-m} + (\omega/\omega_p)^{1-n}} \tag{11}$$

The quasi-dc dispersion has been observed experimentally at frequencies below 1 Hz in which some of the charges are weakly bound and partially free to move (Ramdeen et al. 1984; Rowe et al. 1988; Jonscher 1991). For frequencies less than characteristics rate ω_c, the quasi-dc dispersion is represented by

$$\chi(\omega) \propto \chi(0)(i\omega/\omega_c)^{-p} = \chi(0)(\omega/\omega_c)^{-p} \times \{\cos(p\pi/2) - \sin(p\pi/2)\} \tag{12}$$

and for frequencies greater than the characteristics rate

$$\chi(\omega) \propto \chi(0)(i\omega/\omega_c)^{n-1} = \chi(0)(\omega/\omega_c)^{n-1} \times \{\sin(n\pi/2) - i\cos(n\pi/2)\} \tag{13}$$

The equivalent presentation for this behavior can be written as

$$\chi''(\omega) = \cot(n\pi/2)\chi'(\omega) \propto \omega^{n-1} \quad \omega \gg \omega_c \tag{14}$$

$$\chi''(\omega) = \tan(p\pi/2)\chi'(\omega) \propto \omega^{-p} \quad \omega \ll \omega_c \tag{15}$$

The exponent n in dipolar response defines the degree to which the displacements of a dipole to those of its environment form a cluster and is also a measure of the structural order of the cluster (Dissado et al. 1987). The exponent m is a measure of the extent to which one cluster can affect the others and can be taken as the efficiency of dipolar displacement between clusters. Therefore, as the value of m approaches unity, it describes a situation in which the disturbance is spread almost homogeneously over the system as equilibrium is approached. In quasi-dc process, p is a measure of the efficiency of the charge transport between clusters. The deviation of p from unity defines the degree of inefficiency of the charge transport in the system and is a measure of the degree of long-range homogeneity.

In the dipolar case, it is seen that in the high frequency range of the loss peak ($>\omega_P$), $\chi'(\omega)$ and $\chi''(\omega)$ exhibit a constant ratio, independent of frequency. The quasi-dc relaxation is characterized by two independent processes, below and above a certain characteristic frequency ω_c without any loss peak. In the quasi-dc process, the real and imaginary parts of the susceptibility increase steadily with decreasing frequency, with a small exponent value of p at frequency $<\omega_c$, followed by a flat loss behavior above ω_c (Jonscher 1990b).

The Dissado-Hill theory (Dissado and Hill 1982; Pathmanathan et al. 1985; Dissado et al. 1987) explained the dipolar and quasi-dc response in terms of cluster models. They assumed that the two independent processes in the dipolar and quasi-dc response above and below ω_P or ω_c are due to the intra-cluster and inter-cluster exchange mechanisms, respectively.

Another type of dielectric response has been found experimentally and is attributed to diffusion (Dissado and Hill 1984; Doyle and Jonscher 1986). This type of dielectric response has been expressed as

$$\chi(\omega) \propto \chi(\omega_d)(i\omega/\omega_d)^{-s} \qquad s \cong 0.5 \tag{16}$$

which defines a diffusion susceptibility of magnitude $\chi(\omega_d)$ at frequency ω_d.

The parameters used in the Cole-Cole, Davidson-Cole, and Havrilak-Negami functions have no physical significance, and the value has been determined by empirical means. On the other hand, the exponent used in the universal law or in dipolar and quasi-dc response

TABLE 1. *Values of the different exponent and circuit element used in equivalent circuit modeling.*

	Quasi-dc						Diffusive			
MC (%)	C_1	C_2	p	n	f_1	f_2	C_s	s	C	R
Longitudinal										
99.8	1.7e8	7.4e7	0.93	0.06	280	1e-3	7.6e15	0.29	6.3e4	
14.5	1.2e5	9.5e2	0.06	0.42	18	6e-2			1.6e4	7e-4
4.1	8.5e-2	1.2e-1	0.008	0.46	2e-2	4e-3			4.2e-3	1.5e0
Radial										
98.2	1.3e7	9.7e6	0.99	0.05	280	1e-3	1.2e13	0.52	6.3e12	
14.1	2.7e4	3.3e2	0.04	0.42	1e1	6e-2			4.4e3	2e-3
4.3	6.4e-2	6.1e-2	0.01	0.40	1.5e-2	4e-4			4.2e-2	1.5e0
Tangential										
98.5	2.4e7	1.4e7	0.45	0.04	280	1e-3	1.6e13	0.43	7.3e12	
14.2	3.1e5	8.5e2	0.005	0.4	40	6e-2			1.1e4	8e-4
3.8	8.8e-2	1.2e-1	0.006	0.42	2.5e-2	4e-3			3.5e-3	1.8e0

has physical significance regarding the degree of ordering in the system.

The dielectric behavior so far discussed involves dielectric loss due only to the polarization mechanism. Many of the dielectric materials exhibit loss with the conduction of charge carriers and add to the total loss. This conduction loss is given by

$$\epsilon_\sigma'' = \frac{\sigma}{\omega} \tag{17}$$

where ϵ'' is the dielectric loss factor and σ is the electrical conductivity of the material. It is characteristic of dc conduction that ϵ'' shows a slope of –1 at low frequency with ϵ' (dielectric constant) independent of frequency.

MATERIALS AND METHODS

Dielectric properties, such as dielectric constant and dielectric loss factor of rubber wood *(Hevea brasiliensis,* Muell. Agr.), were measured by using a Dielectric Spectrometer consisting of Chelsea Dielectric Interface (CDI 4c/L-4, Dielectric Instrumentation, UK) and Frequency Response Analyzer (SI 1255, Schlumberger Technologies, UK). Rubber wood was supplied by the Farm Department of University Putra Malaysia. The measurements were carried out for three anisotropic directions—longitudinal, radial, and tangential at frequencies from 10^{-2} to 10^5 Hz. Specimens were prepared in the form of discs of 35–40 mm in diameter and 3.0–3.5 mm in thickness. For measuring the dielectric properties at different moisture contents, specimens were fully soaked in water for a sufficiently long time to achieve full saturation. After that, the specimens were weighed, and measurement was carried at room temperature. They were then dried in air to reduce the moisture. This cycle of measuring, drying, and weighing was repeated until the specimens showed no change of weight by drying. Finally the specimens were dried in an electric oven at 100 ± 3°C for 24 h, and oven-dry weight was taken. Moisture content of the specimens was determined based on the oven-dry weight.

RESULTS AND DISCUSSION

Equivalent circuits have been determined using different combinations of quasi-dc (12 and 13), diffusive (16), nondispersive capacitance and resistance to fit the experimental results at different MC and grain directions. The values of each component and exponents for these models at different moisture content are presented in Table 1. The working expressions for these models are presented in the Appendix.

The change of moisture content above fiber saturation point (FSP) does not contribute much to the dielectric properties in all grain

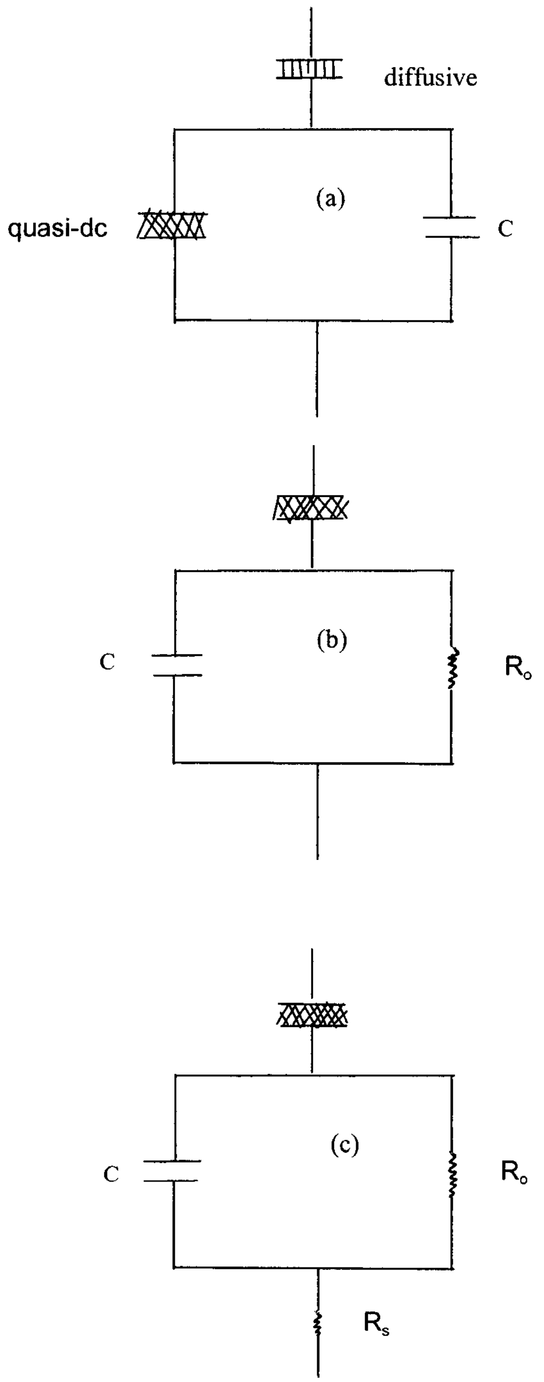

FIG. 1. Equivalent circuit for fitting the experimental results at low frequency.

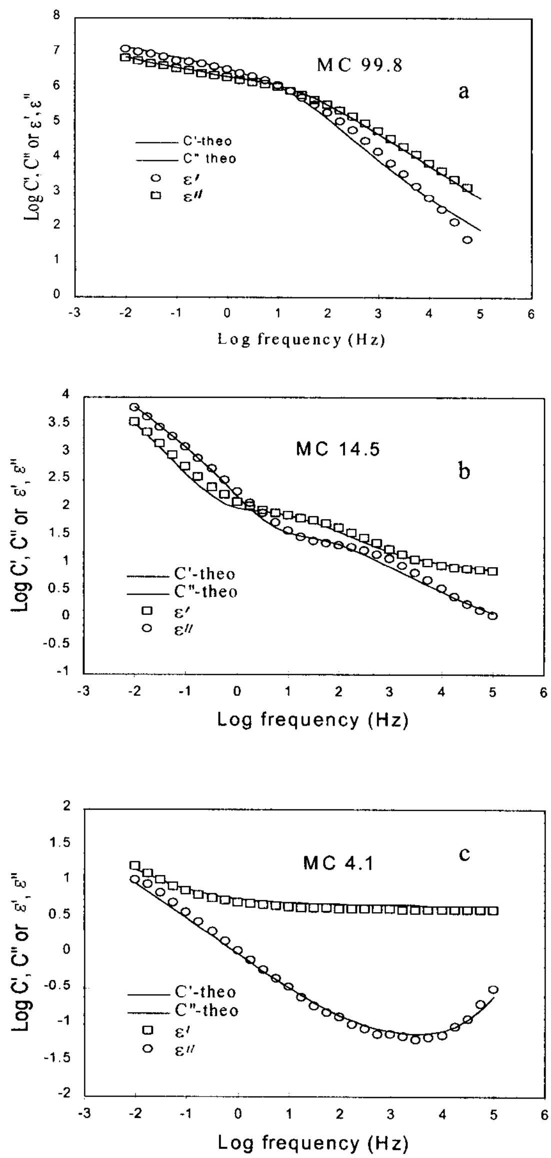

FIG. 2. Experimental and theoretical value obtained from equivalent circuit in longitudinal direction.

directions, as reported earlier (Kabir et al. 1998a). Above FSP (25–30% MC), the dielectric mechanism can be expressed by diffusive (16) in series with the parallel combination of quasi-dc and nondispersive capacitance. The equivalent circuit is shown in Fig. 1(a). The experimental and theoretical values are presented in Figs. 2(a), 3(a), and 4(a) for longi-

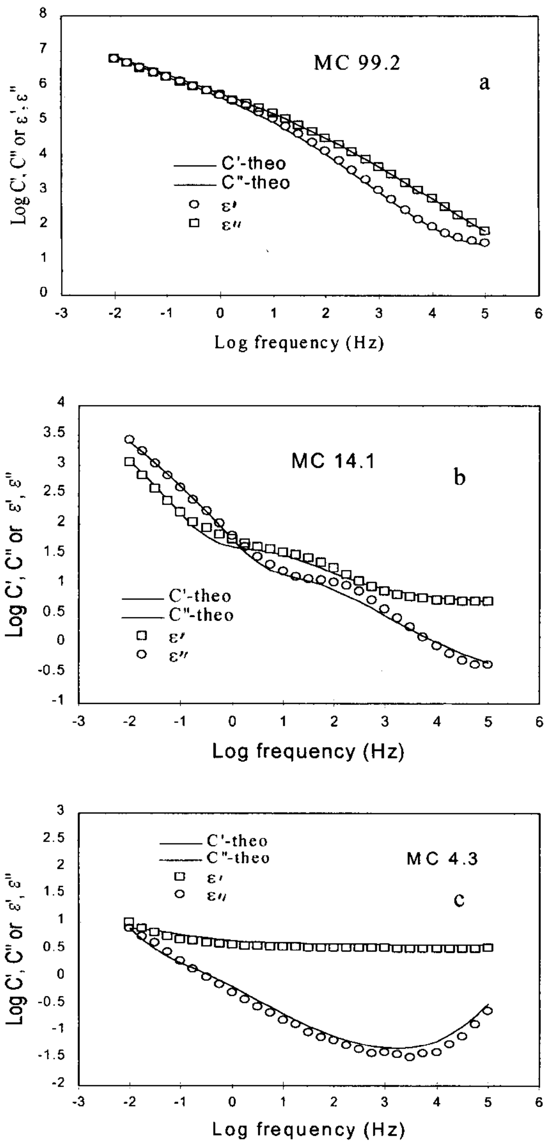

FIG. 3. Experimental and theoretical value obtained from equivalent circuit in radial direction.

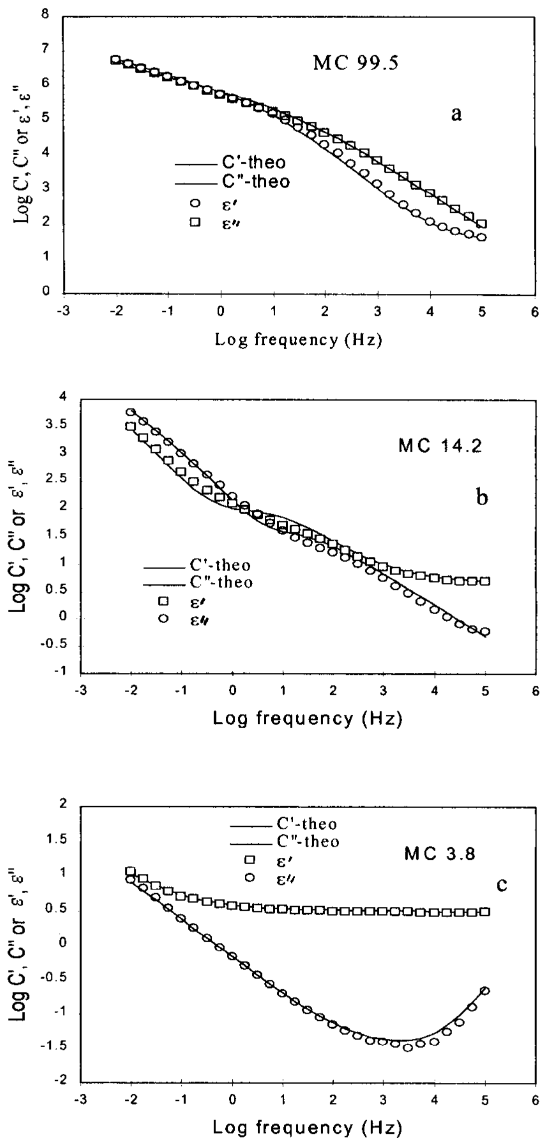

FIG. 4. Experimental and theoretical value obtained from equivalent circuit in tangential direction.

tudinal, radial, and tangential directions, respectively. It is shown that the theoretical values are in good agreement with the experimental ones for all grain directions. Regardless of the grain direction, the values of p lie between 0.45 and 0.99, whereas n ranges from 0.04 to 0.06 for this MC (Table 1). In the free water region, diffusion may occur between the vessel and cell wall. The addition of quasi-dc mechanism to the overall processes may be explained by a cluster model (Dissado et al. 1987).

Below a characteristic frequency (Zaidi and Jonscher 1987), the charge transport takes place between the cluster, i.e., in between the amorphous and crystalline parts of the cellulose. At a frequency greater than characteristic frequency, transport of charge takes place within the amorphous region because the amorphous part of the cellulose is highly hygroscopic compared to the crystalline region and lignin. The high value of p, which is defined as the efficiency of the charge transport between the clusters, suggested that major transport of charge occurred between the amorphous and crystalline portion of the cellulose at high MC.

Below FSP the response can be expressed by quasi-dc in series with the parallel combinations of nondispersive capacitance and resistance as shown in Fig. 1(b). The experimental and theoretical values are presented in Figs. 2(b, c), 3(b, c), and 4(b, c) for longitudinal, radial, and tangential directions, respectively. The values of the exponent n and p are in the ranges of 0.4–0.46 and 0.005–0.06, respectively (Table 1). The lower value of n at these MC levels compared to higher MC means that the transport of charge takes place within the cluster, i.e., within the amorphous or crystalline region. At low frequency and low MC (<7%), the presence of dc conductivity was observed, as shown in Table 1. Finally, when MC is less than 10%, a small resistance was used for fitting the experimental results (Fig. 1c). This is due to the presence of electrode resistance at very low MC. Millany and Jonscher (1980) also reported the presence of series resistance at about 1 MHz.

CONCLUSIONS

Dielectric properties of wood were affected significantly by moisture content for all anisotropic directions. Above FSP the variation of moisture content does not affect the dielectric constant and dielectric loss factor of wood significantly—an indication of a single dielectric mechanism. Below FSP the decrease of moisture content increased the dielectric properties accordingly. Three different dielectric processes, which are the combination of quasi-dc, diffusive and nondispersive capacitance and resistance, either in series or in parallel, were identified for wood at various moisture contents, namely 1) above fiber saturation point, 2) below fiber saturation point, and 3) at very low moisture content or for oven-dried wood. At moisture content above FSP, equivalent circuit can be expressed by diffusive in series with a parallel combination of quasi-dc and nondispersive capacitance. Below fiber saturation point, equivalent circuit consists of quasi-dc in series with a parallel combination of nondispersive capacitance and resistance. A series resistance was observed for oven-dried wood or wood with low moisture contents. The quasi-dc behavior can be explained by the transport of charge between or within the cluster of amorphous or crystalline region.

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APPENDIX

Working expression for equivalent circuit modeling

According to Fig. l(a):

The diffusive element can be expressed by

$$C_s^* = C_s(i\omega)^{-s} \tag{18}$$

$$= R_0\frac{\omega^{s-1}}{R_0C_s}\left[\sin\frac{s\pi}{2} - i\cos\frac{s\pi}{2}\right]$$

$$= R_0\left(\frac{f}{f_s}\right)^{s-1}\left[\sin\frac{s\pi}{2} - i\cos\frac{s\pi}{2}\right];$$

$$C_s = \frac{(2\pi f_s)^{s-1}}{R_0} \tag{19}$$

Quasi-dc behavior can be represented by

$$C_1^* = C_1(i\omega)^{-p} \qquad \omega \ll \omega_c \tag{20}$$

$$C_2^* = C_1(i\omega)^{n-1} \qquad \omega \gg \omega_c \tag{21}$$

The parallel combination of quasi-dc and capacitance can be expressed as

$$Y^* = i\omega C_1^* + i\omega C_2^* + i\omega C \tag{22}$$

$$= C_1(i\omega)^{1-p} + C_2(i\omega)^n + i\omega C$$

$$= \frac{1}{R_0}[R_0C_1(i\omega)^{1-p} + R_0C_2(i\omega)^n + i\omega R_0C]$$

$$= \frac{1}{R_0}\left[\left(\frac{f}{f_1}\right)^{1-p}\{\sin p\pi/2 + i\cos p\pi/2\} + \left(\frac{f}{f_2}\right)^n\{\cos p\pi/2 + i\sin p\pi/2\} + i\left(\frac{f}{f_0}\right)\right] \tag{23}$$

where

$$C_1 = \frac{(2\pi f_1)^{p-1}}{R_0} \tag{24}$$

$$C_2 = \frac{(2\pi f_2)^{-n}}{R_0} \tag{25}$$

$$C = \frac{(2\pi f_0)}{R_0} \tag{26}$$

From Eqs. (19) and (23), C* = C' – iC" can be calculated.

According to Fig. l(b):

$$Y_q^* = i\omega C_1^* + i\omega C_2^* \tag{27}$$

$$= \frac{1}{R_0}[R_0C_1(i\omega)^{1-p} + R_0C_2(i\omega)^n]$$

$$= \frac{1}{R_0}\left[\left(\frac{f}{f_1}\right)^{1-p}\{\sin p\pi/2 + i\cos p\pi/2\} + \left(\frac{f}{f_2}\right)^n\{\cos p\pi/2 + i\sin p\pi/2\}\right] \tag{28}$$

$$Y_1^* = G_0 + i\omega C$$

$$= \frac{1}{R_0}[1 + i\omega R_0C]$$

$$= \frac{1}{R_0}[1 + i\omega\omega_0] \tag{29}$$

where

$$C = \frac{(\omega_0)}{R_0} \tag{30}$$

$$Y^* = Y_q^* + Y_1^* \tag{31}$$

From Eq. (31), C* = C' – iC" can be determined.