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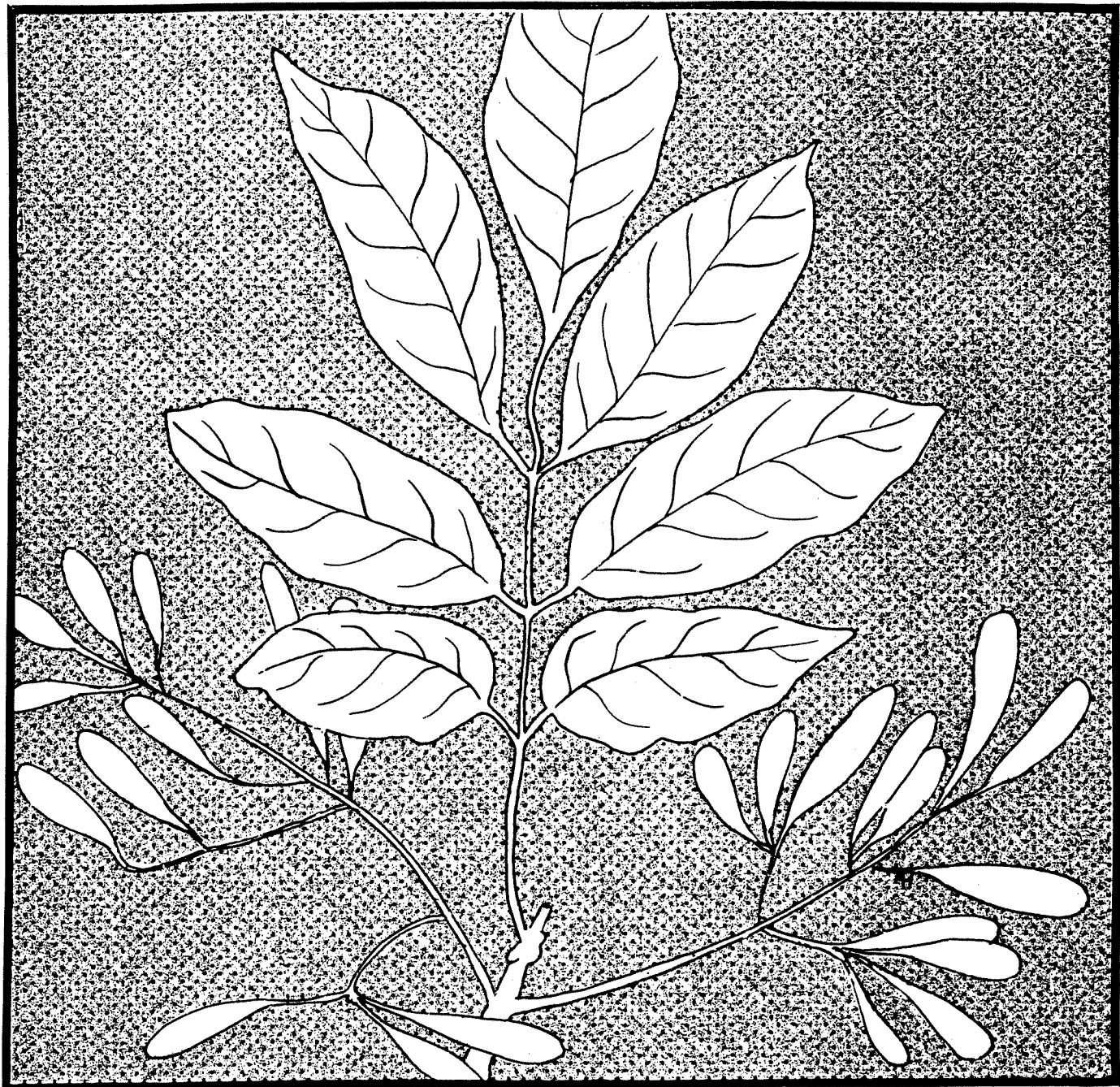
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# Green Ash Volume and Weight Tables

Bryce E. Schlaegel



## SUMMARY

Volume and weight tables were constructed from a 70-tree sample of green ash (*Fraxinus pennsylvanica* Marsh.) taken in the Mississippi Delta. The tables present cubic-foot volume, green weight, and dry weight of bole wood, bole wood plus bark, and total tree excluding leaves above a 1-foot stump as predicted from the allometric model  $\ln Y = b_0 + b_1 \ln(D^2H)$ . Merchantable bole volume and weight estimates can be made to any outside bark diameter limit. More precise bole estimates can be obtained from equations using upper bole diameters taken at relative heights of 25, 33, and 50 percent of total tree height.

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## INTRODUCTION

Green ash (*Fraxinus pennsylvanica* Marsh.) is distributed over most of the United States and southern Canada east of the Rocky Mountains (Little 1979) (fig. 1). It is most commonly found on alluvial soils along rivers and streams. On good sites it can develop a large, clear bole, reach a height of 110 feet, and have a dbh in excess of 30 inches. Commonly the trees reach heights of 60 to 80 feet with dbh ranging from 22 to 26 inches.

In recent years there has been considerable demand for volume and weight tables for bottomland hardwood species. This paper is the sixth in a series of six giving both volume and weight tables for some of the major bottomland hardwood species. The other five species in the series are willow oak (*Quercus phellos* L.) (Schlaegel 1981), Nuttall oak (*Q. nuttallii* Palmer) (Schlaegel and Willson 1983), overcup oak (*Q. lyrata* Walt.) (Schlaegel 1984a), sugarberry (*Celtis laevigata* Willd.) (Schlaegel 1984b), and sweetgum (*Liquidambar styraciflua* L.) (Schlaegel 1984c).

## METHODS

The data were collected from 12 natural bottomland hardwood stands in west-central Mississippi. Stands were either even- or uneven-aged with a mixed species composition. Each stand was measured for species composition and diameter distribution; 70 trees, ranging in dbh from 1 through 31 inches, were chosen for destructive sampling. The sample consisted of growing stock trees from both the overstory and understory with healthy crowns and no visible signs of disease or decay. When available, three trees were selected from each 1-inch diameter class. Cutting was done throughout the year.

Stump height and total height were measured on each felled sample tree. Disks about 1-inch thick were cut from the bole at regular intervals from the stump to the top of the tree. These were then sealed in separate polyethylene bags for laboratory determination of moisture content and specific gravity. The first disk was at stump height and the remainder at 5-foot intervals measured from ground level for trees 5

intervals measured from ground level for trees 5 inches dbh and larger and at 3-foot intervals for trees smaller than 5 inches dbh. All limbs, including leaves, were weighed on 65 of the trees.

In the laboratory, both wood and bark moisture content and specific gravity were determined as follows:

1. Wood and bark were separated with a hammer and chisel.
2. Each component was weighed green.
3. Both wood and bark were soaked in water for at least an hour to ensure complete swelling.
4. Volumes were obtained by immersion (Heinrichs and Lassen 1970).
5. Wood and bark were dried in a forced-air oven at 105°C for at least 48 hours.
6. Wood or bark moisture content =  
$$\frac{\text{green weight} - \text{ovendry weight}}{\text{ovendry weight}}$$
7. Specific gravity =  
$$\frac{\text{ovendry wood or bark weight (g)}}{\text{green wood or bark volume (cm}^3)}$$

Average tree moisture content and specific gravity were calculated from weighted averages of the disk moisture contents and specific gravities; each disk specific gravity or moisture content was weighted by its squared average diameter.

Taper functions (Schlaegel 1981, Schlaegel and Willson 1983) of the form

$$Y = b_1(X^1 - 1) + b_2(X^2 - 1) + \dots + b_p(X^p - 1)$$

where

$$Y = d/D,$$

d = diameter at height h on the bole,

$$D = \text{dbh},$$

$$X = h/H,$$

h = height from ground to a specific measurement point,

$$H = \text{total height},$$

b<sub>i</sub> = coefficients estimated for each tree by linear regression; i = 1, 2, ..., p; 4 ≤ p ≤ 7,

were calculated for each tree, both inside and outside bark. Cubic-foot volumes were obtained for each tree by integrating the taper function to heights of specific top diameters to obtain both total and merchant-

Bryce E. Schlaegel is Principal Mensurationist at the Southern Hardwoods Laboratory, maintained at Stoneville, Mississippi, by the Southern Forest Experiment Station, Forest Service—USDA, in cooperation with the Mississippi Agricultural and Forestry Experiment Station and the Southern Hardwood Forest Research Group.

able bole volumes. Bole green and dry weights were calculated from estimated tree volume and weighted average tree density and moisture content.

Allometric regression equations were used to estimate volume, green weight, and dry weight of bole wood, bole wood plus bark, and bole wood plus bark plus limbs.

## RESULTS

Characteristics of the trees used in this study are:

	Average	Range
Age (years)	50	14 - 168
Dbh (in)	8.0	0.9 - 30.6
Total height (ft)	51	14 - 110
Wood moisture content (percent)	57	40 - 141
Bark moisture content (percent)	115	45 - 236
Wood specific gravity	0.563	0.416 - 0.694
Bark specific gravity	0.456	0.259 - 0.610
Wood + bark green density (lb/ft)	55.95	43.46 - 72.00
Wood + bark dry density (lb/ft)	33.84	26.23 - 40.55
Wood + bark moisture content (percent)	66	46 - 177
Wood + bark specific gravity	0.542	0.420 - 0.649

Individual tree volumes and weights were fitted to the allometric model:

$$\ln(Y) = b_0 + b_1 \ln(D^2 H) \quad (1)$$

where

$Y$  = the volume or weight variable of interest,

$\ln$  is a natural logarithm;

$b_0$  and  $b_1$  are coefficients estimated from the data.

Estimates of the coefficients  $b_0$  and  $b_1$  are presented in table 1 for predicting cubic foot volume and green and dry weight of bole wood, bole bark, total bole, and total tree excluding leaves. Additional statistics presented are the component average, fit index, regression standard error of estimate ( $S_e$ ) based on residuals after converting to actual units, and coefficient of variation (C.V.) of predictions in arithmetic units. Fit index, which is similar to  $R^2$ , is used to judge equation efficiency when the dependent variable has been transformed (Farrar 1978) and is calculated in untransformed units from the total and residual sums of squares.<sup>1</sup> The fit index and  $R^2$  are equal when a simple linear regression analysis is performed on an untransformed dependent variable.

Fit indices range from 0.887 for dry total tree weight to 0.975 for bole wood volume. A fit index of 1.0 indicates that predictions can be made without

<sup>1</sup>FI =  $\{1 - [\sum (Y_i - \hat{Y}_i)^2] / [\sum (Y_i - \bar{Y})^2]\}$

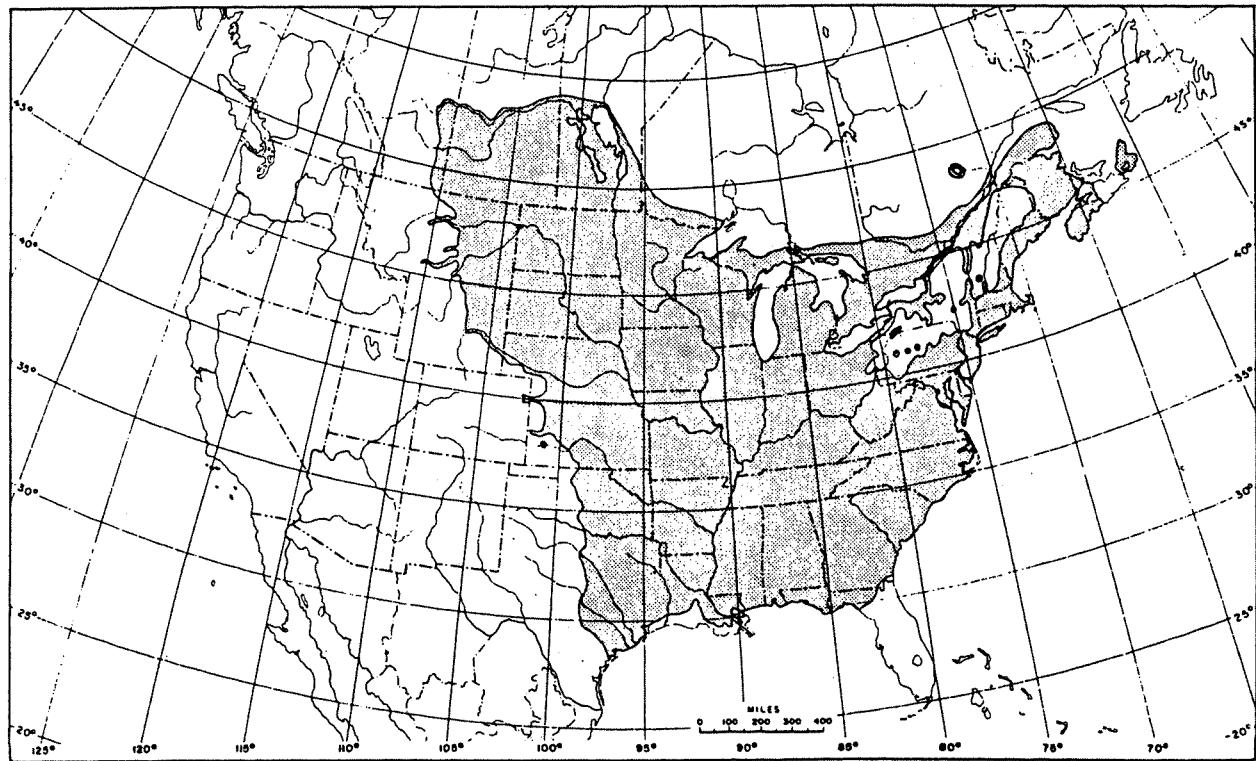


Figure 1.—The range of green ash (Fowells 1965). (F-506594)

error with the data used to fit the model; a value close to 1 is desirable. Coefficients of variation, which are indices of relative precision of prediction, range from 62.6 percent for dry total tree weight to 29.3 percent for bole wood volume.

Using measures of individual tree dbh and total height, estimates can be made using table 1 for tree volume (given in table 2), green weight (given in table 3), and dry weight (given in table 4) for bole wood, bole wood plus bark, and for bole wood plus bark plus limbs excluding leaves.

The equations were developed using trees from a limited geographic range compared to the total range. Therefore, predictions may be less precise when the equations are applied in other parts of the range. Using an upper bole diameter measurement in addition to dbh as described later should alleviate this problem.

### MERCHANTABLE BOLE ESTIMATES

The merchantable bole is defined as the tree bole from a 1-foot stump to a specified top diameter, ignoring limbs. Merchantable bole volume or weight can be expressed as a proportion of the total bole by a generalized form of the logistic model

$$R_I = (1.0 + e^{f(X)})^{-1} \quad (2)$$

where

$R_I$  = ratio of merchantable bole volume or weight to total bole volume or weight,  
where  $I$  = top outside bark diameter of merchantable bole,

$$X = (D - d_t)/D,$$

$D$  = tree dbh outside bark, and

$d_t$  = outside bark top diameter.

For a specific top diameter, the ratio of merchantable bole to total bole is the same for volume, green weight, dry weight, and inside and outside bark (Schlaegel and Willson 1983).

Model 2 was fitted to the overcup oak data for trees  $\geq 4.6$  inches dbh, with  $f(X) = b_0 + b_1 X + b_2 X^2 + b_3 X^3 + b_4 X^4$ . Inverting (2) and taking the natural logarithm results in

$$\ln\left(\frac{1-R_I}{R_I}\right) = b_0 + b_1 X + b_2 X^2 + b_3 X^3 + b_4 X^4;$$

this was solved using multiple linear regression techniques giving:

$$R_I = [1.0 + \text{EXP}(0.77188 + 11.26543 X - 73.62613 X^2 + 111.38621 X^3 - 59.39594 X^4)]^{-1} \quad (3)$$

with fit index = 0.921 and  $\hat{S}_e = 0.065$ ; EXP is the base of the natural logarithm and equals 2.71828.

Volume or weight to any top diameter ( $V_I$ ) is found by multiplying the ratio estimate obtained from (3) by a total bole volume or weight estimate from either table 2, 3, or 4:

$$V_I = R_I \cdot V_T,$$

where  $V_T$  is the total bole volume or weight of interest.

### IMPROVED TOTAL BOLE ESTIMATES

The precision of the merchantable bole estimate depends upon accuracy in measuring total bole. Total bole estimates can be significantly improved by measuring one or two additional diameters. Tables 5 and 6 give regression statistics for total bole volume and weight under two different model forms:

$$\ln(Y) = b_0 + b_1 \ln(D \cdot D_{1/3} \cdot H) \quad (4)$$

and

$$\ln(Y) = b_0 + b_1 \ln[(D \cdot D_{1/3} + D^2_{1/4}) \cdot H] \quad (5)$$

where  $D$ ,  $H$ , and  $Y$  are as defined previously, and  $D_{1/3}$ ,  $D_{1/2}$ , and  $D_{1/4}$  are diameters measured outside the bark at one-third, one-half, and one-fourth the total height.

Statistics from tables 5 and 6 show the significant ( $\alpha = 0.05$ ) improvement over the previous  $D^2H$  equations of table 1. Measuring one additional diameter at one-third the tree height reduces  $\hat{S}_e$  for all bole components from 31 percent for dry bole bark to 63 percent for bole wood volume.

Measuring two upper bole diameters at one-fourth and one-half total height will result in further significant increases in precision. Improvements over the  $D^2H$  model range from 46 to 81 percent for bole wood and total bole and from 38 to 49 percent for bark.

Taking additional bole measures means a higher inventory cost, since more time will be spent at each tree, but most of the inventory cost is in traveling to the candidate tree. Data presented in this paper allow total and merchantable bole estimates to be made with high reliability. Each user has to decide when increased precision is needed and whether the need offsets the higher cost.

### LOCAL VOLUME TABLES

There are many occasions when the only tree variable available is dbh, or perhaps only a rough estimate of tree volume or weight is needed. In these cases a local volume table will suffice. Table 7 gives the statistics needed for estimating by the local volume table technique. However, it must be realized that estimation precision may be considerably lessened. The local volume or weight model is

$$\ln(Y) = b_0 + b_1 \ln(D^2). \quad (6)$$

Precision is significantly ( $\alpha = 0.05$ ) reduced when only dbh is used for prediction, except for total tree and dry bole wood.

### CALCULATING CONFIDENCE LIMITS

Confidence limits ((1 -  $\alpha$ ) percent confidence limits) of individual predictions can be calculated using statistics presented in table 8. Clark et al. (1980) used the technique of Land (1972) to estimate approximate confidence limits for lognormal means of scarlet oak:

$$Y_{UL} = EXP \left\{ \ln(Y) \pm Z_{\alpha/2} \left[ S_{y-x}^2 \left[ \frac{1}{n} + \frac{(X - \bar{X})^2}{\sum(X - \bar{X})^2} \right] + \frac{S_{y-x}^2}{2(n+1)} \right]^{1/2} \right\}$$

where:

$Y_{UL}$  = upper and lower limits for Y,

EXP = base of natural logarithm; EXP = e = 2.71828,

Y = predicted volume or weight of component,

$Z_{\alpha/2}$  = value for the upper  $\alpha/2$  percentage point from the standard normal table,

$S_{y-x}$  = standard error of estimate for prediction equation,

n = number of observations used to develop the equation,

$\bar{X}$  = sample mean of  $\ln(X)$ ,

$\sum(X - \bar{X})^2$  = corrected sum of squares for  $\ln(X)$ ,

X =  $\ln(D^2H)$ ; D = dbh and H = total height of tree for which Y is predicted.

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Table 1.—Regression statistics for predicting green ash tree volumes and weights using dbh and total height<sup>1</sup>

Component	Average Y	$b_0$	$b_1$	Fit index <sup>2</sup>	$\hat{S}_e^3$	C.V. percent <sup>4</sup>
Volume (cubic feet)						
Bole wood	14.4	-5.817	0.93265	0.975	4.22	29.3
Bole bark	2.5	-6.743	0.85307	0.939	0.99	40.4
Total bole	16.9	-5.502	0.91740	0.973	4.99	29.6
Total tree, excluding leaves	22.7	-5.371	0.92436	0.947	9.60	42.3
Green weight (pounds)						
Bole wood	717	-1.616	0.90470	0.939	322	44.9
Bole bark	127	-2.335	0.80534	0.953	45	35.3
Total bole	844	-1.246	0.88369	0.945	355	42.0
Total tree, excluding leaves	1138	-1.104	0.88814	0.908	632	55.5
Dry weight (pounds)						
Bole wood	469	-2.123	0.91416	0.907	263	56.1
Bole bark	77	-3.536	0.87524	0.941	32	41.1
Total bole	546	-1.905	0.90737	0.917	285	52.3
Total tree, excluding leaves	733	-1.759	0.91023	0.887	459	62.6

<sup>1</sup>Volumes and weights of trees from a 1-foot stump to the tree tip, using  $\ln(Y) = b_0 + b_1 \ln(D^2H)$ ; D = dbh, H = total tree height.

<sup>2</sup>Fit index =  $1.0 - [\sum(Y - \hat{Y})^2 / \sum(Y - \bar{Y})^2]$ , using untransformed values.

<sup>3</sup> $\hat{S}_e = [\sum(Y - \hat{Y})^2 / (n - 2)]^{1/2}$ , using untransformed values.

<sup>4</sup>Coefficient of variation =  $100 \hat{S}_e / \bar{Y}$ .

Table 2.—*Cubic-foot volume for green ashbole wood, bole wood plus bark, and total tree*<sup>1</sup>

Dbh <i>inches</i>	Total height in feet														
	10	20	30	40	50	60	70	80	90	100	110	120	130		
1	0.03 <sup>2</sup>	0.05	0.07	0.09											
	0.03 <sup>3</sup>	0.06	0.09	0.12											
	0.04	0.07	0.11	0.14											
2	0.09	0.18	0.26	0.34	0.42										
	0.12	0.23	0.33	0.43	0.53										
	0.14	0.27	0.39	0.51	0.62										
3		0.38	0.55	0.72	0.89										
		0.48	0.69	0.90	1.11										
		0.57	0.82	1.07	1.32										
4		0.65	0.94	1.23	1.52	1.80									
		0.81	1.18	1.53	1.88	2.22									
		0.96	1.40	1.83	2.24	2.66									
5			1.43	1.87	2.30	2.73	3.15								
			1.77	2.31	2.83	3.34	3.85								
			2.11	2.76	3.39	4.01	4.63								
6				2.01	2.63	3.23	3.83	4.43							
				2.47	3.22	3.95	4.67	5.38							
				2.96	3.86	4.75	5.62	6.48							
7					3.50	4.31	5.11	5.90	6.68						
					4.27	5.24	6.20	7.14	8.07						
					5.14	6.31	7.47	8.62	9.75						
8						4.49	5.53	6.56	7.57	8.57					
						5.46	6.70	7.92	9.12	10.30					
						6.57	8.08	9.56	11.0	12.50					
9							5.60	6.89	8.17	9.43	10.7				
							6.78	8.32	9.83	11.30	12.8				
							8.17	10.00	11.90	13.71	15.5				
10								8.39	9.94	11.5	13.0	14.5			
								10.10	11.90	13.7	15.5	17.3			
								12.20	14.40	16.7	18.8	21.0			
11									10.0	11.9	13.7	15.5	17.3		
									12.0	14.2	16.4	18.5	20.6		
									14.6	17.2	19.9	22.5	25.1		
12									11.8	14.0	16.1	18.3	20.4	22.5	
									14.1	16.7	19.2	21.7	24.2	26.6	
									17.1	20.2	23.3	26.4	29.4	32.4	
13									13.7	16.2	18.7	21.2	23.7	26.1	
									16.3	19.3	22.2	25.1	28.0	30.8	
									19.8	23.5	27.1	30.6	34.1	37.6	
14										18.6	21.5	24.3	27.2	30.0	
										22.1	25.5	28.8	32.1	35.3	
										26.9	31.0	35.1	39.1	43.1	
15											21.2	24.4	27.7	30.9	34.1
											25.1	28.9	32.7	36.4	40.1
											30.6	35.2	39.9	44.5	49.0
16											23.9	27.6	31.2	34.9	38.5
											28.3	32.6	36.8	41.0	45.1
											34.4	39.7	44.9	50.1	55.2
17											26.7	30.9	35.0	39.0	43.1
											31.6	36.4	41.1	45.8	50.5
											38.5	44.4	50.3	56.0	61.8
18											29.8	34.4	38.9	43.4	47.9
											35.1	40.4	45.7	50.9	56.0
											42.8	49.4	55.9	62.3	68.7

Table 2.—Cubic-foot volume for green ash bole wood, bole wood plus bark, and total tree—(Continued)

Dbh inches	Total height in feet												
	10	20	30	40	50	60	70	80	90	100	110	120	130
19	cubic feet												
	32.9	38.0	43.0	48.0	53.0								
	38.7	44.6	50.4	56.2	61.9								
	47.3	54.6	61.7	68.8	75.9								
20	36.2	41.8	47.4	52.9	58.3	63.7							
	42.5	49.0	55.4	61.7	68.0	74.2							
	52.0	60.0	67.9	75.7	83.4	91.1							
21	39.7	45.8	51.9	57.9	63.9	69.8							
	46.5	53.6	60.6	67.5	74.4	81.1							
	56.9	65.7	74.3	82.8	91.3	99.7							
22	43.3	49.9	56.6	63.1	69.7	76.1	82.6						
	50.7	58.4	66.0	73.5	81.0	88.4	95.7						
	62.1	71.6	81.0	90.3	99.5	109.0	118.0						
23	47.0	54.3	61.5	68.6	75.7	82.7	89.7						
	55.0	63.3	71.6	79.8	87.9	95.9	104.0						
	67.4	77.7	87.9	98.0	108.0	118.0	128.0						
24	50.9	58.8	66.5	74.3	81.9	89.6	97.1	105					
	59.5	68.5	77.4	86.3	95.0	104.0	112.0	121					
	72.9	84.0	95.1	106.0	117.0	128.0	138.0	149					
25	54.9	63.4	71.8	80.1	88.4	96.6	105	113					
	64.1	73.8	83.4	93.0	102.0	112.0	121	130					
	78.6	90.6	103.0	114.0	126.0	138.0	149	161					
26	59.1	68.2	77.3	86.2	95.1	104	113	121					
	68.9	79.3	89.7	99.9	110.0	120	130	140					
	84.5	97.5	110.0	123.0	136.0	148	160	173					
27	63.4	73.2	82.9	92.5	102	112	121	130					
	73.8	85.0	96.1	107.0	118	129	139	150					
	90.6	104.0	118.0	132.0	145	159	172	185					
28	78.3	88.7	99	109	119	129	139						
	90.9	103.0	114	126	138	149	160						
	112.0	126.0	141	155	170	184	198						
29	83.6	94.7	106	117	127	138	149						
	96.9	110.0	122	134	147	159	171						
	119.0	135.0	150	166	181	196	211						
30	89.1	101	113	124	136	147	159						
	103.0	117	130	143	156	169	182						
	127.0	144	160	177	193	209	225						
31	94.7	107	120	132	144	157	169						
	110.0	124	138	152	166	180	193						
	135.0	153	170	188	205	222	239						
32	100	114	127	140	153	166	179						
	116	131	146	161	176	190	205						
	143	162	180	199	217	235	253						
33	106	121	135	148	162	176	190						
	123	139	155	170	186	201	217						
	151	171	191	211	230	249	268						
34	113	127	142	157	172	186	200						
	130	147	163	180	196	213	229						
	160	181	202	223	243	263	284						
35	119	135	150	166	181	196	212						
	137	155	172	190	207	224	241						
	169	191	213	235	256	278	299						

<sup>1</sup>Tree volume from a 1-foot stump to the tree tip.

<sup>2</sup>The three vertical figures for each dbh give volumes of bole wood first, bole wood plus bark second, and total tree (bole wood plus bark plus limbs excluding leaves) last.

<sup>3</sup>Boldface numbers span the range of the data.

Table 3.—*Green weight in pounds for green ash bole wood, bole wood plus bark, and total tree'*

Dbh <i>inches</i>	Total height in feet															
	10	20	30	40	50	60	70	80	90	100	110	120	130			
1	1.6 <sup>2</sup>	3.0	4.3	5.6												
	2.2 <sup>2</sup>	4.1	5.8	7.5												
	2.6	4.7	6.8	8.8												
2	5.6	10.5	15	20	24											
	7.5	13.8	20	26	31											
	8.8	16.0	23	30	37											
3		22	31	41	50											
		28	40	52	64											
		33	48	62	75											
4		37	53	69	84	99										
		47	67	87	106	124										
		56	80	103	126	148										
5			79	103	126	148	171									
			100	129	157	184	211									
			119	153	187	219	252									
6			110	143	175	206	237									
			138	178	217	254	292									
			164	212	258	303	348									
7				189	231	273	314	354								
				233	284	334	383	431								
				278	339	399	457	515								
8				241	295	347	399	451								
				296	360	423	485	545								
				353	430	506	580	653								
9				298	365	430	494	558								
				364	443	521	597	672								
				435	530	623	715	805								
10					441	520	598	675	751							
					534	627	719	809	898							
					639	752	862	971	1078							
11						524	618	711	802	892						
						632	743	851	958	1063						
						757	890	1021	1150	1276						
12							614	724	832	939	1044	1149				
							737	866	992	1117	1239	1360				
							884	1039	1192	1342	1490	1636				
13								709	836	962	1085	1207	1328			
								849	998	1143	1286	1427	1567			
								1019	1198	1374	1547	1717	1886			
14									956	1100	1241	1380	1518			
									1137	1303	1466	1627	1786			
									1367	1567	1764	1959	2151			
15										1084	1246	1406	1564	1720		
										1285	1472	1657	1838	2018		
										1545	1771	1994	2214	2432		
16										1218	1400	1580	1758	1933		
										1440	1650	1857	2060	2261		
										1732	1987	2237	2483	2727		
17											1359	1562	1763	1961	2158	
											1603	1837	2067	2293	2517	
											1929	2212	2491	2766	3037	
18												1507	1733	1955	2175	2393
												1773	2032	2286	2537	2785
												2136	2449	2757	3061	3362

Table 3.—Green weight in pounds for green ash bole wood, bole wood plus bark, and total tree—(Continued)

Dbh Inches	Total height in feet												
	10	20	30	40	50	60	70	80	90	100	110	120	130
9						1662	1911	2156	2399	2638			
						1951	2236	2516	2792	3064			
						2351	2696	3035	3370	3700			
20						1824	2097	2366	2632	2895	3156		
						2136	2448	2754	3057	3355	3650		
						2575	2953	3325	3691	4053	4411		
21						1992	2290	2584	2875	3162	3447		
						2328	2668	3002	3332	3657	3978		
						2808	3220	3626	4025	4420	4811		
22						2167	2491	2811	3127	3440	3750	4057	
						2528	2897	3260	3617	3970	4319	4664	
						3050	3498	3938	4372	4801	5225	5645	
23						2348	2700	3047	3389	3728	4064	4397	
						2735	3134	3526	3913	4295	4672	5046	
						3301	3785	4261	4731	5196	5654	6109	
24						2536	2916	3290	3660	4027	4389	4749	5105
						2948	3378	3802	4219	4630	5037	5440	5838
						3560	4082	4596	5103	5604	6098	6588	7074
25						2731	3140	3543	3941	4335	4726	5113	5497
						3169	3631	4086	4534	4977	5414	5847	6275
						3828	4389	4942	5487	6025	6557	7084	7606
26						2932	3370	3803	4231	4654	5073	5489	5901
						3396	3892	4379	4860	5334	5803	6266	6726
						4104	4706	5298	5883	6460	7030	7595	8155
27						3139	3609	4072	4530	4983	5432	5877	6318
						3631	4160	4681	5195	5702	6203	6699	7190
						4388	5032	5666	6290	6908	7518	8122	8720
28						3854	4349	4838	5322	5801	6276	6748	
						4437	4992	5540	6080	6615	7143	7667	
						5368	6044	6710	7368	8019	8664	9302	
29						4107	4634	5155	5671	6181	6688	7190	
						4720	5312	5894	6469	7038	7600	8157	
						5713	6432	7142	7842	8535	9221	9900	
30						4367	4927	5481	6030	6573	7111	7645	
						5012	5640	6258	6869	7472	8070	8661	
						6068	6832	7585	8329	9065	9793	10515	
31						4634	5228	5816	6398	6974	7545	8112	
						5311	5976	6632	7279	7918	8551	9178	
						6432	7241	8040	8829	9609	10381	11145	
32						4907	5538	6160	6776	7387	7992	8592	
						5617	6321	7014	7699	8375	9045	9708	
						6805	7662	8506	9341	10166	10983	11792	
33						5188	5855	6513	7164	7810	8449	9084	
						5931	6674	7406	8129	8843	9550	10250	
						7187	8092	8984	9866	10737	11600	12454	
34						5476	6180	6875	7562	8243	8918	9588	
						6253	7036	7808	8569	9323	10068	10806	
						7578	8533	9474	10403	11322	12231	13133	
35						5771	6512	7245	7969	8687	9398	10104	
						6581	7406	8218	9020	9813	10597	11374	
						7979	8984	9974	10953	11920	12878	13827	

<sup>1</sup>Tree weight from a 1-foot stump to the tree tip.<sup>2</sup>The three vertical figures for each dbh give weights of bole wood first, bole wood plus bark second, and total tree (bole wood plus bark plus limbs excluding leaves) last.<sup>3</sup>Boldface numbers span the range of the data.

Table 4.—*Dry weight in pounds for green ash bole wood, bole wood plus bark, and total tree<sup>1</sup>*

Dbh <i>inches</i>	Total height in feet												
	10	20	30	40	50	60	70	80	90	100	110	120	130
1	1.0 <sup>2</sup>	1.8	2.7	3.5									
	1.2 <sup>3</sup>	2.3	3.3	4.2									
	1.4	2.6	3.8	5.0									
2	3.5	6.6	9.5	12.4	15								
	4.2	7.9	11.5	14.9	18								
	4.9	9.3	13.4	17.0	21								
3		14	20	26	32								
		17	24	31	38								
		19	28	37	45								
4		23	34	44	54	64							
		28	40	52	64	76							
		33	47	62	76	89							
5			51	66	81	96	110						
			60	78	96	113	130						
			71	93	113	134	154						
6			71	92	113	134	154						
			84	109	134	158	182						
			99	129	158	187	215						
7				122	150	177	204	231					
				145	177	209	240	271					
				171	209	247	284	321					
8				156	192	226	261	294					
				184	225	266	306	345					
				218	267	315	363	410					
9				194	238	281	323	365					
				228	279	329	379	428					
				270	331	391	449	508					
10					288	340	392	443	493				
					338	399	459	518	576				
					401	473	545	615	684				
11					343	405	466	527	587				
					402	474	545	616	685				
					477	563	648	731	814				
12					402	475	547	618	688	758			
					471	555	639	721	802	883			
					559	659	759	857	954	1050			
13					465	550	633	715	796	877			
					544	642	739	834	928	1021			
					646	763	878	991	1104	1215			
14						630	725	819	912	1004			
						735	845	954	1061	1168			
						873	1005	1135	1263	1390			
15						714	822	929	1035	1139			
						833	958	1081	1203	1323			
						990	1139	1286	1432	1576			
16						804	925	1045	1164	1282			
						936	1077	1215	1352	1488			
						1113	1281	1447	1610	1773			
17						898	1034	1168	1301	1432			
						1045	1202	1357	1510	1661			
						1243	1431	1616	1798	1979			
18						997	1147	1296	1444	1590			
						1159	1333	1505	1674	1842			
						1380	1587	1793	1996	2196			

Table 4.—Dry weight in pounds for green ash bole wood, bole wood plus bark, and total tree—(Continued)

Dbh inches	Total height in feet												
	10	20	30	40	50	60	70	80	90	100	110	120	130
19						1100	1267	1431	1594	1755			
						1279	1470	1660	1847	2032			
						1522	1752	1978	2702	2424			
20						1208	1391	1572	1751	1928	2103		
						1403	1614	1822	2027	2231	2432		
						1671	1923	2172	2417	2661	2902		
21						1321	1521	1719	1914	2107	2299		
						1533	1763	1990	2215	2437	2657		
						1827	2102	2373	2642	2908	3171		
22						1438	1656	1871	2084	2295	2503	2711	
						1668	1919	2166	2410	2652	2891	3129	
						1988	2288	2583	2875	3165	3452	3736	
23						1560	1796	2030	2260	2489	2715	2940	
						1808	2080	2348	2613	2875	3134	3392	
						2156	2480	2801	3118	3432	3743	4051	
24						1686	1942	2194	2443	2690	2935	3178	3419
						1954	2247	2536	2822	3106	3386	3664	3940
						2329	2680	3027	3369	3708	4044	4378	4708
25						1817	2092	2364	2632	2899	3163	3424	3684
						2104	2420	2731	3039	3344	3646	3946	4243
						2509	2887	3260	3629	3994	4356	4715	5072
26						1952	2248	2539	2828	3114	3398	3679	3958
						2259	2598	2933	3264	3591	3915	4237	4556
						2695	3101	3501	3898	4290	4679	5064	5447
27						2092	2408	2721	3030	3337	3640	3942	4241
						2419	2782	3141	3495	3846	4193	4537	4879
						2886	3321	3750	4175	4595	5011	5424	5834
28						2574	2908	3239	3566	3891	4213	4533	
						2972	3355	3733	4108	4479	4847	5212	
						3548	4007	4460	4909	5354	5796	6234	
29						2744	3101	3453	3802	4148	4492	4833	
						3168	3576	3979	4378	4774	5166	5555	
						3782	4271	4755	5233	5708	6178	6645	
30						2920	3299	3674	4045	4414	4779	5142	
						3369	3802	4231	4656	5076	5493	5907	
						4023	4543	5057	5566	6071	6571	7068	
31						3100	3503	3901	4295	4686	5074	5460	
						3575	4036	4491	4941	5388	5830	6269	
						4271	4823	5368	5909	6444	6975	7503	
32						3286	3712	4134	4552	4966	5378	5786	
						3787	4275	4757	5234	5707	6176	6641	
						4525	5110	5688	6260	6828	7391	7949	
33						3476	3927	4373	4816	5254	5689	6121	
						4005	4520	5030	5535	6035	6531	7023	
						4786	5404	6016	6621	7221	7816	8407	
34						3671	4147	4619	5086	5549	6008	6464	
						4228	4772	5310	5843	6371	6894	7414	
						5053	5706	6352	6991	7624	8253	8877	
35						3870	4373	4870	5362	5851	6335	6816	
						4456	5030	5597	6159	6715	7267	7814	
						5327	6015	6696	7370	8038	8700	9358	

<sup>a</sup>Tree weight from a 1-foot stump to the tree tip.

<sup>b</sup>The three vertical figures for each dbh give weights of bole wood first, bole wood plus bark second, and total tree (bole wood plus bark plus limbs excluding leaves) last.

<sup>c</sup>Boldface numbers span the range of the data.

Table 5.—Regression statistics for predicting green ash bole volumes and weights using two bole diameter measurements and total height<sup>1</sup>

Component	Average Y	$b_0$	$b_1$	Fit index <sup>2</sup>	$\hat{S}_e$	C.V. percent <sup>4</sup>
Volume (cubic feet)						
Bole wood	14.4	-5.868	0.97894	0.997	1.57*	10.9
Bole bark	2.5	-6.804	0.89660	0.975	0.63*	25.8
Total bole	16.9	-5.556	0.96315	0.996	1.86*	11.0
Green weight (pounds)						
Bole wood	717	-1.671	0.94999	0.978	193*	26.9
Bole bark	127	-2.394	0.84651	0.982	28*	21.7
Total bole	844	-1.301	0.92811	0.981	208*	24.6
Dry weight (pounds)						
Bole wood	469	-2.183	0.96029	0.960	171*	36.6
Bole bark	77	-3.605	0.92033	0.971	22*	29.0
Total bole	546	-1.967	0.95330	0.967	180*	33.0

<sup>1</sup>Volumes and weights of trees from a 1-foot stump to the tree tip, using  $\ln(Y) = b_0 + b_1 \ln(D \cdot D_{1/4} \cdot H)$ ;  $D = \text{dbh}$ ,  $H = \text{total tree height}$ ,  $D_{1/4} = \text{bole diameter outside bark at } \frac{1}{4}H$ .

<sup>2</sup>Fit index =  $1.0 - [\sum(Y - \hat{Y})^2 / \sum(Y - \bar{Y})^2]$ , using untransformed values.

<sup>3</sup> $\hat{S}_e = [\sum(Y - \hat{Y})^2 / (n - 2)]^{1/2}$ , using untransformed values.

<sup>4</sup>Coefficient of variation =  $100 \hat{S}_e / \bar{Y}$ .

\*Significantly more precise by the F test than D·H model;  $\alpha = 0.05$ .

Table 6.—Regression statistics for predicting green ash bole volumes and weights using three bole diameter measurements and total height<sup>1</sup>

Component	Average Y	$b_0$	$b_1$	Fit index <sup>2</sup>	$\hat{S}_e$	C.V. percent <sup>4</sup>
Volume (cubic feet)						
Bole wood	14.4	-6.609	1.00941	0.999	0.90+	6.2
Bole bark	2.5	-7.490	0.92503	0.981	0.56*	22.8
Total bole	16.9	-6.286	0.99323	0.999	0.97+	5.7
Green weight (pounds)						
Bole wood	717	-2.392	0.97969	0.987	151+	21.0
Bole bark	127	-3.042	0.87346	0.988	23*	17.9
Total bole	844	-2.007	0.95724	0.989	159+	18.8
Dry weight (pounds)						
Bole wood	469	-2.914	0.99049	0.973	141*	30.1
Bole bark	77	-4.312	0.94977	0.976	20*	26.0
Total bole	546	-2.693	0.98336	0.979	144*	26.4

<sup>1</sup>Volumes and weights of trees from a 1-foot stump to the tree tip, using  $\ln(Y) = b_0 + b_1 \ln(D \cdot D_{1/4} + D_{1/2} \cdot H)$ ;  $D = \text{dbh}$ ,  $H = \text{total tree height}$ ,  $D_{1/4}$  and  $D_{1/2}$  = bole diameters outside bark at  $\frac{1}{4}H$  and  $\frac{1}{2}H$ , respectively.

<sup>2</sup>Fit index =  $1.0 - [\sum(Y - \hat{Y})^2 / \sum(Y - \bar{Y})^2]$ , using untransformed values.

<sup>3</sup> $\hat{S}_e = [\sum(Y - \hat{Y})^2 / (n - 2)]^{1/2}$ , using untransformed values.

<sup>4</sup>Coefficient of variation =  $100 \hat{S}_e / \bar{Y}$ .

\*Significantly more precise by the F test than D·D<sub>1/4</sub>·H model;  $\alpha = 0.05$ .

\*Significantly more precise by the F test than D·H model;  $\alpha = 0.05$ .

Table 7.—Local volume and weight equations for green ash<sup>1</sup>

Component	Average Y	$b_0$	$b_1$	Fit index <sup>2</sup>	$\hat{S}_e^3$	C.V. percent <sup>4</sup>
Volume (cubic feet)						
Bole wood	14.4	-3.060	1.17920	0.940	6.48*	45.0
Bole bark	2.5	-4.222	1.07848	0.885	1.36*	55.6
Total bole	16.9	-2.791	1.15993	0.938	7.57*	44.9
Total tree, excluding leaves	22.7	-2.644	1.17048	0.928	11.14	49.2
Green weight (pounds)						
Bole wood	717	1.062	1.14335	0.904	405*	56.5
Bole bark	127	0.036	1.01914	0.923	57*	44.9
Total bole	844	1.367	1.11714	0.915	441*	52.2
Total tree, excluding leaves	1138	1.518	1.12431	0.900	660	58.0
Dry weight (pounds)						
Bole wood	469	0.589	1.15466	0.861	321	68.5
Bole bark	77	-0.953	1.10694	0.901	41*	53.3
Total bole	546	0.784	1.14630	0.873	352*	64.6
Total tree, excluding leaves	733	0.935	1.15150	0.871	489	66.8

<sup>1</sup>Volumes and weights of trees from a 1-foot stump to the tree tip, using  $\ln(Y) = b_0 + b_1 \ln(D^2)$ ;  $D = \text{dbh}$ .

<sup>2</sup>Fit index =  $1.0 - [\sum(Y - \hat{Y})^2 / \sum(Y - \bar{Y})^2]$ , using untransformed values.

<sup>3</sup> $\hat{S}_e = [\sum(Y - \hat{Y})^2 / (n - 2)]^{1/2}$ , using untransformed values.

<sup>4</sup>Coefficient of variation =  $100 \hat{S}_e / \bar{Y}$ .

\*Significantly less precise by the F test than D<sup>2</sup>H model;  $\alpha = 0.05$ .

Table 8.—Statistics for estimating confidence bounds for green ash predictions for the bole and total tree

Independent variable - X	n	Mean of X	Corrected sum of squares for X <sup>1</sup>	S <sub>y,x</sub>		
				Standard error of estimate for tree components		
Bole predictions						
Ln(D <sup>2</sup> )	70	3.155	312.623	Volume	0.2213	0.3014
				Green weight	0.2610	0.3127
				Dry weight	0.2977	0.3569
Ln(D <sup>2</sup> H)	70	6.930	501.821	Volume	0.1502	0.2603
				Green weight	0.1949	0.2942
				Dry weight	0.2257	0.3279
Ln(D · D <sub>1/2</sub> · H)	70	6.647	456.642	Volume	0.0789	0.1994
				Green weight	0.1329	0.2464
				Dry weight	0.1594	0.2696
Ln[(D · D <sub>1/2</sub> + D <sup>2</sup> <sub>1/2</sub> )H]	70	7.179	429.668	Volume	0.0593	0.1773
				Green weight	0.1162	0.2283
				Dry weight	0.1378	0.2466
Total tree predictions						
Ln(D <sup>2</sup> )	65	3.001	290.532	Volume	0.2374	
				Green weight	0.2596	
				Dry weight	0.2963	
Ln(D <sup>2</sup> H)	65	6.738	467.493	Volume	0.1841	
				Green weight	0.2088	
				Dry weight	0.2335	

<sup>1</sup> $\Sigma(X - \bar{X})^2$  in base e logarithmic units.

## APPENDIX

### Examples Using the Equations

In this section examples are presented to illustrate use of the equations. The following tabulation gives data used to illustrate equation use:

$$D = 21.7 \text{ in.}$$

$$H = 75.0 \text{ ft.}$$

$$D_{1/4} = 15.5 \text{ in.}$$

$$D_{1/4} = 14.8 \text{ in.}$$

$$D_{1/4} = 11.5 \text{ in.}$$

Bole wood volume ( $V$ ) is calculated using  $X = D^2 H$  and the parameter estimates from table 1 as follows:

$$\begin{aligned} \ln(V) &= b_0 + b_1 \ln(D^2 H) \\ &= -5.817 + 0.93265 \ln(21.7 \times 21.7 \times 75.0) \\ &= -5.817 + 0.93265 \ln(35316.75) \\ &= -5.817 + 0.93265 (10.472) \\ &= -5.817 + 9.767 \end{aligned}$$

$$\ln(V) = 3.950$$

$$V = e^{3.950}; \text{bole wood volume} = 51.9 \text{ ft}^3.$$

To calculate total green bole weight ( $W$ ) using  $X = D \cdot D_{1/4} \cdot H$  and the parameter estimates from table 5:

$$\begin{aligned} \ln(W) &= b_0 + b_1 \ln(D \cdot D_{1/4} \cdot H) \\ &= -1.301 + 0.92811 \ln(21.7 \times 14.8 \times 75.0) \\ &= -1.301 + 0.92811 \ln(24087.00) \\ &= -1.301 + 0.92811 (10.089) \\ &= -1.301 + 9.364 \end{aligned}$$

$$\ln(W) = 8.063$$

$$W = e^{8.063}; \text{total bole green weight} = 3175 \text{ lb.}$$

To calculate bole dry weight ( $W$ ) using  $X = [(D \cdot D_{1/4} + D_{1/4}^2)H]$  and the parameter estimates from table 6:

$$\begin{aligned} \ln(W) &= b_0 + b_1 \ln[(D \cdot D_{1/4} + D_{1/4}^2)H] \\ &= -2.693 + 0.98336 \ln[(21.7 \times 11.5 + 15.5 \times 15.5) \times 75.0] \\ &= -2.693 + 0.98336 \ln[(249.55 + 240.25) \times 75.0] \end{aligned}$$

$$= -2.693 + 0.98336 \ln[489.80 \times 75.0]$$

$$= -2.693 + 0.98336 \ln[36735.00]$$

$$= -2.693 + 0.98336 (10.511)$$

$$= -2.693 + 10.336$$

$$\ln(W) = 7.643$$

$$W = e^{7.643}; \text{total bole dry weight} = 2086 \text{ lb.}$$

Merchantable bole estimates to, say, an 8-inch top are calculated using equation 3 to estimate the merchantable portion of the bole and then multiplying that proportion ( $R8$ ) by the predicted bole estimate of interest. Letting  $DI = 8.0$ ,  $X = [(21.7 - 8.0)/21.7]$ , or  $X = 0.631$ , and from (3):

$$\begin{aligned} R8 &= [1.0 + \exp(0.77188 + 11.26543(0.631) - 73.62613(0.631)^2 + 111.38621(0.631)^3 - 59.39594(0.631)^4)]^{-1} \\ &= [1.0 + \exp(0.77188 + 11.26543(0.631) - 73.62613(0.39816) + 111.38621(0.25124) - 59.39594(0.15853))]^{-1} \\ &= [1.0 + \exp(0.77188 + 7.10849 - 29.31498 + 27.98467 - 9.41604)]^{-1} \\ &= [1.0 + \exp(-2.86598)]^{-1} \\ &= [1.0 + 0.05693]^{-1} \\ &= [1.05693]^{-1} \end{aligned}$$

$$R8 = 0.946.$$

Then bole wood volume to an 8-inch top ( $V8$ ) can be calculated:

$$\begin{aligned} V8 &= (R8)(\text{predicted total bole wood volume}) \\ &= (0.946)(51.9) \\ &= 49.1 \text{ ft}^3. \end{aligned}$$

Of course, total bole volumes or weights can be calculated using any one of the three sets of parameter estimates given in tables 1, 5, or 6. Then any of these total bole calculations can be used with the estimate of merchantable proportion to calculate merchantable bole values. The choice of which set of equations to use is left to the user, who must balance inventory precision against inventory cost.

Schlaegel, Bryce E. Green ash volume and weight tables. Res. Pap. SO-206. New Orleans, LA: U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station; 1984. 14 p.

A sample of 70 trees from the Mississippi Delta is used to construct volume and weight tables for green ash.

Additional keywords: *Fraxinus pennsylvanica*, merchantable bole estimates, biomass.