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DIAMETER DISTRIBUTIONS IN NATURAL YELLOW-POPLAR STANDS

by

Charles E. McGee and Lino Della-Bianca



Cover photo: A Southern Appalachian yellow-poplar stand with a wide variety of diameters.

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U.S. Department of Agriculture

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by

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and

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EXCELLENT STANDS of yellow-poplar grow on fertile, well-drained coves, lower slopes, and bottom lands in the Southern Appalachian mountains. These elite stands make up only a small portion of the mountain forest, but, because of their productivity, they are of great importance to the mountain economy.

Foresters and timber owners need information that will help them plan the intensive management these stands should have. This paper presents the diameter distributions found in pure, unthinned stands of yellow-poplar. These distributions represent the natural development of the number of trees in specific size classes in relation to stand age, site index, and total number of stems per acre. If what has happened in the past is a good indicator of what will happen in the future, the stand descriptions should be very valuable for answering questions that must be decided in stand management planning. For instance, will it pay to thin a yellow-poplar stand on a moderate site at age 30, or will the trees to be thinned be too small to make a merchantable cut? Of course, in yellow-poplar stands the production of high-value veneer logs outweighs most every other consideration. Therefore, another important question is, will there be enough veneer-size stems on the site at age 60 to justify a harvest cut, or will the number of veneer-size trees increase enough to warrant waiting until age 70?

Admittedly, these data will not answer all questions, and someday more accurate criteria based on remeasurement of managed stands will probably become available. Meanwhile, the value of these diameter distributions is already being proved.

Methods

Plot Selection

Data for this study were obtained from 141 circular $\frac{1}{4}$ -acre plots established in the Appalachian Mountains of North Carolina (93 plots), Virginia (31 plots), and Georgia (17 plots). Site index, stand age, and number of trees per acre, by 1-inch diameter classes, were obtained on each plot.

To be included in the study, a stand had to have 75 percent or more of its overstory in yellow-poplar. All the stands were even-aged and ranged from 17 to 76 years of age. Site index at age 50 ranged from 75 to 150 feet, and basal area varied from 44 to 208 square feet per acre. The maximum and minimum number of yellow-poplar trees per acre, by stand age and site index, are shown in table 1.

Sampled stands were free of disease and insects and showed no evidence of past cutting. Trees were well distributed over the plot.

TABLE 1.—Minimum and maximum observed numbers of yellow-poplar trees per acre by age and site index classes.

Site index (Feet at age 50)	Age (years)						
	< 21	21-30	31-40	41-50	51-60	61-70	> 70
< 81		220					
		220					
81-90	228	240		140	72		
	228	320		168	196		
91-100	160	256	100	96	72	68	
	380	256	172	208	232	140	
101-110	176	136	48	84	100		
	364	252	272	272	192		
111-120	152	152	136	132	88		
	252	332	212	156	176		
121-130	216	236	124	96	152	136	
	216	396	256	188	152	136	
131-140	164	140					
	328	204					
> 140	184						
	184						

Analysis

Analysis of the data to determine diameter distributions followed procedures developed by Clutter and Bennett for planted slash pine.¹

Our objective was to present a table of the number of trees per acre by 1-inch diameter classes for various combinations of age, site index, and total number of trees per acre. Basal area per acre was used in the initial computations, rather than number of trees, because basal area allowed a more accurate description of the number of trees in the larger diameter classes.

The curve form used to determine the proportion of basal area for trees in any 1-inch diameter class was:

$$f(D_i) = \frac{\Gamma(\alpha + \beta + 2)}{\Gamma(\alpha + 1) \Gamma(\beta + 1)} \left(\frac{D_i - D_{min}}{D_{max} - D_{min}} \right)^\alpha \left(1 - \frac{D_i - D_{min}}{D_{max} - D_{min}} \right)^\beta$$

where,

$f(D_i)$ = relative frequency of basal area per acre for diameter D_i ;

α and β = parameters to be estimated from the data;

D_{max} = maximum diameter of trees in the stand;

D_{min} = minimum diameter of trees in the stand (D_{min} was set at 4.5 inches, the smallest trees measured).

¹ Clutter, Jerome L., and Bennett, Frank A. Diameter distributions in old-field slash pine plantations. Ga. Forest Res. Coun. Rep. 13, 9 pp., illus. 1965.

The basal area in any 1-inch diameter class was converted to number of trees per class per acre by the relationship:

$$N_i = P_i \frac{BA}{Bi}$$

where,

N_i = number of trees in the i^{th} diameter class;

P_i = proportion of the total basal area per acre that lies in the i^{th} diameter class ($i = 5, 6, \dots, 27$);

BA = total basal area per acre;

Bi = basal area per tree for the midpoint tree in the i^{th} diameter class ($i = 5, 6, \dots, 27$).

The predicted values of α , β , and D_{max} were obtained from the following equation:

$$\begin{aligned} \alpha &= 13.02839 - 0.02546 (\text{Age}) - \left(\frac{66.84692}{\text{Age}} \right) \\ &\quad - 4.43671 (\text{Logarithm of number of trees}) \\ &\quad - 0.04678 \left(\frac{\text{Site index} \times \text{number of trees}}{1000} \right) \end{aligned}$$

$R^2 = 0.476$ Standard error = 0.582

$$\beta = 1.21499 - 0.12504 \left(\frac{\text{Age} \times \text{site index}}{1000} \right)$$

$R^2 = 0.129$ Standard error = 0.450

$$\begin{aligned} D_{max} &= 9.38123 + 2.41398 \left(\frac{\text{Age} \times \text{site index}}{1000} \right) \\ &\quad - 0.35928 \left(\frac{\text{Age} \times \text{number of trees}}{1000} \right) \end{aligned}$$

$R^2 = 0.700$ Standard error = 2.09

Actual values of α , β , and D_{max} were plotted over calculated values. No bias was apparent in any case.

Results and Discussion

The expected number of trees by 1-inch diameter classes was calculated for various combinations of age, site index, and total number of trees per acre. These values are presented in tables 2 through 6. Although the tables are useful, they have several limitations. Neither the tables nor the equations involved in their development describe changes in basal area and number of trees that occur on a given site with time. Such changes can be determined only by periodic remeasurements of permanent plots. To apply the tables, one must assume that a certain number of trees per acre will be present at a given time.

We also recognize that managed stands of the future will not have diameter distributions exactly like those reported here for unmanaged stands, because cleanings and thinnings will remove smaller size classes. It is to be hoped there will be more large trees than are shown in the tables.

Because of these limitations the use of the tables for predicting the performance of any particular managed yellow-poplar stand is restricted. Nevertheless, the growth of these stands is the best available indicator we have of future stand behavior, and the availability of diameter distributions will allow much more sophisticated stand management planning than has been possible in the past. The choice of future thinning schedules and rotation ages can now be based on stem size as well as on total volume and basal area.

Careful study of the tables provides considerable information relevant to thinning certain types of yellow-poplar stands. For instance, should the owner of a yellow-poplar stand on site 100 land plan on thinning at age 30? For this example we

will assume a stand density of 250 stems per acre.² Under these site and stand stipulations, a thinning from below would consist mostly of 5- and 6-inch trees and would reduce the basal area of the stand below 87 square feet per acre. The decision whether or not to plan on a 30-year thinning in this stand must be left with the individual landowner, but we do recommend further study of the 40-year diameter distribution before a final decision is made.

² McCarthy, E. F. Yellow poplar (*Liriodendron tulipifera* L.) characteristics, growth, and management. U.S. Dep. Agr. Tech. Bull. 356, 58 pp., illus. 1933. Table 17 in this publication supplies average number of yellow-poplar stems found in natural stands in relation to age and site index.

TABLE 2.—Diameter distributions for pure natural yellow-poplar stands by age and stand density per acre on site index 90.

Total trees (Number)	Basal area (Square feet)	Number of trees per diameter class (inches)																					
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Age 20																							
100	33	15	21	19	16	12	9	5	3														
150	38	48	37	25	17	11	7	4	1														
200	43	95	47	26	15	9	5	2	1														
250	48	145	53	26	14	7	4	1	--														
300	54	194	57	26	13	6	3	1	--														
350	61	242	61	26	12	6	2	1	--														
Age 30																							
100	51	3	8	13	14	15	14	12	10	7	4												
150	61	12	23	25	24	21	17	13	9	5	1												
200	70	29	39	36	30	24	18	13	8	3	--												
250	77	51	56	46	35	26	18	11	6	1	--												
300	84	77	72	54	39	27	17	10	4	--	--												
350	90	105	87	62	42	28	17	8	1	--	--												
Age 40																							
50	44	--	--	1	2	4	5	6	6	7	7	6	4	2									
100	67	1	4	8	10	12	12	12	12	11	9	6	3	--									
150	82	5	14	18	20	20	18	16	14	11	8	5	1	--									
200	91	14	27	30	29	26	22	19	15	10	6	2	--	--									
250	98	28	42	42	37	31	25	19	14	9	3	--	--	--									
300	106	44	59	53	44	35	27	19	13	6	--	--	--	--									
350	113	62	75	64	51	39	28	19	10	2	--	--	--	--									
Age 50																							
50	55	--	--	1	1	2	3	4	5	5	6	6	6	5	4	2							
100	82	1	3	5	7	9	10	11	11	10	10	9	7	5	2	--							
150	98	4	10	14	16	17	17	16	14	13	11	9	6	3	--								
200	110	10	21	25	25	24	22	19	17	14	11	8	4	--	--								
250	117	21	34	36	34	30	26	22	18	14	10	5	--	--	--								
300	123	34	49	48	42	36	30	23	18	12	7	1	--	--	--								
Age 60																							
50	68	--	--	1	2	2	3	3	4	4	5	5	5	5	5	4	2						
100	98	1	2	4	6	7	8	9	9	9	9	8	7	6	4	2	--						
150	116	3	8	12	14	14	14	14	13	12	10	9	7	5	1	--							
200	125	9	19	22	22	22	20	19	16	14	12	10	8	5	2	--	--						
250	130	19	31	33	31	28	25	22	18	15	12	9	6	1	--	--	--						
Age 70																							
50	80	--	--	1	1	2	2	3	3	4	4	4	4	4	4	4	4	2					
100	114	1	2	4	5	6	7	7	8	8	8	7	7	7	6	5	4	1	--				
150	132	3	8	11	12	13	13	12	12	11	11	10	9	8	7	5	4	1	--				
200	141	9	18	20	20	19	18	17	15	14	12	11	9	8	6	4	--	--					
250	146	19	30	30	28	26	23	20	17	15	13	11	9	6	3	--	--	--					

Potential stem sizes can have a definite bearing on selecting rotation ages. Tables 2 through 6 show how stem sizes are distributed in older natural stands. For example, land with site index 100 and 100 stems per acre produced 15 trees 19 inches d.b.h. or larger in 60 years. In 70 years, 27 trees larger than 18 inches were produced. Each owner can decide whether the increase in veneer-size trees is worth waiting 10 years to harvest.

The information presented here describes the development of yellow-poplar stands under natural unmanaged conditions. The stands described in the tables represent the last opportunity foresters will have to observe natural stand development over a wide range of conditions, because most yellow-poplar stands in the future will come under management at an early age.

TABLE 3.—*Diameter distributions for pure natural yellow-poplar stands by age and stand density per acre on site index 100.*

Total trees (Number)	Basal area (Square feet)	Number of trees per diameter class (inches)																						
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Age 20																								
100	35	13	20	19	16	12	9	6	4	1														
150	41	43	36	26	18	12	8	5	2	--														
200	46	84	48	29	18	11	6	3	1	--														
250	52	129	56	30	17	10	5	2	1	--														
300	58	173	63	31	17	9	5	2	--	--														
350	65	214	71	34	17	9	4	1	--	--														
Age 30																								
100	56	2	7	11	13	14	14	12	11	8	6	2												
150	68	9	20	23	22	20	18	14	11	8	4	1												
200	77	23	35	34	30	25	20	15	10	6	2	--												
250	87	40	50	44	36	28	21	15	10	5	1	--												
300	94	60	66	54	42	31	22	15	8	2	--	--												
350	103	79	81	64	48	34	23	14	7	--	--	--												
Age 40																								
50	48	--	--	1	2	3	4	5	6	6	6	6	6	6	4	1								
100	74	1	4	6	9	10	11	11	11	9	8	6	3	--										
150	91	4	11	16	18	18	17	16	15	13	10	8	4	--	--									
200	104	11	22	26	27	25	22	20	16	13	10	6	2	--	--									
250	113	21	35	38	35	31	27	22	17	13	8	3	--	--	--									
300	122	32	49	49	43	37	30	24	18	12	6	--	--	--	--									
350	131	44	63	60	52	43	34	25	17	10	2	--	--	--	--									
Age 50																								
50	62	--	--	1	1	2	2	3	4	4	5	5	6	6	5	4	2							
100	95	1	2	4	6	7	8	9	10	10	10	9	8	7	6	3	--							
150	113	3	8	12	13	15	15	15	14	13	12	10	9	7	4	--	--							
200	127	8	17	21	22	22	21	19	17	15	13	11	8	5	1	--	--							
250	137	14	28	31	31	29	26	23	20	17	14	10	6	1	--	--	--							
300	147	23	39	42	39	36	31	27	22	18	13	8	2	--	--	--	--							
Age 60																								
50	76	--	--	1	1	2	2	3	3	4	4	4	5	5	5	5	4	3						
100	115	--	2	3	5	6	7	7	8	8	8	8	8	8	7	6	5	4	--					
150	137	2	6	10	11	12	13	13	12	11	11	10	9	7	6	4	--	--						
200	150	6	14	18	19	19	18	16	15	14	12	10	9	7	4	--	--	--						
250	159	13	24	27	27	26	24	22	20	17	15	13	10	8	4	--	--	--						
Age 70																								
50	92	--	--	1	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	3	1	
100	136	--	2	3	4	5	5	6	6	7	7	7	7	7	7	6	6	6	6	5	4	--	--	
150	157	2	6	9	10	11	11	11	11	10	10	9	9	8	7	6	6	3	--	--	--	--	--	
200	169	7	14	16	17	17	16	15	14	13	12	10	9	8	7	6	2	--	--	--	--	--		
250	179	13	23	25	25	23	22	20	18	16	15	13	11	10	8	6	2	--	--	--	--	--	--	

TABLE 4.—*Diameter distributions for pure natural yellow-poplar stands by age and stand density per acre on site index 110.*

Total trees (Number)	Basal area (Square feet)	Number of trees per diameter class (inches)																					
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Age 20																							
100	37	11	18	18	16	13	10	7	5	2													
150	44	38	35	26	19	13	9	6	3	1													
200	50	74	48	30	20	13	8	5	2	--													
250	56	113	58	34	20	12	7	4	2	--													
300	63	150	68	37	21	13	7	3	1	--													
350	72	184	78	41	23	13	7	3	1	--													
Age 30																							
100	60	2	6	10	12	13	13	12	11	9	7	4	1										
150	74	8	17	20	21	20	18	15	12	10	6	3	--										
200	86	18	30	32	29	25	21	17	13	9	5	1	--										
250	97	31	44	42	36	30	24	18	13	8	4	--	--										
300	107	45	59	52	43	35	26	19	13	7	1	--	--										
350	117	59	72	63	51	39	29	20	12	5	--	--	--										
Age 40																							
50	54	--	--	1	2	2	3	4	5	5	6	6	6	5	4	1							
100	83	1	3	5	7	9	10	10	11	10	10	9	7	5	3	--							
150	104	3	9	13	15	16	16	16	15	13	12	10	7	4	1	--							
200	117	8	18	23	24	24	22	20	18	15	12	9	6	1	--	--							
250	129	15	29	33	33	30	27	24	20	16	12	8	3	--	--	--							
300	142	23	40	43	41	37	32	27	22	17	12	6	--	--	--	--							
350	154	30	51	53	50	44	38	31	24	17	10	2	--	--	--	--							
Age 50																							
50	70	--	--	1	1	2	3	3	4	4	5	5	5	5	5	4	3						
100	109	--	2	3	5	6	7	8	8	9	9	8	8	8	7	6	4	1	--				
150	131	2	6	9	12	13	13	14	13	13	12	11	10	9	7	5	1	--					
200	148	5	13	17	19	20	20	19	17	16	14	13	11	8	6	2	--	--					
250	161	10	22	26	27	27	25	23	21	19	16	14	11	7	2	--	--	--					
300	175	16	30	35	35	34	31	28	24	21	18	14	10	4	--	--	--	--					
Age 60																							
50	89	--	--	1	1	1	2	2	2	3	3	4	4	4	4	4	4	2					
100	133	--	1	2	4	5	6	6	7	7	7	8	8	7	7	6	4	2	--				
150	161	2	5	8	9	10	11	11	11	11	11	10	9	9	8	7	5	2	--				
200	177	4	11	15	16	17	17	16	16	15	14	13	12	10	9	8	6	1	--	--			
250	190	9	19	22	24	23	22	21	20	18	16	15	13	11	9	7	1	--	--	--			
Age 70																							
50	109	--	--	1	1	1	1	1	2	2	2	3	3	3	3	4	4	4	4	4	4	3	
100	159	--	1	2	3	4	5	5	6	6	6	6	6	7	6	6	6	5	5	3	--		
150	184	2	5	7	8	9	9	10	10	10	10	9	9	8	8	7	7	6	5	2	--		
200	209	4	10	13	14	15	15	14	14	13	13	12	11	10	10	9	8	7	6	2	--	--	
250	220	9	17	20	21	21	20	19	18	16	15	14	13	12	11	9	8	6	1	--	--	--	

TABLE 5.—Diameter distributions for pure natural yellow-poplar stands by age and stand density per acre on site index 120.

Total trees (Number)	Basal area (Square feet)	Number of trees per diameter class (inches)																					
		5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Age 20																							
100	39	10	17	17	16	13	10	8	5	3	1												
150	46	33	33	26	20	15	10	7	4	2	--												
200	54	64	47	32	22	15	10	6	3	1	--												
250	62	97	59	37	23	15	9	6	3	1	--												
300	70	128	70	42	26	16	10	6	2	--	--												
350	80	154	82	48	29	18	11	6	2	--	--												
Age 30																							
100	67	1	5	8	10	12	12	12	11	10	8	6	4	1									
150	82	6	15	18	19	19	18	16	13	11	8	5	2	--									
200	95	14	26	29	28	25	22	18	15	11	8	4	--	--									
250	108	24	38	39	35	31	26	21	16	11	7	2	--	--									
300	121	34	50	49	43	36	30	23	17	11	6	1	--	--									
350	134	42	62	59	52	43	34	26	18	11	3	--	--	--									
Age 40																							
50	59	--	--	1	1	2	3	3	4	5	5	5	6	5	5	4	1						
100	93	1	2	4	6	8	9	9	10	10	9	9	8	7	5	3	--						
150	114	2	8	11	14	15	15	15	14	13	12	11	9	7	4	--	--						
200	133	6	15	19	21	22	21	20	18	16	14	12	9	6	1	--	--						
250	148	11	23	28	29	29	27	24	22	19	15	12	8	3	--	--	--						
300	163	16	32	37	38	36	32	29	25	21	16	12	6	--	--	--	--						
350	180	20	40	46	46	43	39	34	29	23	17	11	2	--	--	--	--						
Age 50																							
50	79	--	--	1	1	1	2	3	3	3	4	4	5	5	5	5	4	3	1				
100	122	--	1	3	4	5	6	7	8	8	8	8	8	7	7	6	5	1	--				
150	149	1	5	8	10	11	12	12	12	12	11	11	10	9	7	5	2	--	--				
200	172	4	10	14	16	17	18	17	17	16	15	14	12	11	9	7	3	--	--				
250	188	7	17	21	24	24	23	22	21	19	18	16	14	11	8	4	--	--	--				
300	206	10	23	29	31	31	29	28	26	23	20	18	15	11	6	--	--	--	--				
Age 60																							
50	98	--	--	1	1	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	3	
100	154	--	1	2	3	4	4	5	6	6	6	7	7	7	7	7	6	5	4	--			
150	185	1	4	6	8	9	10	10	10	10	10	10	9	9	9	8	7	6	4	--	--		
200	208	3	8	12	14	14	15	15	15	14	14	13	12	11	11	10	8	7	4	--	--	--	
Age 70																							
50	121	--	--	--	--	1	1	1	1	2	2	2	3	3	3	3	4	4	4	4	4	4	
100	189	--	1	2	2	3	3	4	4	5	5	5	6	6	6	6	6	6	6	6	5	2	
150	223	1	3	5	7	7	8	8	9	9	9	9	8	8	8	8	7	7	6	5	1	--	

TABLE 6.—Diameter distributions for pure natural yellow-poplar stands by age and stand density per acre on site index 130.

McGee, Charles E., and Della-Bianca, Lino
1967. Diameter Distributions in Natural Yellow-
Poplar Stands. Southeast. Forest Exp.
Sta., U. S. Forest Serv. Res. Pap. SE-25,
7 pp.

Diameter distributions obtained from 141 pure, natural unthinned yellow-poplar stands in the Appalachian Mountains of Virginia, North Carolina, and Georgia are presented in tables. The distributions are described in relation to stand age, site index, and total number of trees per acre, and are useful for stand management planning.

McGee, Charles E., and Della-Bianca, Lino
1967. Diameter Distributions in Natural Yellow-
Poplar Stands. Southeast. Forest Exp.
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