U.S. Department of Agriculture
Forest Service
Research Paper SO-154

Growth and Yield Estimation for Loblolly Pine in the West Gulf

PAUL A. MURPHY AND HERBERT S. STERNITZKE

Southern Forest Experiment Station

SUMMARY

An equation system is developed to estimate current yield, projected basal area, and projected volume for merchantable natural stands on a per-acre basis. These estimates indicate yields that can be expected from woods-run conditions.

1979

Growth and Yield Estimation for Loblolly Pine in the West Gulf

PAUL ***A. MURPHY***
AND
HERBERT S. *STERNITZKE*

INTRODUCTION

Loblolly (*Pinus taeda* L.) is the keystone of the southern pine forest products industry. Except in Florida, where slash pine (*P. elliottii* Engelm.) prevails, it dominates in the Atlantic and Gulf coastal States south of New Jersey, accounting for nearly half the total southern pine inventory. About a third of the loblolly resource is west of the Mississippi River (Sternitzke and Nelson 1970).

According to the most recent State-by-State field inventories conducted by the Forest Service as part of the nationwide Forest Survey, standing volume of southern pine on commercial forest land[1] west of the Mississippi is more than 22 billion cubic feet (table 1); nearly 13 billion is loblolly pine (table 2).

Table' 1.— *Volume by state of southern pine growing stock on commercial forest land west of the Mississippi River*

State	Date of Survey	Growing Stock
		--------*Million*-------- *cubic feet*
Arkansas	1969	6,192.2
Louisiana'	1974	6,779.4
Missouri	1972	305.0
Oklahoma	1976	1,002.4
Texas	1975	8375.2
All States		22354.2

[1]Excludes southeast Louisiana parishes.

'Forest land either producingor capableof producinycropsof industrial wood and not withdrawn from timber utilization.

West of the Mississippi Valley, loblolly pine is restricted to the Coastal Plain in Arkansas, Louisiana, southeast Oklahoma, and east Texas. The main factor limiting the northern extension of its natural range probably is temperature. Also, damage by snow and sleet may be a factor. The western extension of loblolly probably is limited by lack of precipitation (USDA Forest Service 1965).

Information on growth and yield of natural stands of loblolly pine in the West Gulf Coastal Plain is deficient. Older studies do not provide information on stands not fully or normally stocked. More recent studies were conducted in other regions of the South and sampled pure, uniformly stocked stands, but stands encountered in practice do not approach the pure, ideal conditions sampled in these studies. Many are not uniformly stocked, most have a significant hardwood component, and their yields are less.

The growth and yield relationships represented here have been observed on inventory plots throughout the West Gulf. Inventory data have more unexplained variation than do data from controlled studies, and variables tend to be clustered rather than dispersed throughout their range of possible values. This type of information, however, has the advantage of depicting actual conditions.

Data

The data are from remeasured plots of statewide surveys made by the Forest Service. The interval between surveys is about 10 years. Sample plots are at the intersections of a 3-mile-square grid. Information was collected according

Paul A. Murphy is Principal Mensurationist. Southern Forest Experiment Station, Monticello, Arkansas, and Herbert S. Sternitzke is Principal Resource Analyst, Southern Forest Experiment Station, Forest Service-USDA, New Orleans, Louisiana.

to standardized inventory procedures." Plots have lo-point samples spaced 66 feet apart over about 1 acre.

Plots within the Coastal Plain of the West Gulf region were screened by use of these criteria: (1) at least half the basal area in trees 5 inches and larger in the initial survey had to be loblolly pine; (2) all 10 points at each location had to be recovered in the remeasurement tally; (3) site quality information had to be collected from loblolly pine sample trees; (4) plots had to be free from catastrophic mortality, (5) sampled stands had to be even-aged and natural in origin and (6) plots had to be undisturbed by logging or other activity between measurements. Dates of surveys are Louisiana, 1964 and 1974; east Texas, 1965 and 1975; and Arkansas,:' 1969 and 1978.

The geographical distribution of 145 sample plots that met these criteria is shown in figure 1. Numbers of plots by age, site, and density are tabulated in table 3. Average stand age is 38; average basal area, 70 square feet per acre; and average site index, 90 feet. The range of data is from 17 to 137 square feet of basal area at remeasurement; site index varies from 68 to 127 feet; and stand age ranges from 18 to 85 years at remeasurement.

These stand and tree statistics were calculated: (1) site index from equations developed by Farrar (1973); (2) stand age at first measurement and at remeasurement; (3) basal area in square feet per acre of loblolly pine trees 5 inches and larger at the first measurement and of the same trees at remeasurement; and (4) solid wood volume from a l-foot stump to a 4-inch top, outside bark, of trees included in the basal area calculations. The data were then summarized into plot statistics according to Beers and Miller (1964).

'Renewable Resources Evaluation Staff. Forest resources inventory work plan. Suppl., For. Serv. Handb. 4809.11, 83 p. U.S. Dep. Agric. For. Serv., South. For. Exp. Stn.

'Includes only counties in the Coastal Plain.

Estimation **in Equation Systems**

Much recent research in the growth and yield of even-aged natural timber stands has used data from remeasured permanent plots. This information has prompted interest in estimating not only current volumes but also future yields. Some variables used to predict stand volume, such as site quality, are assumed to be constant. Others, such as stand age, change by a known amount. Others, such as basal area, change by an unknown amount, and their future values must be estimated. The result is an interdependent set of prediction equations.

Clutter (1963) fitted a yield model, a basal area growth model, and a volume growth model by least squares. He derived basal area projection and volume projection equations by integrating the growth functions to produce a five-equation system for estimating growth and yield of loblolly pine. Buckman (1962), Curtis (1967), and Moser and Hall (1969) also have developed equations based on the concept that the yield function is the integral form of the growth function.

Remeasurements on the same plot introduce the problem of correlated errors, which violates one of the basic assumptions of the standard linear model. Sullivan and Clutter (1971) modified Clutter's cubic yield equation and projected basal area equation by combining them into a single function. Parameters were es-

Table 2.— *Volume by State und diumetrr class of loblolly pine growing stock on commercial forest land west of the Mississippi River*

State	Date of Survey	All classes	5.0-6.9	7.0-8.9	9.0-10.9	11.0-12.9	13.0-14.9	15.0-16.9	17.0-18.9	19.0-20.9	21.0-28.9	29.0- and larger
		Million cubic feet										
Arkansas	1969	2603.8	250.2	283.5	357.9	398.5	374.1	313.9	246.8	173.5	196.7	8.7
Louisiana]	1974	4,804.6	383.3	614.0	650.4	690.4	668.9	523.7	470.6	319.8	449.1	34.4
Oklahoma	1976	63.7	3.7	6.7	10.1	6.2	10.3	2.8	5.0	10.2	8.7	___
'Texas	1975	5,186.8	339.9	545.2	721.8	808.0	779.9	655.1	513.3	367.2	429.9	26.5
All States		12,658.9	977.1	1,449.4	1,740.2	1,903.1	1,833.2	1.495.5	1,235.7	870.7	1,084.4	69.6

(Diameter Class: inches at breast height)

'Excludes southeast Louisiana parishes.

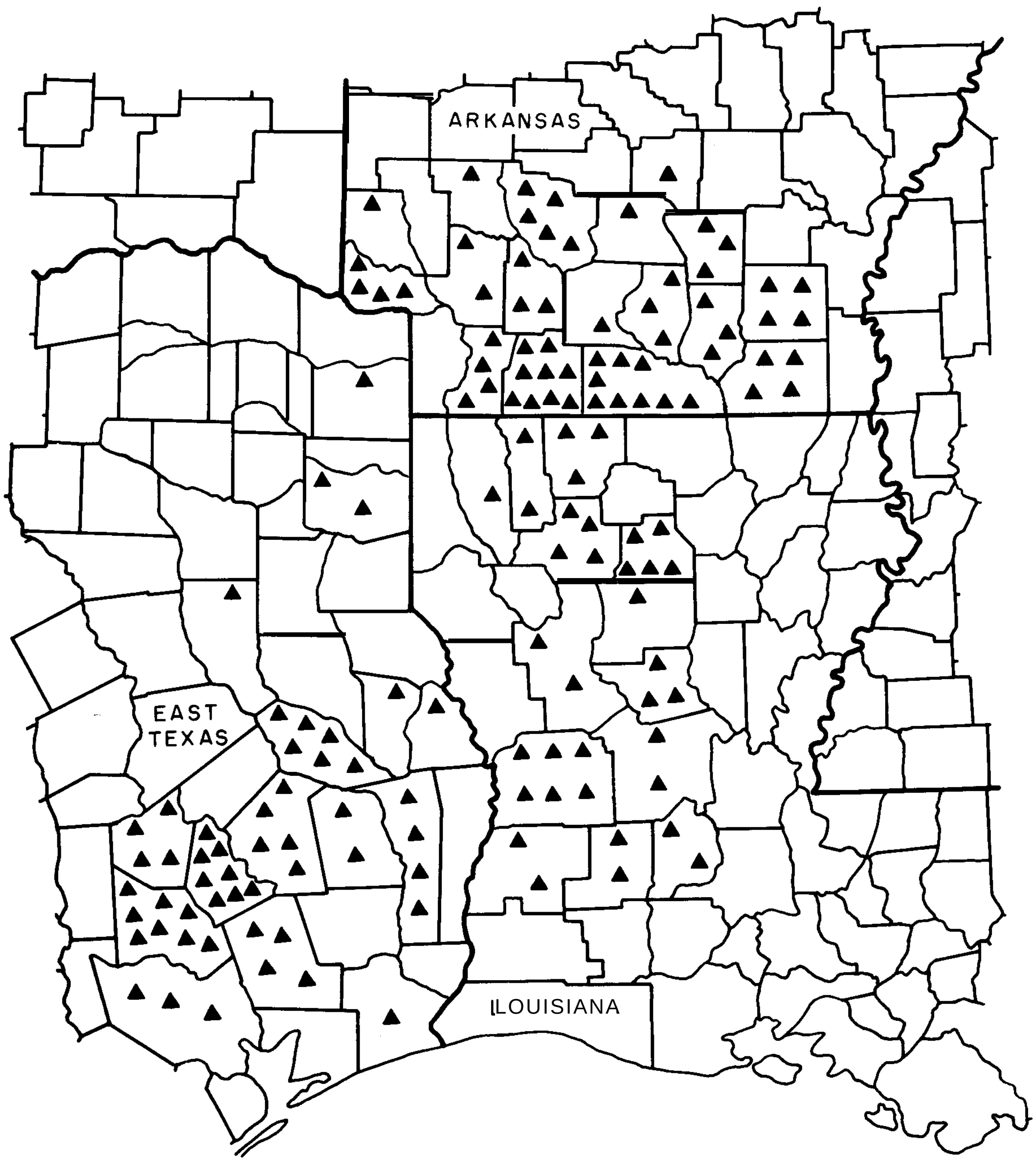

Figure 1.—*Geographical distribution of loblolly pine sample plots in Arkansas, Louisiana, and Texas.*

Table 3.— *Plot distribution by age, site index, and loblolly basal area at remeasurement*

Age	Site index	Basal area per acre						
years	*feet*	*square feet*						
		≤ 20	21-40	41-60	61-80	x1-100	100+	Total
1 l-20	70	—	----					0
	80		—	—	—		—	0
	90	----				----		0
	100		1	1		—	—	2
	110+	----	----	—	—	—	—	0
	Total	0	1	1	0	0	0	2
21-30	70	1						1
	80	----	3		1	1	----	5
	90	1	3	4	2	....	....	10
	100	—	1	2	3	1	3	10
	110+	----	2	2	3	2	—	9
	Total	2	9	8	9	4	3	35
31-40	70	—	4	3	2	1	—	10
	80	1	2	9	2	3	1	18
	90	—	4	6	10	5	1	26
	100		----		3	4	2	9
	110+			----			1	1
	Total	1	10	18	17	13	5	64
41-50	70		1			1		2
	80	—	1	1	1	2	2	7
	90	----		2	3	—	7	12
	100				2	1	2	5
	110+	----	----	—	—	—	—	0
	Total	0	2	3	6	4	11	26
51-60	70	—	1	—	1	—	—	2
	80	—	—		1	—	1	2
	90	----	----	—	1	2	—	3
	100		—		3	—	1	4
	110+			—				0
	Total	0	1	0	6	2	2	11
61+	70	----		1			----	1
	80	----		—	2	1	—	3
	90	—	—	—	1	1	1	3
	100	----	—	—	—	—	—	0
	110+	----	----	----	—	----	—	0
	Total	0	0	1	3	2	1	7
All ages	70	1	6	4	3	2	0	16
	80	1	6	10	7	7	4	35
	90	1	7	12	17	8	9	54
	100	0	2	3	11	6	8	30
	110+	0	2	2	3	2	1	10
	Total	3	23	31	41	25	22	145

timated by maximum likelihood. The fact that errors between two measurements of the same plot were correlated was taken into account in the maximum likelihood procedure. Seegrist and Arner (1978) extended this maximum likelihood procedure to more than one remeasurement.

Furnival and Wilson (1971) dealt with the problems of estimating parameters in a system of equations to predict growth and yield of forest stands. They pointed out that using ordinary least squares may not be an appropriate means of estimation: often it is necessary to treat avariable as both independent and dependent; coefficients in one equation may be related to coefficients in other equations; and residuals from equations may be correlated. They described several simultaneous equation techniques widely used in econometrics that might be applied to growth and yield equation systems. These techniques explicitly treat the problems of equation systems. One such technique, three-stage least squares (3SLS), is applied here to an equation system that estimates cubic yield and basal area of even-aged stands of loblolly pine.

Model Development

The equations are those of Clutter (1963) and Sullivan and Clutter (1971). In modified form, they are:

$$(1)\quad E(\ln B_2) = \ln(B_1)A_1/A_2 + \alpha_1(1-A_1/A_2) + \alpha_2(1-A_1/A_2)\ln(B_1) + \alpha_3(1-A_1/A_2)\ln(B_1)^2$$

$$(2)\quad E(\ln V_2) = \beta_0 + \beta_1 S + \beta_2 A_2^{-1} + \beta_3(\ln B_2)$$

where

A_i	=	stand age at the ith occasion
B_i	=	basal area in square feet per acre of loblolly pine trees 5 inches in diameter and greater at breast height
s	=	site index in feet at base age 50
V_i	=	cubic volume (inside bark) per acre of loblolly pine trees 5 inches in diameter and greater at breast height from a 1-foot stump height to a 4-inch top diameter, outside bark
α_i, β_i	=	parameters to be estimated.

The symbol E(. ..) denotes expected value of the quantity within the parantheses. The form of the basal area equation (1) was suggested by Seegrist.[4]

Variables in simultaneous equation models may be designated by standard econometric terminology (Furnival and Wilson 1971, Theil 1971, Pindyck and Rubinfeld 1976) as exogenous (independent) or endogenous (dependent). The variables stand age A_i and site S in equation (1) are uncontrolled covariates and exogenous to the equation system. The variables $\ln(B_i)$ and $\ln(V_i)$ are endogenous variables. The original set of relationships, equations (1) and (2), are the structural equations.

The estimation procedure depends upon whether unique estimates of parameters can be obtained. If so, the system is said to be exactly identified. If more than one estimate is possible, the system is overidentified. When none is possible, it is unidentified. The degree of identification can be determined by changing the system of equations to its reduced form, where the exogenous variables occur on the right side, and the endogenous variables occur on the left. The reduced form of equation (2) is:

$$(3)\quad E(\ln V_2) = \lambda_0 + \lambda_1 S + \lambda_2 A_2^{-1} + \lambda_3(A_1/A_2)\ln(B_1) + \lambda_4(1-A_1/A_2) + \lambda_5(1-A_1/A_2)\ln(B_1) + \lambda_6(1-A_1/A_2)\ln(B_1)^2$$

where $\lambda_0 = \beta_0$, $\lambda_1 = \beta_1$, $\lambda_2 = \beta_2$, $\lambda_3 = \beta_3$, $\lambda_4 = \beta_3\alpha_1$, $\lambda_5 = \beta_3\alpha_2$ and $\lambda_6 = \beta_3\alpha_3$.

Equation (1) is in reduced form. Estimates of the model parameters can be determined from those of the reduced form equations. Examination of the reduced form equations shows that there are two sets of estimates for the parameters α_1, α_2, and α_3. One set comes directly from equation (1), and the other is $\alpha_1 = \lambda_4/\lambda_3$, $\alpha_2 = \lambda_5/\lambda_3$, $\alpha_3 = \lambda_6/\lambda_3$.

Since the system is overidentified and the endogenous variables probably are correlated, using ordinary least squares is inappropriate. Two-stage least squares (2SLS) could be used to estimate the parameters of an overidentified system. The basal area equation (1) is fitted using ordinary least squares, and expected values are used as independent variables in fitting equation (2). But using 3SLS is more efficient because it takes account of possible correlation in errors between equations in overidentified systems.

The 3SLS is an extension of 2SLS. Estimates of the second stage are used to obtain estimates of residuals. Residuals are used to obtain estimates of cross-equation correlations. Zellner and Theil (1962) originated this technique, and estimates obtained are consistent. The efficiency of 3SLS ususally exceeds that of the two-stage procedure. Its efficiency is matched by 2SLS only if there are no correlated errors between the system equations (Madansky 1964).

But 3SLS would not estimate the likely serial correlation between original and remeasured observations in equation (2). However, equation (2) can be decomposed into two components — a function that predicts current yield and another that forecasts future volume — so that these three equations can be solved:

$$(4)\quad E(\ln B_2) = \ln(B_1)A_1/A_2 + \alpha_1(1-A_1/A_2) + \alpha_2(1-A_1/A_2)\ln(B_1) + \alpha_3(1-A_1/A_2)\ln(B_1)^2$$

$$(5)\quad E(\ln V_1) = \beta_0 + \beta_1 S + \beta_2 A_1^{-1} + \beta_3\ln(B_1)$$

[4]Personal communication, March 5, 1979.

(6) $E(\ln V_2) = \beta_0 + \beta_1 S + \beta_2 A_2^{-1} + \beta_3 \ln(B_2)$.

Data were evaluated with the Statistical Analysis System (SAS) 3SLS procedure (Barr, Goodnight, Sall, Helwig 1976) that handles side conditions on the parameters.

Results

The results of analyses are shown in table 4. Cross-equation correlations of residuals from the second stage of the analysis are:

	$\ln(B_2)$	$\ln(V_1)$	$\ln(V_2)$
$\ln(B_2)$	1.0	-.34457	-.37965
$\ln(V_1)$	-.34457	1.0	.60611
$\ln(V_2)$	-37965	.60611	1.0

A negative correlation results between the basal area projection (4) and the two yields (5,6). Also, a stronger correlation results between projected basal area and projected volume than between projected basal area and current yield. Residuals between current and projected yields are positively correlated. The same pattern of correlated errors has been reported by Seegrist and Arner[5]. Sullivan and Clutter (1971) observed a positive correlation between residuals of current versus projected yields.

Although refined estimates of parameters might be obtained by calculating the residuals and obtaining new values for cross-equation correlations, Madansky (1964) found that iterated 3SLS offers no practical gain in efficiency.

Whether any advantage results from using simultaneous equation procedures, such as 3SLS, cannot be settled on the basis of the simple equation system we presented here. But as the prediction of growth and yield evolves from reliance upon a single equation to a set of interdependent ones, the applicability of these techniques becomes increasingly relevant. Furnival and Wilson (1971) and Sullivan and Reynolds (1976) have outlined the hazards of applying ordinary least squares to equation systems. Three-stage least squares has an advantage over other simultaneous equation procedures such as maximum likelihood. Software packages are readily available, and extension to more than one remeasurement is straightforward.

Table 4.— *Estimates of model coefficients and their standard errors*

Parameter	Estimate	Standard Error
α_1	1.55265	0.81079
α_2	1.55846	0.48408
α_3	-0.15086	0.07115
β_0	3.02439	0.12324
β_1	0.00515	0.00114
β_2	-20.17542	1.04924
β_3	1.06693	0.02804

[5]Personal communication, March 5, 1979.

Table 5.— *Evaluation of model predictions*

Equation	Mean Value	Criterion			
		$\bar{D}$[1]	RMSE[2]	$\bar{D}$%[3]	RMSE%[4]
	Cubic feet			*Percent*	
Current volume V_1	897	82	193	12	30
Projected volume V_2	1,754	-24	235	1	15
	Square feet			*Percent*	
Basal area B_2	69.8	-1.5	8.2	-1	12

[1]Mean difference $\bar{D} = \sum \frac{(\hat{Y}_i - Y_i)}{n}$

[2]Root mean square error RMSE $= \sqrt{\frac{1}{n}\Sigma(\hat{Y}_i - Y_i)^2}$

[3]Percentage of mean difference $\bar{D}\% = \frac{1}{n}\sum\left(\frac{\hat{Y}_i - Y_i}{Y_i}\right) \times 100$

[4]Percentage of root mean square error $\text{RMSE}\% = \sqrt{\frac{1}{n}\Sigma\frac{(\hat{Y}_i - Y_i)^2}{Y_i^2}} \times 100$

where n = number of observations, Y_i = actual value, and $\hat{Y}_i$ = predicted value.

Our yield equations accounted for 90.7 and 92.9 percent of the variation about mean cubic-foot volume for current V_1 and projected V_2 stand volumes. The basal area equation explained 91.3 percent of the variation about mean projected basal are B,.

Table 5 provides information on the prediction performance of the equation system. The mean difference indicates that the current volume equation overestimates volume, and the projected volume and basal area equations give slight underestimates on the average. The root mean square errors penalize large deviations from the observed values more than average differences. Percentage of mean difference and percentage of root mean square error depict error as a proportion of observed values.

Application of the equations is straightforward. Projected basal area can be predicted with the equation

$$(7)\quad \ln(B_2) = \ln(B_1)A_1/A_2 + 1.55265(1\text{-}A_1/A_2) + 1.55846(1\text{-}A_1/A_2)\ln(B_1) - .15086(1\text{-}A_1/A_2)\ln(B_1)^2$$

Current and projected yields can be estimated by

$$(8)\quad \ln(V) = 3.02439 + 0.005155 - 20.17542/A + 1.066931n(B).$$

For example, suppose estimates of current yield and projected volume and basal area in 10 years are wanted for a loblolly pine stand at age 40 with a site index of 85 feet and a basal area of 70 square feet per acre. Current yield estimate is:

$$V_1 = \exp[3.02439 + 0.00515(85) - 20.17542/40 + 1.06693\ln(70)]$$
$$= 1791 \text{ cubic feet.}$$

Projected basal area is:

$$B_2 = \exp[\ln(70)40/50 + 1.55265(1\text{-}40/50) + 1.55846(1\text{-}40/50)\ln(70) - .15086(1\text{-}40/50)\ln(70)^2],$$
$$= 89 \text{ square feet.}$$

Projected volume is:

$$V_2 = \exp[3.02439 + 0.00515(85)\ 20.17542/50 + 1.06693\ln(89)],$$
$$= 2560 \text{ cubic feet.}$$

Periodic annual increment for merchantable volume would be 77 cubic feet. Figure 2 illustrates cubic yields expected for different densities for site index 90; figure 3 shows projected basal area yields for various initial densities; and figure 4 depicts projected volumes for different initial densities for site index 90. The estimates indicate yields that can be expected from extensive areas of woods-run conditions.

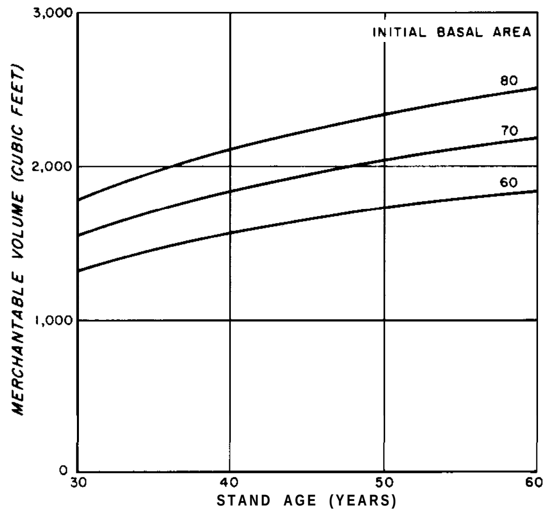

Figure 2.— *Relationship of current yield to basal area and age for natural even-aged loblolly pine stands in the West Gulf Coastal Plain, site index 90.*

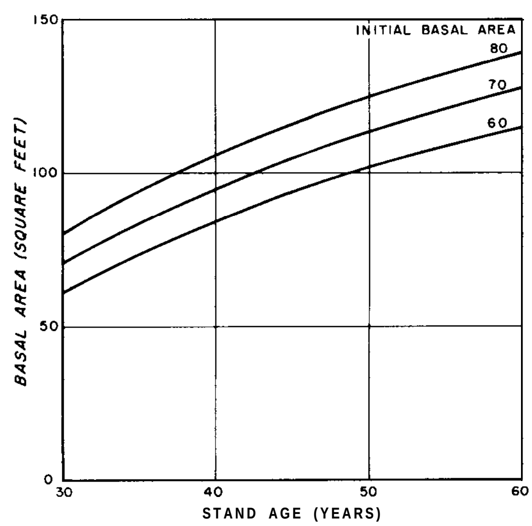

Figure 3.- *Relationship of projected basal area to age and initial basal area at age 30 for natural even-aged loblolly pine stands in the West Gulf Coastal Plain.*

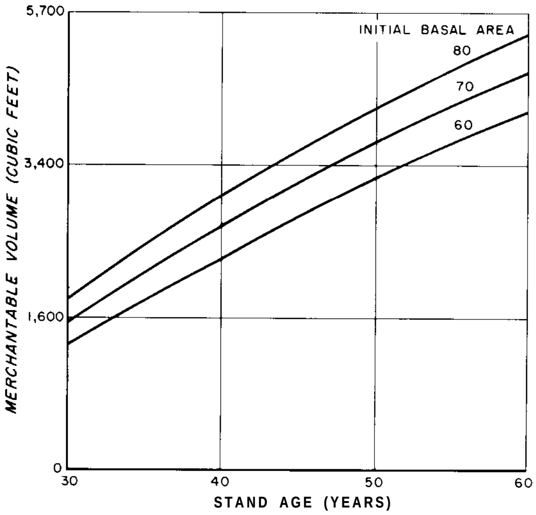

Figure 4.— *Relationship of projected volume to age and initial basal area at age 30 for natural even-aged loblolly pine stands in the West Gulf Coastal Plain, site index 90.*

LITERATURE CITED

Barr, A.J., J. H. Goodnight, J. P. Sall, and J. T. Helwig. ,
1976. A user's guide to SAS 76. SAS Institute, Inc., Raleigh, N. C., 329 p.

Beers, T. W., and C. I. Miller.
1964. Point sampling: research results, theory, and applications. Purdue Univ. Agric. Exp. Stn. Res. Bull. 786, 56 p.

Buckman, R. E.
1962. Growth and yield of red pine in Minnesota. U.S. Dep. Agric. Tech. Bull. 1272, 50 p.

Clutter, J. L.
1963. Compatible growth and yield models for loblolly pine. For. Sci. 9:354-371.

Curtis, R. O.
1967. A method of estimation of gross yield of Douglas-fir. For. Sci. Monog. 13, 24 p.

Farrar, R. M.
1973. Southern pine site index equations. J. For. 71:696-697.

Furnival, G. M., and R. W. Wilson.
1971. Systems of equations for predicting forest growth and yield. In Statistical Ecology 3:43-55. G. P. Patil, E. C. Pielow, W. E. Waters, eds. Penn. State University Press, University Park.

Madansky, A.
1964. On the efficiency of three-stage least squares estimation. Econometrica 32:51-56.

Moser, J. W., and 0. F. Hall.
1969. Deriving growth and yield functions for uneven-aged forest stands. For. Sci. 15:183-188.

Pindyck, R. S., and D. L. Rubenfield.
1976. Econometric models and economic forecasts. 580 p. McGraw-Hill, Inc., New York.

Seegrist, D. W., and S. L. Arner.
1978. Statistical analysis of linear growth and yield models with correlated observations from permanent plots remeasured at fixed intervals. *In* Growth models for long-term forecasting of timber yields. p. 203-223, Pub. FWS-1-78. J. Fries, H. E. Burkhart, T. A. Mar, eds., Va. Polytech. Inst. State Univ., Blacksburg.

Sternitzke, H. S., and T. C. Nelson.
1970. The southern pines of the United States. Econ. Bot. 24:142-150.

Sullivan, A. D., and J. L. Clutter.
1971. A simultaneous growth and yield model for loblolly pine. For. Sci. 18:76-86.

Sullivan, A. D., and M. R. Reynolds, Jr.
1976. Regression problems from repeated measurements. For. Sci. 22:382-385.

Theil, H.
1971. Principles of econometrics. 736 p. John Wiley and Sons, Inc., New York.

U.S. Dep. Agric., Forest Service.
1965. Silvics of forest trees of the United States. U.S. Dep. Agric. For. Serv. Agric. Handb. 271, 762 p.

Zellner, A., and H. Theil.
1962. Three-stage least squares: simultaneous estimation of simultaneous equations. Econometrica 30:54-78.

MURPHY, PAUL A., AND HERBERT S. STERNITZKE,

1979. Growth and yield estimation for loblolly pine in the West Gulf. U.S. Dep. Agric. For. Serv. Res. Pap. SO-154, 8 p. South. For. Exp. Stn., New Orleans, La.

Presents an equation system developed to estimate current yield, projected basal area, and projected volume for merchantable natural stands on a per-acre basis. These estimates indicate yields that can be expected from woods-run conditions.

Additional keywords: *Pinus taeda*, volume prediction, basal area projection, stand volume, natural stand yields.