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A GRAPHIC CHI-SQUARE TEST FOR TWO-CLASS GENETIC SEGREGATION RATIOS

Abstract.--A chart is presented for testing the goodness of fit of observed two-class genetic segregation ratios against hypothetical ratios, eliminating the need of computing chi-square. Although designed mainly for genetic studies, the chart can also be used for other types of studies involving two-class chi-square tests.

When a geneticist encounters a trait which is expressed qualitatively, he is often interested in comparing the observed frequencies of individuals in each phenotypic class against frequencies expected under one or more classical Mendelian segregation patterns. To determine the goodness of fit of observed to hypothetical ratios, he commonly uses chi-square tests. Although these tests are relatively simple, they are a time-consuming chore. The chart presented here is designed to eliminate this chore for problems involving two phenotypic classes.

METHOD

For two classes of individuals

$$\chi^2 = \frac{(f_1 - F_1)^2}{F_1} + \frac{(f_2 - F_2)^2}{F_2},$$

where f_1 and f_2 are the sample counts of individuals in each class, and F_1 and F_2 are the corresponding hypothetical numbers (Snedecor 1956, p. 19).

Yule and Kendall (1940, p. 68) defined $\phi^2 = \frac{\chi^2}{n}$, where $n = f_1 + f_2 = F_1 + F_2$. By substitution,

$$\phi^2 = \frac{(f_1 - F_1)^2}{nF_1} + \frac{(f_2 - F_2)^2}{nF_2}.$$

Let r and 1-r be the observed proportions and R and 1-R be the corresponding hypothetical proportions of individuals in each class.

Then,

$$f_1 = nr,$$
$$f_2 = n(1-r),$$
$$F_1 = nR,$$
and $$F_2 = n(1-R).$$

Substituting and collecting terms,

$$\phi^2 = \frac{(r-R)^2}{R(1-R)}, \text{ and } \phi = \frac{|r-R|}{\sqrt{R(1-R)}}.$$

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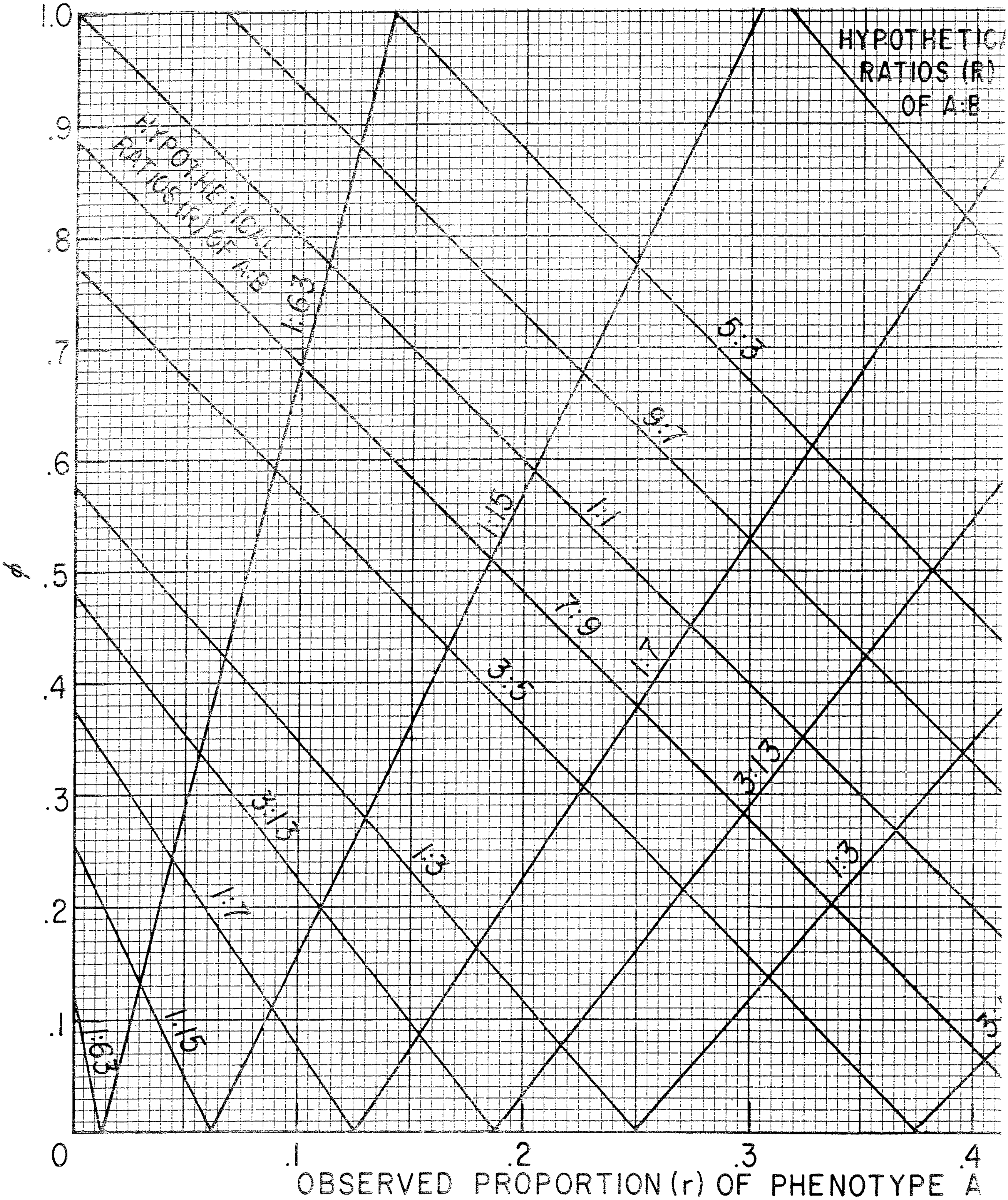
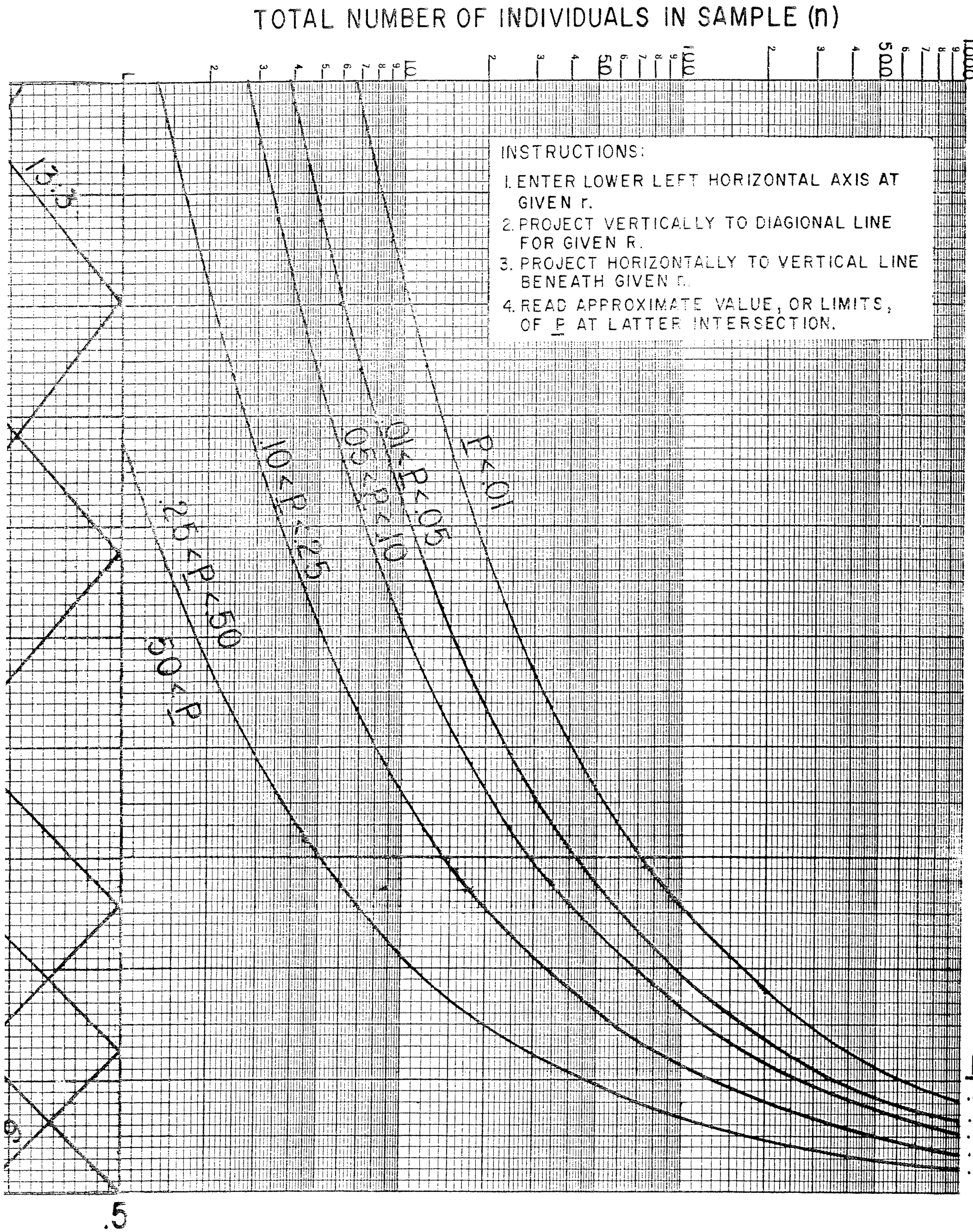
Figure 1.--Chart for testing goodness of fit of observed to hypothetical two-class genetic segregation ratios.




TOTAL NUMBER OF INDIVIDUALS IN SAMPLE (n)
INSTRUCTIONS:
1. ENTER LOWER LEFT HORIZONTAL AXIS AT GIVEN r.
2. PROJECT VERTICALLY TO DIAGIONAL LINE FOR GIVEN R.
3. PROJECT HORIZONTALLY TO VERTICAL LINE BENEATH GIVEN n.
4. READ APPROXIMATE VALUE, OR LIMITS, OF P AT LATTER INTERSECTION.
P<.01
.01<P<.05
.05≤P<.10
.10<P≤.25
.25<P<.50
.50<P
P
.01
.05
.10
.25
.50
.5

Thus, ϕ is independent of n because it is derived solely from the observed and expected proportions in the two classes. Furthermore, for any hypothetical ratio, the ϕ values for observed r are given by a straight line with slope $\mp \frac{1}{\sqrt{R(1-R)}}$. Such lines for various hypothetical genetic ratios are drawn on the left side of figure 1. Lines are shown through the range $r = 0$ to $r = 0.5$ only because solutions for r tested against R, and 1-r tested against 1-R, are identical. For example, a test of an observed ratio of 7:3 against a hypothetical ratio of 3:1 is equivalent to a test of 3:7 against 1:3. Also, the lines are terminated at $\phi = 1$. This restriction prevents use of the chart when sample sizes are small (5 or less) and the observed and hypothetical ratios differ greatly.

The curves on the right side of figure 1 depict the distribution of ϕ with 1 d.f. for various sample sizes. This arrangement permits a direct determination of P, the probability of obtaining a greater value of χ^2 from a population having the hypothetical ratio being tested. The distribution curves were drawn by computing values of ϕ from the equation $\phi = \sqrt{\frac{\chi^2}{n}}$, in which values of χ^2 were taken from the distribution table in Snedecor (1956, p. 28).

The procedure for using figure 1 is outlined directly on the chart and in the following examples. Note that the method does not contain a correction for continuity (Snedecor 1956, p. 217).

EXAMPLES

1. In a sample of 800 seeds, an investigator found that 440 were wrinkled and 360 were smooth (Snedecor 1956, p. 19, example 1.10.1). Genetic theory led him to expect a ratio of 1:1. We shall assume that he would accept the null hypothesis (no difference between observed and expected ratios) if P is found to be greater than 0.05. To make the test, first compute r for the least frequent class (smooth): $r = 360/800 = 0.45$. Then enter the lower left horizontal axis of figure 1 at 0.45; project vertically to the intersection with the diagonal line for a hypothetical 1:1 ratio (this occurs at $\phi = 0.10$); project this point horizontally to the right until it intersects with the vertical line through $n = 800$. Here we see that $P < 0.01$, and the investigator would, therefore, reject the null hypothesis. This result agrees with the conventional method, in which $\chi^2 = 8.00$ and $P < 0.01$ for 1 d.f. (Snedecor 1956, p. 28, table 1.14.1).

2. Squillace and Kraus (1963) reported 23 normal and 11 albino seedlings in a family of slash pine obtained by selfing. The investigators tested the observed ratio against a hypothetical ratio of 3 normal: 1 albino (with acceptance of the null hypothesis when $P > 0.05$). In using the chart, we again take the class of lowest frequency as r. Thus, with $r = \frac{11}{34} = 0.324$, $R = 1:3$, and $n = 34$, we find in figure 1 that $0.25 < P < 0.50$, leading to acceptance of the null hypothesis.

3. This third example is given to show that, although the chart is designed mainly for genetic studies, it can also be used for other types of investigations. Suppose a sampler of public opinion found that 520 voters preferred one candidate while 480 favored his opponent. With a hypothesis ($P > 0.05$) of equal numbers of votes for the two candidates, we shall test $r = 0.48$ against $R = 1:1$, with $n = 1{,}000$. Using figure 1, we note that $0.10 < P < 0.25$, which leads to acceptance of the hypothesis.

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