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Estimating Actual Evapotranspiration for Forested Sites: Modifications to the Thornthwaite Model

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Abstract

A previously coded version of the Thornthwaite water balance model was used to estimate annual actual evapotranspiration (AET) for 29 forested sites between 1900 and 1993 in the Upper Great Lakes area. Approximately 8 percent of the data sets calculated AET in error. Errors were detected in months when estimated AET was greater than potential evapotranspiration. Annual climate variability led to errors in accrued soil water storage, which led to errors in the calculation of AET. Two hydrologically justifiable modifications were made to correct errors resulting from the use of the original coded Thornthwaite model. The first modification allows for soil water accumulation throughout the year, whereas the second allows for oversaturated soil conditions in the spring. The modified program appears to be robust for any temperate climate condition.

Keywords: Hydrology, potential evapotranspiration, Upper Great Lake States, water balance.

Introduction

The most difficult parameter to measure when calculating a site's water balance is actual evapotranspiration (AET), which is a function of precipitation, temperature, solar radiation, soil water storage, wind, canopy and understory interception, and growth rates. Few methods for measuring AET directly are available. Although field studies using lysimeters and air-monitored tents have been somewhat successful in measuring AET in agricultural or open situations, AET cannot be measured directly within forested systems by any practical field method (Brooks and others 1991). Because of their size and complex surface dynamics, trees are not easily measured by either of these methods. One approach for estimating AET on a watershed basis is to use paired watersheds where differences in streamflow following clearcutting are attributed to changes in AET (Hornbeck and others 1970). Unfortunately, the scale, cost, availability of sites, and the time necessary to implement this approach are not appropriate for site-specific studies.

At the site or stand scale, the method commonly used to estimate AET is a water balance that recognizes the relationship between potential evapotranspiration (PET) and AET. The results of equations used to calculate PET, and, thus, AET, are indices, not absolute values. The water budget method uses soil water storage in conjunction with PET and precipitation to estimate AET (Brooks and others 1991). A number of methods have been developed to estimate PET, including the Thornthwaite equation (Thornthwaite and Mather 1955), which is based on temperature and day length. The traditional approach has been to estimate PET using long-term monthly averages of temperature and precipitation as well as latitude (to determine day length), which are easily accessible data. Thornthwaite and Mather (1957) used PET, soil water storage, and precipitation to calculate a monthly site-specific water balance from which they could estimate AET. Simply, AET is the sum of monthly change in soil moisture storage (±) and precipitation. Many people have coded Thornthwaite's approach for computer calculation, but one of the first to do so was P.E. Black of Syracuse University (Black 1966). Dr. Black's program calculates a site's water balance using monthly temperature and precipitation, soil water storage, and latitude. Grigal and Bloom (1985) later modified Black's program to better estimate runoff for sites in Minnesota. Modifications stemmed from the comparison of observed and estimated ratios of runoff to precipitation for six watersheds in eastern and northern Minnesota. The program tended to underestimate runoff. Modifications were made in the evapotranspiration portion of the program to better approximate the measured runoff. For this paper, we will refer to the Grigal and Bloom (1985) modified program as the "original program."

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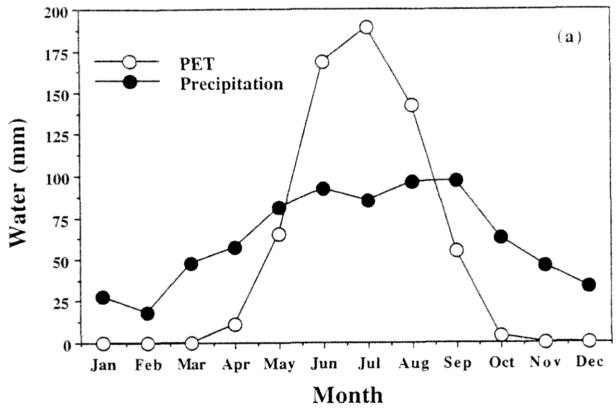
We used the original program to estimate annual AET for 29 sites located in the Upper Great Lakes area. As one environmental variable, AET has been used to predict annual aspen (*Populus tremuloides* Michx.) height growth (Leary and others 1997). Instead of long-term average climatic conditions, we interpolated monthly climatic data for the life of individual trees on each site. We ran 2,726 site-year combinations, and our evaluation of the results showed that, in some cases, annual AET was greater than PET, which is a physical impossibility. A search of occurrences where AET > PET showed that 229 of 2,726, or 8.4 percent of annual results, had been calculated in error. Black (1966) discussed these types but gave no remedy. Although such errors are not prevalent when using long-term averages (fig. 1a), large variability in monthly precipitation during 3 consecutive months, i.e., wet-dry-wet or dry-wet-dry, leads to such errors (fig. 1b). When circumstances lead to such a variable annual climatograph as is shown in figure 1b, the program incorrectly calculates the recovery of soil water. Errors in the original program are not in calculating AET but in the method used to determine accumulated soil water. In this study, we modified the program to properly measure soil water gains and losses for all climatic conditions encountered.



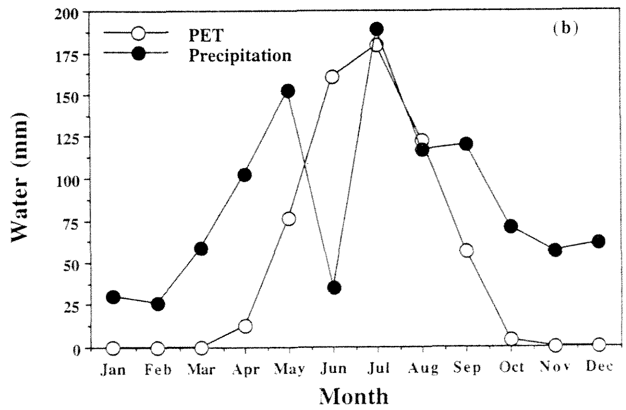


Figure 1—(a) Comparison of long-term average annual precipitation and potential evapotranspiration (PET) (b) with those that are typically encountered when analyzing single years of data.

Program Modifications and Results

Two successive modifications were needed to fully correct for errors in the original program. Before discussing those modifications, however, some parameters need to be defined (Black 1966):

1. PET = potential evapotranspiration calculated with the Thornthwaite equation.

2. P-PET = precipitation less the potential evapotranspiration.

3. ACPWL = accumulated potential water loss, which is the amount of soil water lost when PET exceeds P; i.e., there is less precipitation than potential evapotranspiration. In the calculation of AET, ACPWL is not a factor until P-PET becomes negative. To determine the ACPWL for a particular month, the previous month's ACPWL and the current month's P-PET are summed. In the original program, ACPWL becomes 0 after a month in which PET < P.

4. STRGE = soil storage; this is the maximum soil storage at field capacity (ACPWL = 0). When below field capacity (ACPWL < 0), STRGE is a function of both maximum soil storage and ACPWL.

5. DELTA = the difference between STRGE in successive months when it is less than maximum. When DELTA is negative, then AET < PET; i.e., soil moisture is limiting evapotranspiration. When DELTA is positive, then AET = PET.

6. AET = actual evapotranspiration. This is the sum of available precipitation for the month ± the change in STRGE. When DELTA is positive, AET = PET. When DELTA is negative, AET = precipitation for the month + the absolute value of DELTA.

The original program assumes that once P-PET becomes negative (ACPWL becomes negative), it can not become positive until the end of the hydrologic season. Our modification assumes that soil water deficits are accumulated throughout the year regardless of the positive or negative value of P-PET. As in the original program, we begin to accumulate potential water loss when P-PET becomes negative. However, if P-PET then becomes positive we simply continue to accumulate ACPWL until it reaches zero or the hydrologic season ends. At values greater than zero, ACPWL does not enter into AET calculations (AET = PET). Hydrologically, the scenario we are simulating is more natural than that simulated by the original program. In the original program, 1 mm of excess precipitation (P-PET) following a month with a large soil water deficit will allow PET to equal AET. However, the following month's soil deficit should reflect the large deficits already present in the soil. If one considers the soil as a bucket, the bucket is half full in the first month (soil storage deficit). In the second month, water is taken out (evapotranspiration) at the same rate as it is being put in (precipitation); thus, AET = PET, and the excess precipitation (1 mm) is added to storage. In the third month, the bucket does not begin full, as assumed in the original model; it has only 1 mm more water than it did at the end of the first month. After the first modification, we found that 197 of 2,726 data sets, or 7.2 percent, had still erroneously calculated AET.

We investigated the data sets with errors remaining after the first modification. In all cases, STRGE never fell to field capacity in the spring; it dropped from saturated (soil water + snow water) to undersaturated (soil water), never equilibrating to maximum capacity. After assessing the site-year combinations in our data and those in the literature (Thornthwaite and Mather 1957, Black 1966, Grigal and Bloom 1985), we found all those that calculated AET < PET fell to the exact value of maximum storage sometime in the spring before ACPWL began to accumulate. When temperatures are > -1 °C (Eq.1), the original program calculates STRGE with the following equation:

$$STRGE = 10^{[\log MSW - (0.525/(MSW^{1.0371}) \times ACPWL)]} \quad (1)$$

where

STRGE = soil water storage,
MSW = the maximum soil water, and
ACPWL = the absolute value of accumulated potential water loss.

Equation 1 was derived from soil storage tables given in Thornthwaite and Mather (1957). When ACPWL = 0, STRGE is equal to maximum soil storage; and when ACPWL < 0, soil water storage is less than maximum. In most situations, ACPWL = 0 for some period after mean monthly air temperatures are > -1 °C (when PET begins). For the few circumstances encountered here, ACPWL began to accumulate in the same month that PET began. The original program calculates AET in error for these situations because it calculates changes in storage as soon as ACPWL begins. When ACPWL begins directly after a month when snow is still accumulating (temperature < -1 °C), the original program calculates the change in storage from an oversaturated condition (soil water + snow water) to an undersaturated condition, not from maximum soil water to undersaturated conditions. Our second modification recognizes these conditions and simply calculates ACPWL from maximum soil storage instead of oversaturated soil storage. As with the first modification, the second more nearly simulates natural conditions. Meltwater is not transpired or evaporated to a great extent, especially at rates greater than potential; it either infiltrates, raising the soil to field capacity, or is lost as runoff. With the second modification, 100 percent of the site-year data calculated AET ≤ PET.

Examples of program output for three selected sites show the results of modifications (tables 1, 2, and 3). The modifications did not change AET estimates for years where climate data followed long-term patterns; although the second modification does affect how STRGE (soil storage) is accumulated (table 1). The second data set calculated AET > PET using the original program, but AET ≤ PET after the first and second modifications (table 2). Values for P-PET changed from negative in June to positive in the wet month of July. Thus, using the original program, ACPWL began from zero in August. This error leads to zero storage in the original program and a subsequent overestimation of AET. The first modification corrects this error by continuing to accumulate potential water loss. Again, the change is justified hydrologically because the soil does not fully recover to maximum soil water conditions if PET = AET in the previous month. In table 2, August does not begin with zero ACPWL as assumed in the original program but with the sum of soil water losses and gains from all previous months. The third data set estimates AET incorrectly for both the original and the initially modified version of the program (table 3). Soil water storage is never at the exact maximum value. The DELTA value for April is the difference between soil storage in March and April. The large negative DELTA was calculated in error and led to an overestimate of AET; it actually should have been the difference between maximum soil storage and soil storage for April (table 3). The final modification allows for a reliable estimation of AET.

Table 1—Model calculation of AET in normal climatic year. The original program, first modification, and second modification calculate AET < PET (maximum soil storage = 153 mm, latitude = 46.6° N, year = 1904)

Months and years	Temp	PPT	PET	P-PET	ACPWL	STRGE	DELTA	AET
	°C	mm						
				Original Program				
Jan	-14.2	45	0	45	0	265	0	0
Feb	-16.3	39	0	39	0	305	0	0
Mar	-5.7	59	0	59	0	363	0	0
Apr	1.1	26	4	22	0	153	0	4
May	10.0	97	70	27	0	153	0	70
Jun	15.6	80	179	-98	-98	80	-72	153
Jul	17.4	80	177	-96	-195	42	-37	118
Aug	15.7	104	128	-23	-219	36	-5	110
Sep	11.7	109	52	57	0	93	57	52
Oct	6.2	127	3	124	0	153	60	3
Nov	1.4	27	8	20	0	153	0	8
Dec	-9.4	67	0	67	0	220	0	0
Year	—	861	621	240	—	—	—	518
				First Modification				
Jan	-14.2	45	0	45	0	265	0	0
Feb	-16.3	39	0	39	0	305	0	0
Mar	-5.7	59	0	59	0	363	0	0
Apr	1.1	26	4	22	0	153	0	4
May	10.0	97	70	27	0	153	0	70
Jun	15.6	80	179	-98	-98	80	-72	153
Jul	17.4	80	177	-96	-195	42	-37	118
Aug	15.7	104	128	-23	-219	36	-5	110
Sep	11.7	109	52	57	-162	93	57	52
Oct	6.2	127	3	124	-39	153	60	3
Nov	1.4	27	8	20	-19	153	0	8
Dec	-9.4	67	0	67	0	220	0	0
Year	—	861	621	240	—	—	—	518
				Second Modification				
Jan	-14.2	45	0	45	0	247	0	0
Feb	-16.3	39	0	39	0	286	0	0
Mar	-5.7	59	0	59	0	345	0	0
Apr	1.1	26	4	22	0	153	0	4
May	10.0	97	70	27	0	153	0	70
Jun	15.6	80	179	-98	-98	80	-72	153
Jul	17.4	80	177	-96	-195	42	-37	118
Aug	15.7	104	128	-23	-219	36	-5	110
Sep	11.7	109	52	57	-162	52	16	52
Oct	6.2	127	3	124	-39	118	66	3
Nov	1.4	27	8	20	-19	134	16	8
Dec	-9.4	67	0	67	0	202	0	0
Year	—	861	621	240	—	—	—	518

TEMP = Temperature; PPT = Precipitation, PET = potential evapotranspiration; P-PET = precipitation less potential evapotranspiration; ACPWL = accumulated potential water loss; STRGE = soil storage; DELTA = the difference between STRGE in successive months when it is less than maximum; AET = actual evapotranspiration.

Table 2—Model calculation of AET in abnormal climatic year. The original program calculates AET > PET in August, while the first and second modification calculate AET < PET for all months (maximum soil storage = 153 mm, latitude = 46.6° N, year = 1903)

Months and years	Temp	PPT	PET	P-PET	ACPWL	STRGE	DELTA	AET
	°C	*mm*						
				Original Program				
Jan	-11.0	30	0	30	0	300	0	0
Feb	-10.2	26	0	26	0	326	0	0
Mar	-0.9	59	0	59	0	153	0	0
Apr	4.1	102	13	89	0	153	0	13
May	11.3	152	76	75	0	153	0	76
Jun	14.0	35	160	-124	-124	67	-85	120
Jul	17.8	189	179	11	0	0	-66	179
Aug	15.2	117	122	-5	-5	147	147	264
Sep	12.8	120	56	64	0	153	6	56
Oct	8.0	71	4	67	0	153	0	4
Nov	-3.0	56	0	56	0	209	0	0
Dec	-12.0	61	0	61	0	270	0	0
Year	—	1017	610	407	—	—	—	712
				First Modification				
Jan	-11.0	30	0	30	0	300	0	0
Feb	-10.2	26	0	26	0	326	0	0
Mar	-0.9	59	0	59	0	153	0	0
Apr	4.1	102	13	89	0	153	0	13
May	11.3	152	76	75	0	153	0	76
Jun	14.0	35	160	-124	-124	67	-85	120
Jul	17.8	189	179	11	-114	72	5	179
Aug	15.2	117	122	-5	-120	69	-2	119
Sep	12.8	120	56	64	-56	106	36	56
Oct	8.0	71	4	67	0	153	47	4
Nov	-3.0	56	0	56	0	209	0	0
Dec	-12.0	61	0	61	0	270	0	0
Year	—	1017	610	407	—	—	—	567
				Second Modification				
Jan	-11.0	30	0	30	0	300	0	0
Feb	-10.2	26	0	26	0	326	0	0
Mar	-0.9	59	0	59	0	153	0	0
Apr	4.1	102	13	89	0	153	0	13
May	11.3	152	76	75	0	153	0	76
Jun	14.0	35	160	-124	-124	67	-85	120
Jul	17.8	189	179	11	-114	72	5	179
Aug	15.2	117	122	-5	-120	69	-2	119
Sep	12.8	120	56	64	-56	106	36	56
Oct	8.0	71	4	67	0	153	47	4
Nov	-3.0	56	0	56	0	209	0	0
Dec	-12.0	61	0	61	0	270	0	0
Year	—	1017	610	407	—	—	—	567

TEMP = Temperature; PPT = Precipitation, PET = potential evapotranspiration; P-PET = precipitation less potential evapotranspiration; ACPWL = accumulated potential water loss; STRGE = soil storage; DELTA = the difference between STRGE in successive months when it is less than maximum; AET = actual evapotranspiration.

Table 3--Model calculation of AET in abnormal climatic year. The original program and first modification calculate AET > PET in April while the second modification calculates AET < PET for all months (maximum soil storage = 177 mm, latitude = 47.7°N, year = 1926)

Months and years	Temp	PPT	PET	P-PET	ACPWL	STRGE	DELTA	AET
	°C	*mm*						
				Original Program				
Jan	-11.8	18	0	18	0	259	0	0
Feb	-9.0	15	0	15	0	273	0	0
Mar	-7.0	32	0	32	0	305	0	0
Apr	3.5	1	10	-7	-7	169	-136	138
May	13.2	39	78	-38	-46	136	-32	72
Jun	14.0	83	142	-57	-104	98	-37	121
Jul	19.2	109	171	-61	-166	69	-28	138
Aug	17.7	64	125	-60	-227	49	-19	84
Sep	10.8	114	42	72	0	122	72	42
Oct	4.6	70	2	68	0	177	55	2
Nov	-6.4	44	0	44	0	221	0	0
Dec	-13.7	20	0	20	0	240	0	0
Year	—	610	569	41	—	—	—	597
				First Modification				
Jan	-11.8	18	0	18	0	190	0	0
Feb	-9.0	15	0	15	0	205	0	0
Mar	-7.0	32	0	32	0	237	0	0
Apr	3.5	1	10	-7	-7	169	-67	69
May	13.2	39	78	-38	-46	136	-32	72
Jun	14.0	83	142	-57	-104	98	-37	121
Jul	19.2	109	171	-61	-166	69	-28	138
Aug	17.7	64	125	-60	-227	49	-19	84
Sep	10.8	114	42	72	-154	74	25	42
Oct	4.6	70	2	68	-86	108	35	2
Nov	-6.4	44	0	44	-42	152	0	0
Dec	-13.7	20	0	20	-23	172	0	0
Year	—	610	569	41	—	—	—	529
				Second Modification				
Jan	-11.8	18	0	18	0	190	0	0
Feb	-9.0	15	0	15	0	205	0	0
Mar	-7.0	32	0	32	0	237	0	0
Apr	3.5	1	10	-7	-7	169	-7	9
May	13.2	39	78	-38	-46	136	-32	72
Jun	14.0	83	142	-57	-104	98	-37	121
Jul	19.2	109	171	-61	-166	69	-28	138
Aug	17.7	64	125	-60	-227	49	-19	84
Sep	10.8	114	42	72	-154	74	25	42
Oct	4.6	70	2	68	-86	108	35	2
Nov	-6.4	44	0	44	-42	152	0	0
Dec	-13.7	20	0	20	-23	172	0	0
Year	—	610	569	41	—	—	—	469

TEMP = Temperature; PPT = Precipitation, PET = potential evapotranspiration; P-PET = precipitation less potential evapotranspiration; ACPWL = accumulated potential water loss; STRGE = soil storage; DELTA = the difference between STRGE in successive months when it is less than maximum; AET = actual evapotranspiration.

Conclusion

The Thornthwaite model, which was developed for prediction of AET (Thornthwaite and Mather 1957), programmed by Black (1966), and subsequently modified by Grigal and Bloom (1985), was producing erroneous results when used to estimate AET for 2,726 site-year combinations in the Upper Great Lakes area. During years with variable wet and dry periods, AET was calculated in error because AET > PET. Two hydrologically justifiable modifications made to the soil water storage component of the original program enabled realistic estimates of AET for all data sets. Those modifications allow the program to be robust for any temperate climate.

Acknowledgments

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The program is available in DOS format from R.K. Kolka. Please send program requests to Dr. Randall Kolka, USDA Forest Service, Savannah River Ecology Lab, Drawer E, Aiken, SC 29802.

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