Biomass and Taper for Trees in Thinned and Unthinned Longleaf Pine Plantations

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ABSTRACT. *Longleaf pine (*Pinus palustris *Mill.) trees growing in thinned plantation studies in Louisiana and Texas and unthinned stands from the Louisiana sites were sampled for establishing taper, volume, and specific gravity. Stem analysis data were collected on 147 stems ranging in age from 30 to 50 yr. Analyses of covariance (ANCOVAs) were employed to determine coefficients and to detect differences among treatments, for tree taper and specific gravity. Taper and volume equation coefficients and statistics at specified ages were developed for intermediate plantation ages by examining and aging internal growth rings from the stem sections. Biomass was computed by combining the taper and specific gravity equations. Seemingly unrelated regression (SUR) was used to simultaneously fit the system of four equations composed of specific gravity, taper, volume, and biomass, because of the correlated error structure of these equations. Biomass equations, however, could not be developed for the intermediate ages because specific gravity could not be determined or related to earlier tree ages. South. J. Appl. For. 19(1):29–35.*

The potential for growing longleaf pine (*Pinus palustris* Mill.) in plantations continues to improve. After a long period of declining acreage and loss of favor that came with the infatuation foresters had with rapid early growth, longleaf acreage appears to be stabilizing at about 3.75 million ac (Kelly and Bechtold 1990). On the national forests the acreage is stable at about 96,000 ac, and plantation establishment may increase (K. Stoneking, pers. comm.). This trend accompanies the recognition that a number of risk factors (rust, bark beetles, etc.) associated with more rapidly growing species are reduced for longleaf pine. Good progress has been made on the establishment of longleaf, and prescriptions for hastening the passage through the "grass" stage have been developed; regeneration techniques are well documented, especially in the recent literature (Mann 1969, Croker and Boyer 1975, Barnett et al. 1990). A preliminary analysis of financial returns which considers the premium received for poles has been run on a small set of longleaf pine data from central Louisiana (Busby et al. 1993). The results indicate that longleaf can be profitable if grown for poles, even to ages over 45 yr. Given these trends, forest managers would do well to consider longleaf pine in their mixture of strategies to maintain and improve the productivity of the South's next forest.

Objectives

Developing growth and yield models for plantation longleaf pine entails analyses of data that relate to the form of trees at various points in the development of the stands and their changing competition conditions. The objectives of these analyses are (a) to develop specific gravity relations for the trees by stands, (b) to produce taper equations that yield volume in an integral function form and (c) to obtain biomass estimates for trees given their harvest conditions.

Methods

Study Locations

Data for these analyses were collected in longleaf pine plantations in Louisiana and in Texas (Table 1). Five individual studies were involved. They represent some of the oldest longleaf pine plantation, so age might be an important variable. They represent a relatively small locational range, and all were on cutover forest sites. Their current ages (A_p,

Table 1. **Identification of studies and locations associated with trees included in the longleaf pine plantation biomass analyses.**

Study no. ident.	*n* trees	Location	Treatment	Age (*Ap*)
3.02 LAT1	29	Johnson Tract, Louisiana	thinned	50
3.13 TXT1	12	Yellow Pine, Texas	thinned	45
3.13 TXT2	24	Angelina, Texas	thinned	45
2.29 LAT2	59	Longleaf Tract, Louisiana	thinned	35
3.29 LAU	25	Louisiana	unthinned	35

growing seasons since field planting) range from about 35 yr to almost 55 yr old. A history of frequent fires, prior to planting and after the previous stands were clearcut, controlled woody competition and allowed the pines to be planted without mechanical or chemical site preparation. Some plots were established at the time of planting to test initial stand densities from 250 to 2500 trees/ac. Other plots were installed in existing plantations, 16 or more years old, that showed no evidence of severe insect or disease damage. Most, but not all, of these plots had been burned by prescription. The density of their hardwood and brush understories varied with the frequency and effectiveness of the fires. The unthinned control study consists of 59 plots ranging in size from 0.05 to 0.21 ac. We will refer to thinning levels and controls as treatments in the statistical sense.

Measurement

Measurements on 147 felled trees—125 from thinned and 22 from unthinned plots (Table 2)—were used to develop equations for stem taper, volume, and biomass. Measurements included diameters inside and oustide bark at 0.5, 2.5, 4.5 ft and then at 5 ft intervals up the bole. Other measurements of crown class and length and branches were made, but were not included in this study. These felled trees were selected from an experimental array of thinning conditions. Trees representing the unthinned condition were selected from buffer strips adjacent to the unthinned plots. Thinned stands had been repeatedly cut to residual basal areas of 40 to 80 ft^2/ac for at least 15 yr before the sample trees were felled. Only sound trees that did not fork were measured. Some had been marked for cutting in regularly scheduled thinnings, but the sample also included some high -quality fast-growing trees that ordinarily would have been left to grow. While we feel it is important to know what these studies were and their location, the reader should not confuse the individual tree stem analysis data with the thinning study data. Individual trees were selected from thinnings or near the control. We tested only the difference between these two treatments. There was no attempt to design the selection (statistically) for the possible affect of levels of thinning on tree form.

Disks were removed from the felled trees at 5 ft intervals and at stump height, 2.5 ft, and breast height. These disks were transported to the laboratory, oven-dried, and the bark removed (on larger disks a wedge was removed from the disk). The specific gravity of the wood was calculated by weighing in air and volume determined by immersing the sample in water. Finally the disks were sanded and copied on a photocopier (with rectification scales), and the copies were measured on a dendrograph to determine the ring widths and ages. Breast height age of each sample tree was determined. Even after correction for early height growth, this was rarely the same as the plantation establishment age. Inside bark taper was determined from the disks for reassembled 5-yr interval growth of the individual trees. A total of nearly 3,000 disk measurements constituted the initial data base. A few trees with extraordinary stem profiles[1] were eliminated from the analysis and a subset of trees in the suppressed crown

[1] These trees exhibited major protuberances in the profile, almost certainly a function of a mistake in measuring the height of the disk or misnumbering it.

Table 2. Distribution of felled longleaf pine sample trees representing (unthinned) thinned stand plantations.

	Total height (ft)						
dbh (in.)	40	50	60	70	80	90	Total
				(no. of trees)			
4	5	2					7
5	1	3	4				8
6			8	2			10
7			(1)1	6			(1)7
8			(3)3	(1)5			(4)8
9			(3)	(2)7	2		(5)9
10			(1)	3	(2)4	(1)	(4)7
11			1	(1)5	(2)2	(1)1	(4)9
12				14	(2)7		(2)21
13				9	(1)7	(1)	(2)16
14				2	10	2	14
15				1	4		5
16					1	1	2
17					1	1	2
Total	6	5	(8)17	(4)54	(7)38	(3)5	(22)125

class, whose taper was obviously different, had to be separately analyzed. Thus the final number of observations amounted to 2,281 disk and specific gravity pairs.

Data Analysis

We have chosen to present both a stepwise analysis of component equations and an integrated seemingly unrelated regression (SUR) analysis. Model development depends somewhat on the intermediate steps, and there may be considerable interest in the volume and taper results for different ages as well as in the final integrated biomass equation coefficients obtained by the SUR approach.

Specific Gravity

Previous studies (Parresol and Thomas 1989) have shown that specific gravity can be modeled as a function of height in several tree species. In the initial phases of the study we attempted to look at specific gravity as a function of variables that growth and yield models often cite, i.e., crown length and crown class. Our analyses did not support inclusion of crown length in any of the several functional forms that we tested. However, it did suggest that these variables, if available in a broader range of values than our data, might become important. Indeed, in the case of longleaf pine, age was not significant in all the treatment cases, but the distribution of ages within treatments was uneven, and the additional information gained in including it in the model was insufficient to overcome the advantages of using a simpler model for all the treatments and locations. We analyzed the data using analysis of covariance (ANCOVA) expressed in the following equation:

$$sg_{ijk} = \alpha_{0i} + \alpha_{1i}e^{x_{ij}} + \varepsilon_{ijk} \qquad (1)$$

where *sg* is the specific gravity, treatments (thinning levels) are represented by subscript i, $i = 1,...,5$, x_{ij} represents the jth relative height, i.e., height of observation/total height (both above ground), *e* is the base of natural logarithms, a's are model parameters and e represents associated error terms. The results of the final ANCOVA are presented in Table 3. Several iterations of the analysis were actually performed. We tested a variety of hypotheses using linear contrasts, resulting in three sets of coefficients that were sufficient to describe the data sets; one for the unthinned Louisiana study, one for the two Texas thinning studies, and one for the thinned Louisiana studies. Plots of residuals were checked for departures from regression assumptions and having found none, the values for coefficients of the final location groupings were collated (Table 4).

Taper and Volume

There have been numerous approaches to modeling stem form in recent decades. The majority have concentrated on the simpler coniferous bole form and have become increasingly complex mathematical expressions. Use of trigonometric equations provide a simple expression of taper that is flexible enough to fit both conifer and hardwood bole forms. We have previously reported on the application of trigonometric taper equations to examples from thinned and unthinned slash pine, willow oak and sweet gum (Thomas and Parresol 1991) and compared them with the segmented polynomial approach developed by Max and Burkhart (1976). Trigonometric equations performed equally well and are parsimonious.

After examination of the plots of unit circle trigonometric functions, including nonzero centered transformations (of the form $\sin(x + a\pi)$), and comparison to the plots of tree taper on a relative height and diameter scale, we selected the following taper model:

$$\frac{d^2}{D^2} = (\beta_1(x-1) + \beta_2 \sin(c\pi x) + \beta_3 \cot(\frac{\pi}{2}x)) + \varepsilon \qquad (2)$$

where *d* is diameter inside bark (in.) at a given height, *D* is diameter *inside* bark (in.) at breast height [the left hand side of Equation (2) will be referred to as relative diameter squared] and the β's are model parameters. Arguments for trigonometric functions are expressed in radians. The cotangent function yields values from + • to 0, when *x* (relative height) is scaled from 0 to p/2. We ran a series of nonlinear regressions to determine an appropriate value for *c*. Values of *c* for the longleaf ranged from 1.7 to 1.9. However, we found little practical[2] improvement in the coefficient of determination (R^2) or mean squared error (MSE) over using a fixed

2 We made this judgment based on the performance over the merchantable bole and discounted the minimal differences in top and stump of trees.

Table 3. Specific gravity model statistics for longleaf plantation felled tree study.

Source	*df*	Sum of squares	Mean square	*F*-value/*P*>*f*
Model	8	613.2	76.65	58828 / 0.0[1]
Error	2273	2.96	0.00130	
Total(uncor)	2281	616.16		
Treatment	4	25.37	6.342	4867 / 0.0
Relht*trt	4	3.23	0.808	620 / 0.0
LAT1 v. 2[2]	1		0.0000	0.01 / 0.92
LAU v LAT	1		0.0010	0.80 / 0.37

1 Table values of 0.0 represent some small positive value.
2 LAT1 v. 2 refers to the contrast between the two thinned stand studies 3.02 and 2.29. LAT v. U refers to the contrast between these thinned stands and the unthinned stands identified as study 3.29.

Table 4. Coefficients and associated statistics for specific gravity; $g = a_0 + a_1e^{-x}$ of plantation longleaf in Louisiana and Texas.

	Estimate					
Location	a_0	SE^3	a_1	SE	Root MSE	R^2
LAT1 & 2 thinned	0.395	0.0037	0.184	0.0052	0.0369	0.48
LAU unthinned	0.385	0.0071	0.170	0.0100	0.0377	0.43
TXT1 & 2 thinned	0.364	0.0051	0.228	0.0072	0.0334	0.64

value of $c = 2.0$. Using this fixed value makes the expression take on the value 0 when $x = 1$; that is, the equation is constrained to equal 0 at the tree top. Examination of the deviation of residuals indicated the form was least accurate in the extreme upper portion of the stem. This portion of the bole is also where the least values of both the product and information regarding bole form are found in these relatively mature longleaf pine growing in plantations.

These trees are from plantations established in the 1940s and 1950s. Because of the long term of the study, we suspected that there might have been an effect of age on the taper of these trees. Consequently, we decided to introduce a plantation-age variable for fitting the model given in Equation (2). The modified equation describing taper and including the terms for age was:

$$\frac{d^2}{D^2} = \beta_{1i}(x-1) + \beta_{2i}\frac{\sin(\pi x)}{A_p} + \beta_{3i}\frac{\cot(\pi x/2)}{A_p} + \varepsilon \quad (3)$$

where A_p represents plantation age and other variables are as previously defined. (However, the subscript is a location rather than treatment indicator). The formulation indicates that we chose to look at the effect of location on the taper of the trees. As with development of the specific gravity model, we used an ANCOVA to detect differences in coefficients due to the effect of location. After testing the effect of location on the model, we ran individual regression models on: (1) Louisiana study 3.02 (LAT1), (2) the combined Texas studies 3.13 (TXT1 and TXT2) and central Louisiana 2.29 (LAT2), and (3) the Louisiana study 3.29 (LAU). The coefficients and some summary statistics from those regressions are presented in Table 5.

To obtain volume of the entire stem or portions of the stem, the taper equation is integrated over height or correspondent portions of height using:

$$\hat{v} = k\int_{h_l}^{h_u} d^2 dh \quad (4)$$

where v is volume in ft^3, k is $\pi/576$ (the factor for converting in^2 to ft^2), h_l is lower height limit (ft) and h_u is upper height limit (ft). Performing a change of variable from h to x (relative height) results in:

$$\hat{v} = kH\int_{x_l}^{x_u} d^2 dx \quad (5)$$

where x_l is h_l/H; and x_u is h_u/H and subscripts u and l correspond to the upper and lower height limit for integration, H = total tree height (ft). Introducing the results of our taper model and its parameter space yields:

$$\hat{v} = \frac{kD^2H}{A_p}\int_{x_l}^{x_u} A_p b_1(x-1) + b_2\sin(2\pi x) + b_3\cot(\pi x/2)dx \quad (6)$$

Specifically, integration yields:

$$\hat{v} = \frac{kD^2H}{A_p}\left\{\begin{array}{l}\frac{A_p b_1}{2}(x_u^2 - x_l^2) - A_p b_1(x_u - x_l) - \frac{b_2}{2\pi}[\cos(2\pi x_u) \\ -\cos(2\pi x_l)] + \frac{2b_3}{\pi}\left[\ln\left(\frac{\sin(\pi x_{u/2})}{\sin(\pi x_l/2)}\right)\right]\end{array}\right\} \quad (7)$$

where ln represents natural logarithms. The volume of any segment or the entire tree can be obtained by using Equation (7) with the coefficients from Table 5.

Biomass

We must emphasize that biomass cannot be determined for intermediate ages as were taper and volume. Specific gravity was determined for complete disks only at the final

Table 5. Coefficients for taper model, longleaf pine plantations in Texas and Louisiana.

	Estimate				
Location	b_1	b_2	b_3	$RMSE$	Adj. R^2
LAT1 (thinned)	-1.0280	2.2508	0.1681	0.0811	0.99
LAU (unthinned)	-1.0063	1.4284	0.1168	0.0542	0.99
LAT2/TXT1 & 2 (thinned)	-1.0439	1.4965	0.1152	0.0709	0.99

(cut) age of the trees. We spent considerable effort trying to rationalize methods for obtaining estimates of specific gravity for the disks at earlier ages. Obviously, this cannot be done from the disks themselves as heartwood is known to change in specific gravity over time and using the current density for earlier ages would be absolutely wrong. If some rational method for determining specific gravity for earlier stages in the development of the trees were available, we would attempt to develop age related biomass estimation equations. Unfortunately, we were unable to justify any sampling or other scheme for reconstructing specific gravity of individual disks at prior ages. There were three different final ages for the plantations, but these are completely confounded with the plantation and also could not be separated for this analysis. Nonetheless, we did feel it worthwhile to develop biomass equations for these relatively mature plantations of longleaf pine.

When the objective is to obtain biomass for a tree, a double integration over both cross-sectional area and height can be performed. The integration is somewhat more complex, and at this point requires at the least a partial evaluation by numerical methods. We have been unable to find a closed form integral for the final term in the biomass model. We present the integral in parts.

Let

$$\rho(x,y) = \left[a_0 + a_1 e^{-x}\right] \bullet 62.4$$

also

$$f(x) = \frac{\pi}{576} D^2$$
$$\left[b_1(x-1) + \frac{b_2}{A_p}\sin(2\pi x) + \frac{b_3}{A_p}\cot(\frac{\pi x}{2})\right]$$

then

$$\hat{W} = H\int_{x_l}^{x_u}\int_0^{f(x)}\rho(x,y)dy\,dx$$

where W is the biomass or dry weight of the bole (lb). Integration order does not matter so that we integrate the specific gravity equation first and obtain:

$$\hat{W} = 62.4 \bullet H\int_{x_l}^{x_u} a_0 f(x) + a_1 f(x) e^{-x} dx$$

Combining constants for the specific gravity of water (62.4 lb/ft^3), π, and area conversion (k = 0.34034) and substituting the taper function, f(x), yields a working integral:

$$\hat{W} = \frac{kD^2H}{A_p}\int_{x_l}^{x_u} a_o\left[A_p b_1(x-1) + b_2\sin(2\pi x) + b_3\cot(\frac{\pi x}{2}\right] + a_1 e^{-x}\left[A_p b_1(x-1) + b_2\sin(2\pi x) + b_3\cot(\frac{\pi x}{2})\right]dx \quad (8)$$

A closed form integral for all but the last term of this equation is available from the authors. However, the entire working integral (Equation 8) can be integrated by numerical methods to give quite accurate estimates of wood weight.

Simultaneous Fitting—SUR

The four equations, specific gravity (1), taper (3), volume (7), and biomass (8), the latter three with the plantation age variable, were fit by SUR using the SAS/ETS® MODEL procedure (SAS Institute Inc. 1988). The procedure leads to slightly different values for specific gravity and taper for the final age of the plantations. We did not test to see if these differences were significant. Resulting coefficients are presented in Table 6. Statistics for the SUR fits of integrated biomass, volume, taper and specific gravity are presented in Table 7.

One of the authors was aware of a set of longleaf taper data from Alabama that had been obtained in the early 1980s. These data were collected from both natural stands and plantations across a diameter distribution from about 1 to 24 in. in dbh. Auburn University supplied a copy of the taper data obtained in a cooperative study with the Southern Forest Experiment Station's Forest Inventory and Analysis Unit. We believed that these data might be useful for testing the portability of the coefficients obtained in this analysis to a different part of the range of longleaf pine. Fitting the data proved simple and the fits were very good. We refit data from our study excluding the age variable and based only on diameter breast high outside bark measurements in order to match the conditions for the Auburn study. The results of the

Table 6. Planatation longleaf pine equation coefficients and standard errors (SE) for integrated tree specific gravity, taper, volume and biomass, estimated using SUR.

Location	Coefficient (*SE*)				
	a_0	a_1	b_1	b_2	b_3
LAU1	0.3835	0.1671	-0.7405	0.0299	0.00260
	(0.0052)	(0.0070)	(0.0033)	(0.0024)	(0.0001)
LAT1	0.3937	0.1941	-0.7957	0.0269	0.00333
	(0.0051)	(0.0067)	(0.0031)	(0.0022)	(0.0001)
LAT2	0.3975	0.1686	-0.7782	0.0336	0.00255
	(0.0037)	(0.0048)	(0.0023)	(0.0017)	(0.0001)
TXT1/TXT2	0.3712	0.2149	-0.8154	0.0227	0.00263
	(0.0060)	(0.0077)	(0.0035)	(0.0025)	(0.0001)

Table 7. Root mean square error and adjusted R^2 for simultaneous fit of equations by location.

Location equation	*df*	*RMSE*	Adj. R^2
LAU1			
StemSG	380	0.03781	0.424
Taper	380	0.03845	0.980
Volume	380	0.0836	0.982
Biomass	379	3.4736	0.970
LAT1			
StemSG	487	0.03756	0.498
Taper	487	0.04118	0.981
Volume	487	0.1509	0.984
Biomass	486	6.3570	0.977
LAT2			
StemSG	831	0.03669	0.468
Taper	831	0.04605	0.972
Volume	831	0.07540	0.986
Biomass	830	2.6935	0.984
TXT1/2			
StemSG	577	0.03342	0.633
Taper	577	0.04451	0.976
Volume	577	0.1664	0.978
Biomass	576	6.6076	0.971

comparison were quite clear that coefficients are not transportable. Figure (1) presents the comparison of taper equations from the Louisiana and East Texas locations with the data from the Alabama study. For the West Gulf analysis, there were significant differences ($P < 0.05$) between the Louisiana and East Texas sites, so we did not even bother to run a statistical comparison procedure for the Alabama data. The results from these comparisons tell significantly different stories. By examination of the Louisiana and Texas data we can see that there is probably only a small practical difference between the sites. However, this does not mean that only small differences are to be observed across the range of the species.

Conclusion

We have reported results in the development of taper-volume-biomass components of stems for thinned and unthinned plantation longleaf pine from the West Gulf. We are confident that the data will be useful to managers of today's longleaf plantations, though some modifications may be necessary. We suggest that besides diameter, specific gravity, heights and ages of trees are important to application of the equations. The oldest ages will almost certainly not be typical of newly established stands when they reach a similar age, but the process may help develop useful equation coefficients.

The results are most applicable to the existing plantation studies and should also help illustrate some of the statistical and mathematical techniques for development of models for other locations and conditions. The presentation of both individual regression and SUR results allows potential users apply the separate volume-taper or the biomass equations as appropriate and have coefficients of minimum error in either case.

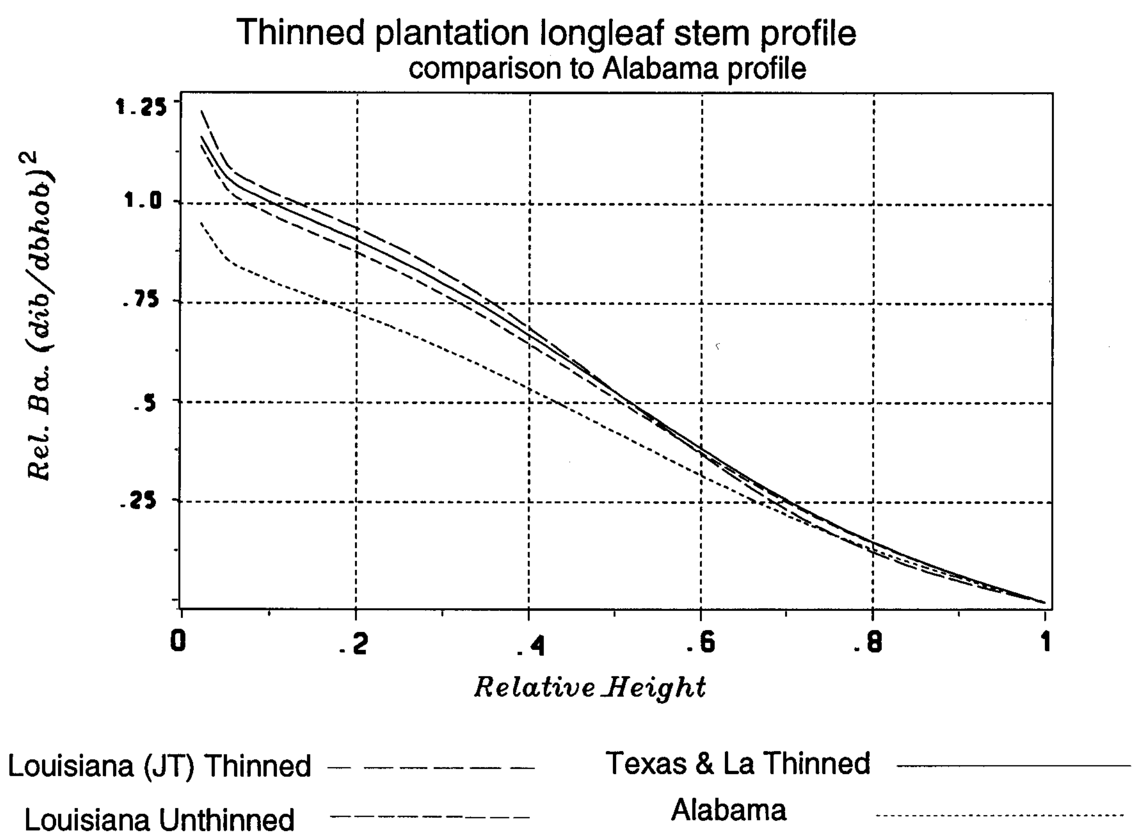

Figure 1. Comparison of longleaf pine taper among selected West Gulf plantation sites and natural stands in Alabama. Note that differences among the West Gulf sites are significant.

For some species of southern pine there is little variation over the range of the tree in taper or specific gravity. We decided to look for some additional data on longleaf pine and found taper and specific gravity measurements from a study done for the Forest Inventory and Analysis Unit in the early 1980s at Auburn University. The results are decidedly different, indicating that for longleaf pine it is possible to get quite disparate taper across the range of this species. However, we also speculate that there could be differences due to the historical development and structure of the tree population of these thinned longleaf pine plantations. The age of trees in our sample is relatively constant, due to the plantation setting. While the trees selected for the construction of biomass equations for longleaf in Alabama by Auburn in the early 1980s is much more variable in age, and most of the Auburn trees did not originate in plantations. While our results don't directly indicate that the equations from this study should not be used in mixed stands of pine or pine-hardwood, the failure to match the Auburn taper should be a warning to users. We believe the main value of our approach is in the comprehensive system for estimating volume or biomass. It will also be useful in projecting the future form of maturing planation grown longleaf pine that may more accurately represent the value of the longleaf in the future southern commercial forest mix.

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