An individual-tree basal area growth model for loblolly pine stands

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Abstract: Tree basal area growth has been modeled as a combination of a potential growth function and a modifier function, in which the potential function is fitted separately from open-grown tree data or a subset of the data and the modifier function includes stand and site variables. We propose a modification of this by simultaneously fitting both a growth component and a modifier component. The growth component can be any function that approximates tree growth patterns, and the logistic function is chosen as the modifier component. This approach can be adapted to a variety of stand conditions, and its application is demonstrated using data from an uneven-aged loblolly pine (*Pinus taeda* L.) study located in Arkansas and Louisiana.

Résumé : La croissance de l'arbre en surface terrière est considérée comme étant le produit de deux fonctions : le potentiel et le réducteur de croissance. Auparavant, les deux fonctions étaient ajustées séparément : le potentiel à partir de données sur les arbres poussant à découvert ou d'un sous-groupe de données de le réducteur à partir de variables du peuplement et de la station. Nous proposons un ajustement simultané des deux fonctions. Toute fonction qui décrit approximativement les patrons de la croissance de l'arbre peut être utilisée pour décrire le potentiel et la fonction logistique est choisie pour représenter le réducteur. Cette approche peut être adaptée à une variété de conditions stationnelles et son application est démontrée par une étude sur les peuplements inéquiennes de *Pinus taeda* L. situés en Arkansas et en Louisiane.

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Introduction

Tree basal area growth is very malleable by silvicultural practices; consequently, modeling diameter or basal area growth and its response to stand and site variables has been intensively studied. One approach, exemplified in the PROGNOSIS model, is to develop a "composite" model, which is usually a linear function of tree, stand, and site variables (Wykoff 1990). The other approach (e.g., Hahn and Leary 1979; Leary and Holdaway 1979) has been to look at a growth model as being composed of two components, a potential growth function and a modifier, which multiplied together give an estimate of tree growth.

Two techniques have been used to derive the potential growth function and its coefficients. One is to select a subset of the growth data that represents trees that are relatively unaffected by the competing influence of other trees. Commonly, it is an upper cohort of trees that have been ranked by growth. This approach was used to develop the tree growth simulator FREP (Hahn and Leary 1979; Leary and Holdaway 1979) for Lake State forests and its successors STEMS (Belcher et al. 1982; Holdaway 1984) and TWIGS (Miner et al. 1988). Adaptations of STEMS have been made by Bolton and Meldahl (1990) for U.S. southern forests, by Fairweather (1988) for common tree species in Pennsylvania, by Hilt and Teck (1988) for northern New England, and by Goodwin (1988) for Tasmanian eucalypt forests. Wensel et al. (1987) used a subset of data to derive coefficients for the potential functions of growth models for northern California conifers; they tried a simultaneous solution, but discovered that it confounded the potential and modifier effects.

The second technique is to use open-grown tree data to derive the coefficients for the potential function; for an example, see Smith et al. (1992). Daniels and Burkhart (1975) and later Burkhart et al. (1987) and Amateis et al. (1989) used this technique to derive a potential function for diameter growth using open-grown loblolly pine (*Pinus taeda* L.) trees as part of a plantation growth model.

While the concept of a potential function with a modifier is useful, these two current approaches for deriving a potential function for tree diameter or basal area growth have limitations. Picking a subset of data to use for fitting the potential function is somewhat arbitrary. The use of open-grown tree data evades the shortcoming of using a data subset. However, there is no assurance that the growth patterns of open trees are similar to those in a forest environment, as suggested by Cannell's (1985) review on dry matter partitioning in trees.

This paper describes a composite modeling approach to predict the basal area growth of individual trees directly from tree, stand, and site variables. However, the potential–modifier concept was used as the basis of model development. We illustrate its application by developing a model for uneven-aged loblolly pine stands.

Data

The data come from a study investigating the effects of residual basal area, maximum DBH, and site quality on the growth and development of loblolly stands under

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Table 1. Summary statistics for the 5130 loblolly pine trees used in the analysis.

Variable	Mean	Min.	Max.
Tree basal area growth (cm^2/year)	26.5	0.0	168.2
Tree basal area (cm^2)	475.3	65.7	2613.3
Basal area larger (m^2/ha)*	11.2	0.2	23.5
Stand basal area (m^2/ha)	16.4	8.1	23.6
Quadratic mean diameter (cm)	22.8	16.0	35.9

*The average basal area in trees larger than or equal to the diameter of the subject tree.

uneven-aged silviculture using single-tree selection. Uneven-aged stands are commonly characterized by three variables: maximum diameter, density (usually basal area), and the ratio "q" (Murphy and Farrar 1982). This q is the ratio of trees in a given diameter class to the number of trees in the adjacent, larger diameter class and is affected by the diameter class width. These three values define an uneven-aged stand table. The factors that were tested in this study were maximum DBH (30, 41, and 51 cm), residual basal area (9, 13.5, and 18 m^2/ha), and site quality (site indexes for loblolly pine of less than 25 m, 25 to 27 m, and greater than 27 m, 50-year basis), replicated 3 times for a total of 81 plots. Because the q value is a difficult variable to precisely control and to reduce the complexity of the study, q was fixed at 1.2 for 2.5-cm diameter classes. The plots are square, 0.2 ha in size, and surrounded by a 18-m isolation strip. The plots were installed and cut to the prescribed treatments over a 3-year period beginning in 1983. Hardwoods were injected with herbicide, and any shortleaf pines (*Pinus echinata* Mill.) that were present were harvested. All live trees 9.1 cm in DBH and larger were measured immediately and 5 years after treatment. At both measurements, tree DBHs were measured with a diameter tape, and heights of a subset of trees were measured with a clinometer. The data used here are from this first 5-year growth period.

West (1980) found that diameter increment and basal area increment were equal to one another for predicting future tree diameter; we chose tree basal area increment as our measure of growth. The following variables were calculated for the surviving trees of the growth period: (1) ΔB, average annual tree basal area growth (cm^2); (2) B_t, average tree basal area during the period (cm^2); and (3) B_l, the average basal area (m^2/ha) in trees larger than or equal to the diameter of the subject tree. Stand variables included the following: (1) B_s, basal area (m^2/ha) and (2) D_q, quadratic mean diameter (cm). All stand variables are means for the period. Data set statistics are presented in Table 1.

Model derivation and analysis

We adapted the concept of a potential function and a modifier to the following conceptual equation:

[1] predicted growth = growth × modifier

where growth is the component that defines tree growth and modifier is the component that adjusts growth depending upon tree, site, and stand factors. There is a subtle but significant difference between eq. 1 and the conventional potential–modifier approach. In the latter, the potential represents the maximum growth that can be attained, while no such claim is made for the former. A similar approach was used to model the basal area growth of longleaf pine (*Pinus palustris* Mill.) trees in even-aged stands (Quicke et al. 1994).

The first step is to select the growth function. To build a biological constraint into the complete model (eq. 1),

Fig. 1. Scatterplots of residuals versus predicted values and the growth and logistic components of eq. 3.

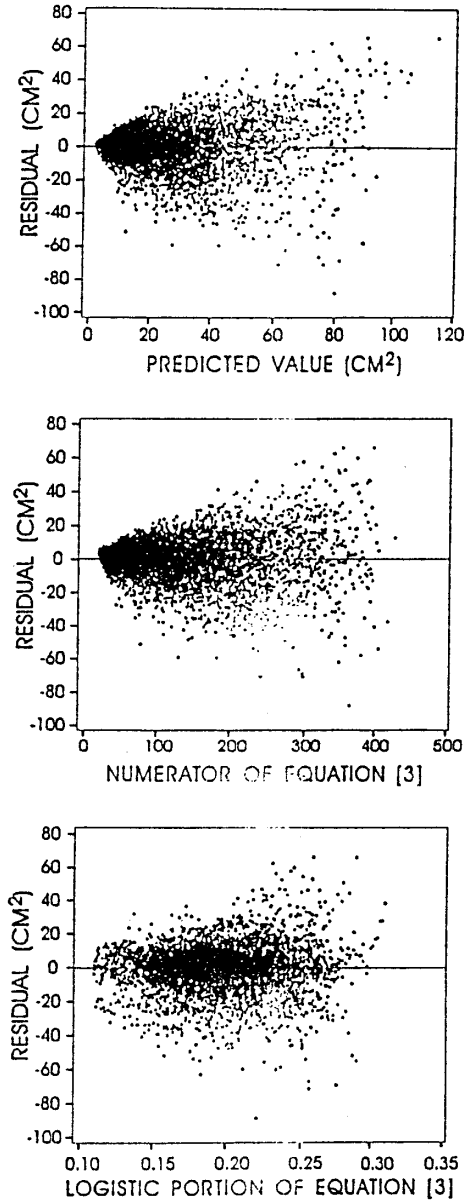


Table 2. Parameter estimates and associated statistics for tree basal area growth eq. 3 for uneven-aged loblolly pine stands.

Coefficient*	Estimate	Approximate standard error	T ratio	Approximate $P > \lvert T \rvert$
b_1	510.98	58.395	8.75	0.0001
b_2	−0.000 700 55	0.000 073 45	−9.54	0.0001
c_1	0.026 769	0.003 334 4	8.03	0.0001
c_2	0.041 854	0.002 414 4	17.34	0.0001
c_3	0.010 847	0.002 499 3	4.34	0.0001

*$\Delta B = \{b_1[1 - \exp(b_2 B_t)]\}/\{1 + \exp[c_1 B_l + c_2 B_s + c_3 D_q]\}$, where ΔB is the average annual basal area growth (cm²), B_t is the average tree basal area (cm²), B_l is the average basal area in trees larger than or equal to the diameter of the subject tree (m²/ha), B_s is the average stand basal area (m²/ha), and D_q is the average quadratic mean DBH (cm).

the growth function should ideally reach a maximum and then decline with increasing tree size. We started with the Chapman–Richards function because of past success in using it as a potential growth function. However, severe convergence problems were encountered in fitting the Chapman–Richards function with nonlinear least squares, even after using champion tree data to constrain the equation, as recommended by Shifley and Brand (1984). The problem was that basal area growth did not exhibit any maximum value and subsequent decline, a classical pattern for the Chapman–Richards function. Although the data did not exhibit a maximum for basal area growth within its range, the growth function should at least have an asymptote so that growth is bounded. This requirement prompted the selection of the following function:

$$[2] \quad b_1[1 - \exp(b_2 B_t)]$$

where the b_i's are coefficients to be determined and the other terms are as previously defined. To correct for heterogeneous error variance, weighting with $1/B_t$ was used. Equation 2 and all subsequent equations were fitted by nonlinear ordinary least squares using the SAS procedure MODEL (SAS Institute Inc. 1988). Good results were obtained with this function. No trends were observed in the residuals, and the fit index $[1 - \Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - y_{mean})^2]$ and the root mean square error were 0.65 and 0.60 cm², respectively.

The next step is to specify the modifier function. Ideally, the modifier function is constrained within the interval (0, 1). A function naturally bounded by this interval is the logistic. The combination of the growth eq. 2 with the logistic as a modifier results in the following equation:

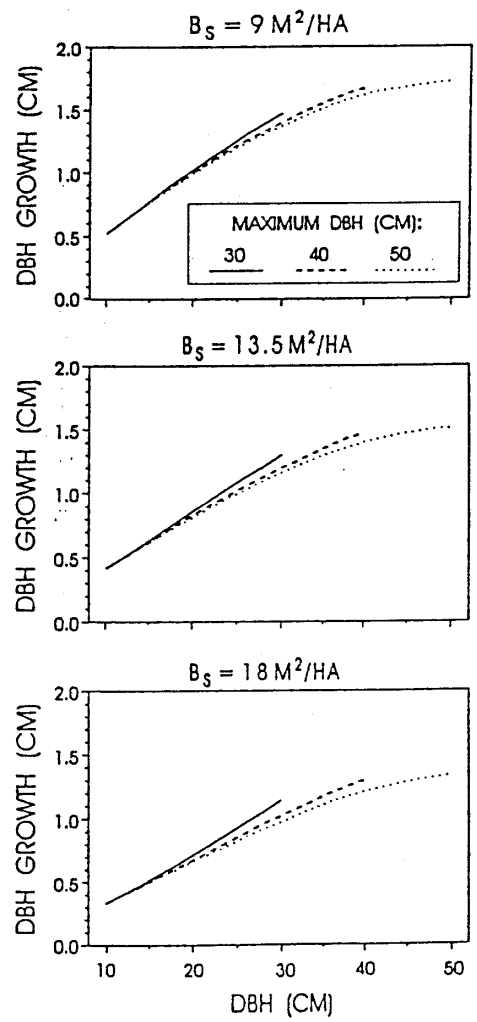
$$[3] \quad \Delta B = \frac{b_1[1 - \exp(b_2 B_t)]}{1 + \exp[c_1 B_l + c_2 B_s + c_3 D_q]}$$

where the c_i's are coefficients to be estimated and the other terms are as previously defined. Equation 3 was fitted to the data using nonlinear ordinary least squares and using the estimates from eq. 2 as starting values for b_1 and b_2.

Results and discussion

The fit index and root mean square error for eq. 3 are 0.69 and 0.56 cm², respectively. The coefficient estimates and their associated statistics are shown in Table 2. All coefficients

Fig. 2. Predicted annual diameter growth for loblolly pine trees in uneven-aged stands (structures are defined by the specified stand basal area and maximum diameter with a q of 1.2 for 2.5-cm DBH classes).



had probabilities for the t-ratio (the coefficient estimate divided by its standard error) of less than 0.001. The positive signs on the coefficients in the logistic portion of eq. 3 indicate that as these variables are increased, predicted tree basal area growth will decrease; these effects are logical and consistent with our knowledge of stand dynamics. No apparent trends are noticeable in the scatterplots of the residuals versus predicted values (Fig. 1). Scatterplots were also done for the two components of eq. 3, the growth and logistic portions, and no trends were observed (Fig. 1).

Application of the model is straightforward. The required information is a tree list that specifies the DBHs of individual trees on a per hectare basis. The stand basal area, quadratic mean DBH, and the basal area in trees larger than or equal to the DBH of the subject tree can be calculated from the tree list. The annual basal area increment for each tree can be calculated using the equation for one year. To project tree growth for subsequent years, the DBH of each tree needs to be updated by adding the annual growth to the current DBH, and stand basal area, quadratic mean DBH, and basal area in trees the same size or larger than the subject tree must be recalculated. This updating is done for each year of the projection period. Of course, additional equations would be needed to describe other components of tree or stand growth (such as survival) to make a comprehensive stand or tree projection system.

As an illustration, loblolly pine tree basal area growth for 1 year in uneven-aged stands was calculated for different stand structures. Plotting predicted values to observe how tree growth is affected by the different variables in the equation is complex, because stand structure affects tree growth. This complexity can be reduced by specifying stand structures using stand basal area, maximum diameter, and a q value. Given these values, the quadratic mean stand diameter and the basal area in trees equal to or larger than the subject tree can be computed for each diameter class following the procedure of Murphy and Farrar (1982). The stand basal areas, maximum diameters, and the q value specified in the uneven-aged loblolly pine study design were used to generate the graphs in Fig. 2.

Tree diameter growth decreases for a given tree diameter class when stand basal area is increased with other factors being held constant (Fig. 2). Increasing the maximum diameter while holding stand basal area constant decreases growth for a given tree diameter. This results because increasing the maximum diameter increases competition from trees larger than the subject tree.

Conclusion

Equation 3 provides a viable alternative to modeling tree diameter or basal area growth. Moreover, the approach described here might work for a variety of functions for the growth component in eq. 2. Modeling the effect of stand and site factors is facilitated by using the logistic function as the modifier; the model can easily be modified by adding variables to the logistic function. Because the growth component and the modifier component of the growth function are fitted simultaneously, their effects are confounded, as previously noted by Wensel et al. (1987). Therefore, the growth component cannot be analyzed in isolation and does not have the same interpretation where it is fitted separately to open-grown tree data or a subset of faster growing trees.

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