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STOCHASTIC PRICE MODELS AND OPTIMAL TREE CUTTING: RESULTS FOR LOBLOLLY PINE

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ABSTRACT. An empirical investigation of stumpage price models and optimal harvest policies is conducted for loblolly pine plantations in the southeastern United States. The stationarity of monthly and quarterly series of sawtimber prices is analyzed using a unit root test. The statistical evidence supports stationary autoregressive models for the monthly series and for the quarterly series of opening month prices. In contrast, the evidence supports a non-stationary random walk model for the quarterly series of average prices. This conflicting result is likely an artifact of price averaging. The properties of these series significantly affect the forms of optimal price-dependent harvest rules and expected returns. Further, the results have implications for conclusions about market efficiency and the performance of a fixed rotation age.

KEY WORDS: Forest management, optimal harvesting, time-series analysis.

1. Introduction. Theoretical and numerical investigations of the stochastic tree-cutting problem have demonstrated the critical dependence of the optimal harvest rule on the specification of the price process. In discrete-time, numerical results depend on stationarity. With stationary price models, optimal harvesting follows a reservation price policy in which cutting takes place when price is above the historical average (Norstrøm [1975], Lohmander [1988], Brazee and Mendelsohn [1988], Haight and Smith [1991]). This type of policy holds for both single– and multiple– rotation formulations regardless of fixed costs. With a non-stationary random walk model, optimal harvesting depends on fixed costs: with none, the policy is a fixed rotation age; otherwise, the policy is price dependent (Thomson [1992]).

In continuous time, Clarke and Reed [1989] show that a fixed rotation age is optimal when price follows geometric Brownian motion (i.e., the logarithm of price grows linearly with additive error), there are no fixed costs, and there is a single rotation.

There is debate about which price process is appropriate for stumpage markets. Thomson [1992] argues for a random walk model in which the current price contains no information about future prices (i.e., no price autocorrelation). A random walk model is consistent with an informationally efficient market (Fama [1970]), which is generally accepted for most assets. On the other hand, Clarke and Reed [1989] conjecture that stumpage markets are not characterized by frictionless, continuous trading, and thus prices are likely autocorrelated. On this basis, Clarke and Reed model price with a definition of geometric Brownian motion that approximates a continuous stochastic process with autocorrelated errors.

Adding to the debate are mixed results from tests of the efficiency of stumpage markets (Washburn and Binkley [1990]). Based on quarterly and annual series of average prices in the southeastern United States, these authors accept the hypothesis that stumpage markets are efficient. However, with monthly price observations they reject market efficiency and support the conjecture that short-term prices are autocorrelated.

To provide empirical evidence in support of either stationary or non-stationary models for stumpage prices, we analyze series of price observations using a statistical test for stationarity. Further, we show that the construction of the price series affects the outcome of the test. Finally we show, via numerical optimization in a discrete-time setting, how the choice between stationary and non-stationary price models together with fixed costs and revenues affects optimal harvest policies and expected economic returns.

First, we analyze monthly and quarterly series of loblolly pine stumpage prices from the Piedmont region of North Carolina, USA. Two quarterly series are constructed using opening month prices and average prices. We analyze a quarterly series of opening month prices because, compared to the series of average prices, it more accurately represents how timber sellers sample the market to determine when to cut. The series are tested for stationarity prior to model estimation. While the monthly series and the quarterly series of opening month prices are stationary and autoregressive, the series of quarterly average prices is not. This conflicting result is likely an artifact of price averaging and not an indication of underlying differences in market behavior depending on the interval of observation.

In the second part, we use numerical optimization to determine the effects of these price models on optimal harvest policies and to assess the cost of using the wrong policy. Of particular interest is the performance of a fixed rotation age, which is optimal in some situations (Clarke and Reed [1989], Thomson [1992]). The decision model is for a single rotation: the problem is to choose the optimal clearcut strategy for a mid-rotation stand that maximizes its expected present value. Revenue includes the value of the harvested trees and the value of bare land, which is independent of price and time and known with certainty. Optimal strategies are also computed with fixed costs assessed in each period before clearcut. Although the results are consistent with previous studies, general conclusions about the behavior of optimal harvesting in a discrete-time setting await theoretical analyses.

2. Stumpage price models. Time-series models are developed using loblolly pine sawtimber prices (\$(1988)/mbf, International) for the Piedmont region of North Carolina as reported in Timbermart South. Price observations are available in monthly intervals between January 1977 and March 1988. Models of logarithms of prices are developed for the monthly series (135 observations) and for two different quarterly series, each with 45 observations. The quarterly series of opening month prices is constructed by sampling the monthly series in quarterly intervals. The quarterly series of average prices is obtained by averaging monthly prices in quarterly intervals. Averages are computed using real prices and then transformed to the logarithmic scale. We use the logarithmic scale because we assume that the underlying process has a non-homogeneous variance that is proportional in size to the observed price. Furthermore, the observations generated with a logarithmic model are non-negative.

The form of the price model depends on the stationarity of the underlying stochastic process. The process is said to have weak-form stationarity if its mean and covariance functions do not depend on time. If true, the process can be modeled by an equation with fixed coefficients that can be estimated directly from the observations, which should exhibit no regular behavior over time. If the process is not stationary, it is more difficult to model, although taking differences in the prices may produce a stationary process (Box and Jenkins [1970]).

The standard way to determine stationarity is to examine the sample autocorrelation function (ACF). The ACF for a stationary series decreases rapidly as the number of lags increases. The ACF for each undifferenced price series shows exponential decay with significant autocorrelations up to five lags. The differenced monthly series has significant autocorrelations at lag one; the differenced quarterly series show no significant autocorrelations. For borderline cases such as these, there is a question about whether to difference the data.

Said and Dickey [1984] describe a formal test for stationarity (see also Dickey et al. [1986]). The test is designed to accept a model for differenced data unless the undifferenced data present statistically significant evidence to the contrary. The null hypothesis is an autoregressive model for differenced data. Letting $q(t)$, $t = 0, \ldots, T$, represent a series of log prices and $\Delta q(t) = q(t) - q(t-1)$, $t = 1, \ldots, T$, represent the price differences, the null hypothesis is:

$$H_0 : \Delta q(t) = \alpha_0 + \sum_{j=1}^{k} \alpha_j \Delta q(t-j) + \varepsilon(t),$$

where α_j are coefficients for k lagged difference terms, $\varepsilon(t)$ is a normally distributed random error with zero mean and variance σ^2, and $\mathrm{E}[\varepsilon(t)\varepsilon(s)] = 0$ for $t \neq s$. The alternative is an autoregressive model for the undifferenced data:

$$H_1 : q(t) = b_0 + \sum_{j=1}^{l} b_j q(t-j) + \varepsilon(t),$$

where b_j are coefficients for l lagged price terms.

The procedure for testing H_0 (the so-called augmented Dickey-Fuller test) is applied when the order of the autoregression in H_0 is unknown. The test is performed by estimating the coefficients in the equation:

$$\Delta q(t) = b_0 + b_1 q(t-1) + \sum_{j=1}^{m} \alpha_j \Delta q(t-j) + \varepsilon(t).$$

The null hypothesis is accepted if the test-statistic $\hat{\tau}_\mu$ for $\hat{b}_1$ is not statistically different from zero. The formula for $\hat{\tau}_\mu$ is the same as the t-statistic for $\hat{b}_1$; however, because the null hypothesis is a non-stationary model, the distribution of τ_μ is not the Student-t distribution even in the limit. Therefore, the probability levels for the t-statistic are not appropriate for testing the significance of $\hat{\tau}_\mu$. Instead, critical values for $\hat{\tau}_\mu$ are found in Fuller [1976, Table 8.5.2].

The limit theory underlying the test does not specify the order m for the number of lagged difference terms. Following the procedure of Said and Dickey [1984], we fit regressions with $m = 1, \dots, 5$ assuming that an autoregression of order 5 gives a sufficient approximation of the data. Then, we use a standard regression F-test (e.g., Neter and Wasserman [1974], p.88) to determine whether the coefficients for groups of parameters are simultaneously equal to zero. In each of the augmented Dickey-Fuller tests below, the coefficients for additional lagged variables are not significantly different from zero and therefore not included in the regressions. The results of the F-test are consistent with the results of the Akaike information criteria, which has also been used to determine the number of lagged difference terms (see Lee and Siklos [1991]).

2.1 Monthly price model. We begin the investigation by regressing the price difference $\Delta q(t)$ on $1, q(t-1), \Delta q(t-1)$, which yields:

$$\Delta\hat{q}(t) = \underset{(.286)}{.902} - \underset{(.059)}{.187}q(t-1) - \underset{(.085)}{.226}\Delta q(t-1) \quad \text{with } \hat{\sigma}^2 = .009.$$

The numbers in parentheses are standard errors of the coefficients, and $\hat{\sigma}^2$ is the regression mean square error. Testing H_0, we compute $\hat{\tau}_\mu$ for $\hat{b}_1$ $(-.187/.059 = -3.17)$, which is less than the critical value at the .05 probability level (-2.89) (Fuller [1976, Table 8.5.2]) and thus significantly different from zero. This is strong evidence against the null hypothesis.

The next step is to estimate an autoregressive model for the undifferenced data. The partial autocorrelations for the monthly data are significant for the first two lags suggesting the second-order model:

$$\hat{q}(t) = \underset{(.040)}{.895} + \underset{(.085)}{.589}q(t-1) + \underset{(.085)}{.226}q(t-2) \quad \text{with } \hat{\sigma}^2 = .009, \tag{1}$$

which has significant coefficients at the .05 probability level (using standard t-tests). Additional lags are not significant. Therefore, statistical evidence supports model (1) for the underlying stochastic process for the monthly logarithms of prices.

Evidence of autocorrelation in the undifferenced series of the logarithms of monthly prices has implications for long-term forecasts, market efficiency, and optimal harvest policies. Forecasts with model (1) approach the mean of the series of logarithms of prices regardless of the level of the most recent price observations. Because current and past prices are used to predict future prices, model (1) is not consistent with the necessary condition for an efficient market. As a result, the predictive power of past prices may be used to construct adaptive harvest policies that time timber harvests to periods of high prices and thus increase the likelihood of higher returns.

2.2 Quarterly model of opening month prices. Similar to the monthly price series, there is strong evidence against the null hypothesis for the quarterly series of opening month prices. Regressing $\Delta q(t)$ on $1, q(t-1)$ yields:

$$\begin{aligned} \Delta\hat{q}(t) = \; & \underset{(.579)}{1.815} - \underset{(.120)}{.378}q(t-1) \quad \text{with } \hat{\sigma}^2 = .015. \end{aligned}$$

Testing H_0, we compute $\hat{\tau}_\mu$ for $\hat{b}_1$ $(-.378/.120 = -3.15)$, which is less than the critical value at the .05 probability level (-2.95) (Fuller [1976, Table 8.5.2]) and thus significantly different from zero. Quarterly series of second or third month prices produce the same results.

The alternative is an autoregressive model for the undifferenced data. The partial autocorrelations are significant for the first lag suggesting the first-order model:

$$\hat{q}(t) = \underset{(.046)}{1.813} + \underset{(.120)}{.623}q(t-1) \quad \text{with } \hat{\sigma}^2 = .015, \tag{2}$$

which has significant coefficients at the .05 probability level (using standard t-tests). Additional lags are not significant. Therefore, statistical evidence supports model (2) for the underlying stochastic process for the quarterly series of opening month prices.

2.3 Quarterly model of average prices. In contrast to quarterly series of opening month prices, the statistical evidence for the series of quarterly average prices supports the null hypothesis. Regressing $\Delta q(t)$ on $1, q(t-1)$ yields:

$$\Delta \hat{q}(t) = \underset{(.454)}{.995} - \underset{(.094)}{.206}q(t-1) \quad \text{with } \hat{\sigma}^2 = .007.$$

Testing H_0, we compute $\hat{\tau}_\mu$ for $\hat{b}_1$ $(-.206/.094 = -2.19)$, which is greater than the critical value at the .05 probability level (-2.95) (Fuller [1976, Table 8.5.2]) and thus not significantly different from zero.

The next step is to test whether the drift parameter is significant. Regressing $\Delta q(t)$ on 1 yields:

$$\Delta \hat{q}(t) = -\underset{(.015)}{.001} \quad \text{with } \hat{\sigma}^2 = .007.$$

The constant is not significant at the .05 level (using a standard t-test). Therefore, the statistical evidence supports a random walk model without drift for the quarterly series of average prices:

$$\hat{q}(t+1) = q(t) + \varepsilon(t) \quad \text{with } \hat{\sigma}^2 = .007. \tag{3}$$

The random walk model (3) has forecasting properties that are qualitatively different from those of autoregressive models (1) and (2). A one-period forecast with the random walk model depends only on the observed price in period t: $\hat{q}(t+1) = E[q(t+1) \mid q(t), q(t-1), \dots, q(0)] = q(t)$. Likewise, the k-period forecast depends only on the current price: $\hat{q}(t+k) = q(t)$. Therefore, all past information about stumpage price cannot be used to produce a better estimate of the future price than the capitalized current price. The implication for optimal harvesting is that no gain in value can be obtained from using past price movements to play the market in timing timber harvests. This property is consistent with a necessary condition for an efficient market.

We emphasize that the underlying stochastic processes for the two quarterly series depend on how the monthly series is sampled. When the monthly series is sampled at quarterly intervals, the resulting series of opening month prices is autoregressive (equation 2), similar to the monthly series. When the monthly series is averaged in quarterly intervals, the resulting series is a random walk (equation 3). In Section 5 we suggest that the statistical evidence supporting a random walk for the series of quarterly averages is an artifact of averaging the monthly series of autocorrelated prices. Before we get to this discussion, we present numerical results that show how the forms of these price models affect optimal tree-cutting policies.

3. Dynamic programming formulation. Similar to the dynamic programming model described by Norstrøm [1975], the following model assumes that the stochastic price forecast depends only on the current price. Thus, the formulation applies to models (2) and (3) for quarterly prices. It is easily expanded to include an additional price state for the monthly price model (1).

The state descriptor is a discrete variable representing the current market state. Let $m_k(t)$, $k = 1, \ldots, n$, represent n discrete price classes (in the logarithm scale) at the beginning of period t, which equals the age of the stand. The price model is used to estimate discrete transition probabilities $P_{j,k}$ representing the probability of being in price class j in period $t+1$ given $m_k(t)$.

The revenue $R[m_k(t)]$ (\$/ac) obtained from clearcutting in period t depends on the current market state. Stand volume $v(t)$ (mbf/ac) is a deterministic function of stand age. Bare land value L (\$/ac) is the selling price for bare land. The revenue function is:

$$R[m_k(t)] = \exp\{m_k(t)\}v(t) + L$$

For a given bare land value and time horizon T, the optimal harvest strategy is found by solving a recurrence relation for optimal stand value. Define $Z_t[m_k(t)]$ as the expected present value (\$/ac) of the stand in period t and market state $m_k(t)$. Assuming that the decision maker's real discount rate is r and the discount factor is $\delta = (1)/(1+r)$, the recurrence relation for optimal stand value is:

$$Z_t[m_k(t)] = \max\left\{ R[m_k(t)], \delta \sum_j P_{j,k} Z_{t+1}[m_j(t+1)] \right\}.$$

The maximization problem is the choice between clearcutting and no action. Clearcutting is optimal when the revenue $R[m_k(t)]$ is greater than the expected present value of the stand in period $t+1$. The boundary condition in period T assumes that all trees are cut:

$$Z_T[m_k(T)] = R[m_k(T)].$$

The recurrence relation for optimal stand value is solved backwards from period $T-1$ using the boundary condition. The recurrence relation ends in the earliest period in which the stand may be harvested. The solution is either a clearcut or no action decision for each market state in each period.

There is an important difference between the horizon T and the rotation age. The horizon T represents the maximum number of periods a stand is allowed to grow. If a stand reaches period T, it is clearcut. Clearcutting may take place in any period $t < T$. The rotation age depends on the market state and the probability distribution of future market states. The horizon T may be arbitrarily long; for computational efficiency it should be long enough that the likelihood of clearcutting before period T is high.

Optimal harvest strategies for a 30-year-old plantation are determined by solving the recurrence relation backwards from age 50. Harvest revenue is obtained for sawtimber; pulpwood has no value. The variable for stumpage price ranges between \$3/mbf and \$6/mbf in \$0.075 intervals in the logarithm scale. Bare land value is \$550/ac and represents the rotation-start present value of an infinite series of plantations computed using a deterministic stumpage price equal to \$125/mbf (real scale). Prices are in 1988 dollars. The discount rate is 4%.

Monthly and quarterly sawtimber yields (International mbf/ac) for a pure loblolly pine plantation are predicted with the second degree polynomial,

$$v(t) = -16.54 + 1.029t - 0.005220t^2,$$

where t is stand age. The model is constructed with ordinary least squares applied to output from the North Carolina State University Plantation Management Simulator (Hafley and Buford [1985]). The simulator is used to predict annual sawtimber yield from a 30-year-old plantation over a 20-year horizon. At age 30, the plantation has 100 trees/ac and 100 ft^2/ac basal area. The plantation is on site index 65 (25 year basis) land in the North Carolina Piedmont. The volume versus age model fits the data with $R^2 = 0.999$; all coefficients are significant at the 0.05 level.

4. Optimization results.

4.1 Monthly price model. The statistical evidence supports the second-order, autoregressive model (1) for monthly price predictions, and its optimal policy is to harvest when the observed price *is greater than* a reservation price that is conditioned on age and last month's price (Figure 1). For past prices between \$80 and \$160/mbf, reservation prices are practically the same until age 45 when they diverge and approach the level of the past price. The areas below the curves contain the price-age combinations when harvesting should be postponed. The expected present value (EPV) is \$2,277/ac for the 30-year-old stand. The expected rotation age is 36 years.

The rationale for the price-dependent cutting policy is as follows. Stationary autoregressive models produce price paths that fluctuate around the mean of the historical series. It is better to postpone cutting when the price is below average because there is a high probability that a future price will be above average. Conversely, it is better to cut when price is above average because price is likely to drop in the future. The fixed land value influences the level of the reservation prices: as the land value approaches zero, the optimal reservation prices and the expected rotation age increase. There are numerous examples of price-dependent cutting policies of this type from numerical studies with stationary price models (Norstrøm [1975], Lohmander [1988], Brazee and Mendelsohn [1988], Haight and Smith [1991]). Outlines for the proof that reservation price policies are optimal, in general, for discrete-time stationary models are found in Brazee [1987] and Lohmander [1988].

For comparison with the performance of the optimal reservation price policy, we used Monte Carlo simulation to estimate the EPVs of fixed rotation ages when the price process is governed by model (1). The costs of fixed rotation ages are large: the EPV of the optimal rotation age (\$1,832/ac, 34 years) is 20% less than the EPV of the optimal

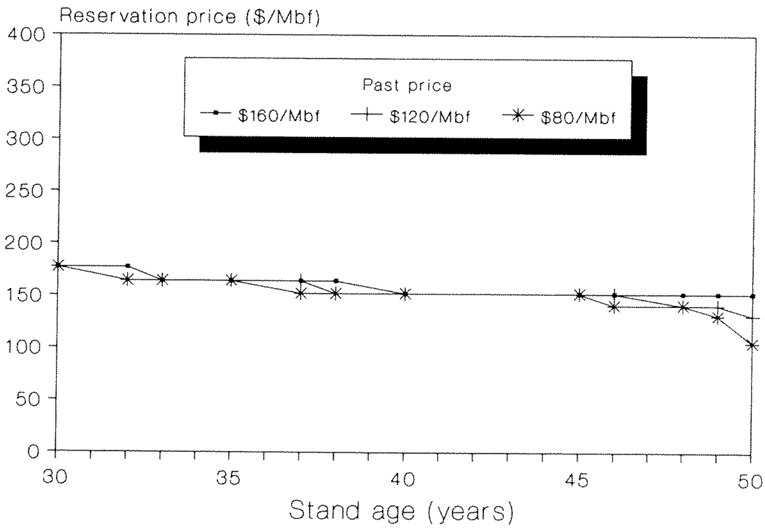

FIGURE 1. Optimal reservation prices for harvest decisions on a monthly interval using the autoregressive model (1).

reservation price policy ($2,277/ac). The cost is due to not using the predictive power of the current and past prices to time the harvest.

4.2 Quarterly model of opening month prices. For the quarterly series of opening month prices, the statistical evidence supports the first-order, autoregressive model (2), and the optimal cutting policy is similar to reservation price policy for the monthly model. The optimal policy is to harvest when the observed price *is greater than* an age-dependent reservation price. Reservation prices decrease with age and approach the mean of the price series ($125/mbf) (Figure 2). The area below the curve contains the price-age combinations when harvesting should be postponed. The expected present value of the 30-year-old stand is $2,183/ac, which is 4% less than the EPV of the optimal policy for the monthly interval. The expected rotation age is 36 years.

Reservation prices for the quarterly model (Figure 2) are lower than those for the monthly model (Figure 1) primarily because of the longer

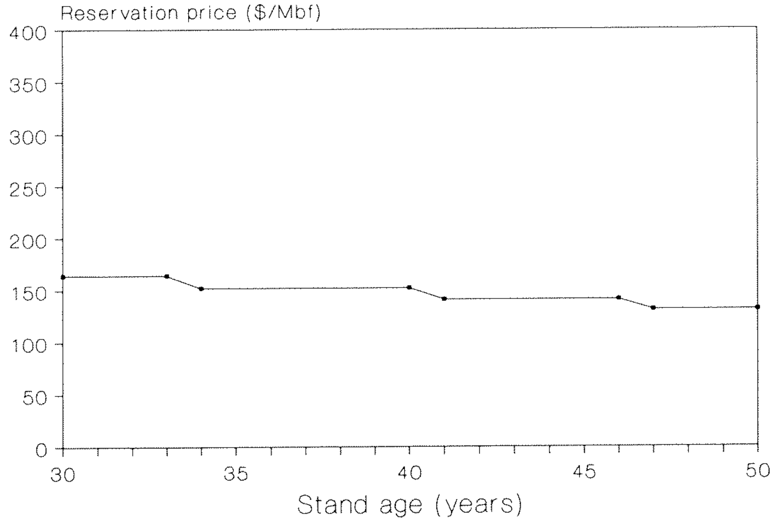

FIGURE 2. Optimal reservation prices for harvest decisions on a quarterly interval using the autoregressive model (2).

time interval between harvest decisions. With fewer chances to evaluate the price, it is less likely that higher prices will be encountered.

We used Monte Carlo simulation to estimate the EPVs of fixed rotation ages when the price process is governed by model (2). Similar to results for the monthly interval, the EPV of the optimal rotation age ($1,786/ac, 34 years) is 18% less than the EPV of the optimal reservation price policy ($2,183/ac).

4.3 Quarterly model of average prices. The optimal harvest policy using the random walk model (3) for the series of quarterly average prices is different from those for the autoregressive models. The optimal policy is to harvest when the observed price *is less than* an age-dependent reservation price. Optimal reservation prices increase with age (Figure 3); the area above and to the left of the curve contains the price-age combinations when harvesting should be postponed. The expected rotation age is 41 years, and for a starting price of $125/mbf, the EPV of the 30-year-old stand is $1,956/ac.

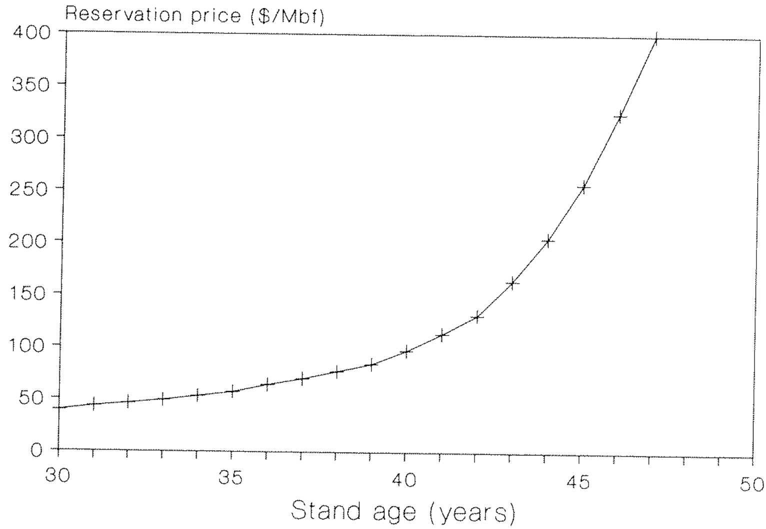

FIGURE 3. Optimal reservation prices for harvest decisions on a quarterly interval using the random walk model (3).

This price-dependent harvest policy is the result of the properties of the random walk model and the fixed land value. With a random walk model, the best estimate of the future price is the current price. Further, for any current price, there is an equal chance that price will rise or fall in the next period. Harvesting is postponed when the observed price is high because the future price is expected to remain high and because the value of the growing stock and growth is high relative to the fixed value of bare land. Conversely, it is better to harvest when price is low because price is expected to stay low and because the value of bare land is greater than the expected return from growing timber. Thomson [1992] produces similar numerical results for a multiple-rotation harvest problem with fixed costs and a random walk model for prices.

We used Monte Carlo simulation to estimate the EPVs of fixed rotation ages when the price process is governed by model (3). The EPV of the optimal rotation age ($1,938/ac, 45 years) is 1% less than the EPV of the optimal reservation price policy ($1,956/ac). The cost is small in comparison to the costs of using fixed rotation ages with the autoregressive models above. The cost is due to not timing the timber harvest according to the levels of the current price and land value.

The fixed land value has an important impact on the optimal policy. With zero land value and no fixed costs, the optimal policy is to ignore price fluctuations and cut at a fixed rotation age. This finding is consistent with results using a deterministic rotation-age model with no fixed costs: the price level does not influence the optimal rotation age (e.g., Clark [1976]). It is also consistent with Clarke and Reed's [1989] findings for a continuous-time cutting problem where price follows geometric Brownian motion.

When timber sellers obtain bids that represent point estimates rather than averages of stumpage price offerings, the quarterly model of opening month prices is a better representation of the stumpage price process than the series of averages. In this case, employing the harvest policy based on the random walk model will provide suboptimal returns when prices are sampled on a quarterly interval. The harvest policy associated with the random walk model (3) performs poorly when the series of quarterly price observations follows the autoregressive model (2). The EPV of the random walk policy ($1,520/ac) is 30% less than the EPV of the optimal policy for the autoregressive model ($2,183/ac).

4.4 Information cost. Using the reservation price policies for the monthly or quarterly intervals requires the monitoring of stumpage prices and a readiness to complete a sale contract. The cost of these activities may affect the choice of decision interval. Therefore, we compute optimal policies using fixed costs (up to $15/ac) that are assessed each period before harvest. While information about the cost of price monitoring is not readily available, it is probably the same order of magnitude as the cost of timber cruising, which averaged between $2 and $3/ac in different regions of the southern United States in 1988.

For the monthly interval, optimal policies are computed using the second-order, autoregressive model (1). With increasing fixed costs, the optimal reservation prices and the expected rotation age decrease. Harvesting is acceptable at lower prices in order to avoid paying the additional costs of price monitoring. The EPVs decrease rapidly and approach $1,752/ac, the value of the 30-year-old stand cut immediately

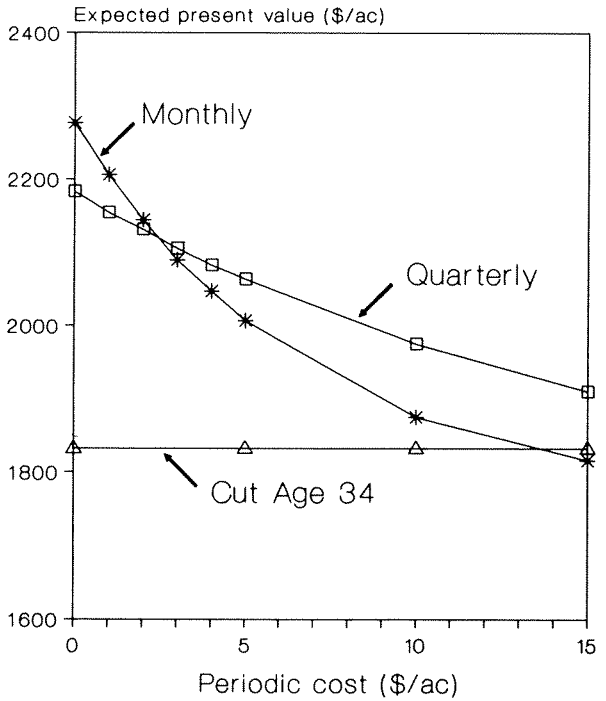

FIGURE 4. The expected present value of optimal harvest policies as a function of the cost of monthly and quarterly decisions.

(Figure 4). With a cost of $15/ac, the expected rotation age is 30.6 years.

For the quarterly interval, optimal policies are computed using the first-order, autoregressive model (2) for opening month prices. The EPVs are greater than those for the monthly interval for costs greater than about $3/ac (Figure 4). With a cost of $15/ac, the expected rotation age is 32.5 years.

In contrast to the reservation price policies, using a fixed rotation age does not require stumpage price monitoring and therefore does not incur the fixed periodic cost. How does the EPV of the optimal rotation age compare? Assuming that the price process is inherently continuous with autocorrelated noise, the best estimate of the EPV of the optimal rotation age is obtained with the autoregressive model (1) for the monthly interval ($1,832/ac, 34 years). With monthly costs less than about $14/ac, the optimal reservation price policy for the monthly interval provides higher expected returns. With quarterly costs less than about $20/ac, the optimal reservation price policy for the quarterly interval provides higher returns.

5. Discussion. The statistical results in Section 2 raise the question of why the stochastic process for the series of quarterly average prices is a random walk while the series of monthly prices is autoregressive. The question is important not only because the forms of the price models affect optimal harvest policies, but also because the price models are used as evidence for market efficiency. For example, Washburn and Binkley [1990] used this evidence to conclude that southern pine stumpage markets are efficient when viewed on a quarterly interval. However, the evidence that the series of quarterly averages is a random walk may be an artifact of averaging an underlying series of autocorrelated prices. Therefore, conclusions about market efficiency based on this evidence may be incorrect.

In an analysis of serial correlation, it is important to consider the impact of the sampling design on the characteristics of the stochastic process. In particular, we are concerned with the implications of averaging an input series on the characteristics of the output series. Assuming that the input series is a random walk, Working [1960] demonstrates that averaging has two important effects on the filtered output series. First, averaging reduces the variance of first differences. Second, averaging induces a spurious first-order serial correlation of first differences of about 0.25. Therefore, when using averaged data, evidence in support of an underlying random walk process should include a test of the hypothesis that first-order serial correlation of first differences is not significantly different than 0.25. The evidence from the series of quarterly average prices for North Carolina supports this hypothesis.

Washburn and Binkley include this test in their analysis of market efficiency. They construct models for the rational expectations equilibrium for stumpage markets and analyze time series of differences between actual and predicted equilibrium rates of stumpage price change. Because period averages are used to calculate rates of price change, the null hypothesis of no serial dependence, and weak-form market efficiency, is a correlation coefficient of 0.25 at the first lag and 0.0 at all further lags. Washburn and Binkley accept the null hypothesis using the quarterly average price data.

Alternatively, the error structures for our time-series model and Washburn and Binkley's equilibrium market model based on quarterly average data may have resulted from an underlying autoregressive price process. In the Appendix, we provide a heuristic proof of the proposition that averaging a stochastic process with positive autocorrelation transforms the series so that it behaves approximately like the original series but with a larger autoregressive coefficient b_1. Averaging in this case preserves random shocks at a rate that exceeds the exponential decay evident in the original series. By increasing the value of b_1 and decreasing the degrees of freedom, averaging a monthly series may create a filtered series that appears to follow a random walk. If the error structures in our time-series model and Washburn and Binkley's market model could have resulted from averaging either an autoregressive series or a random walk, then conclusions about market efficiency based on these models may be incorrect.

In any case, we believe that timber sellers sample the spot or cash market at periodic intervals to determine when to cut, rather than sampling an average market. Consequently, a quarterly series of opening month prices provides a more realistic sampling for constructing a model of quarterly price movements and for determining optimal harvest policies.

Since monthly stumpage prices used in our analysis may represent an average over some higher frequency of price generation, we are concerned that our finding of significant first-order serial correlation in the rate of price change may be spurious. For the monthly North Carolina data, the serial correlation is negative and significantly less than 0.25. Combined with the results of the unit root test, we conclude that the underlying price generation process, which occurs at a higher-than-monthly frequency, is not a random walk.

Is it realistic to conclude that monthly stumpage markets operate with positive feedback? Although a full portrayal of timber market behavior is beyond the scope of this paper, we conjecture that the positive feedback is due to sluggishness in supply response. Time lags of weeks to months can occur between the point when the decision is made to cut, the point when the sale contract is established, and the point when the timber is cut. In a *partial adjustment model* (e.g., Nerlove [1958]), the price forecast governing decisions to cut is the current price, but the actual cut adjusts slowly to a price change. This sluggish adjustment causes prices to monotonically converge to the equilibrium price in the manner of a positive feedback. We note that the same dynamic price behavior could be observed if timber supply is based on adaptive expectations that adjust slowly to new information.

6. Conclusions. Recent investigations of the stochastic tree cutting problem disagree about the appropriate price process. Clarke and Reed [1989] argue that, in a continuous-time setting, the price process should be modeled with a definition of geometric Brownian motion that approximates a continuous time process with autocorrelated errors. Thomson [1992] argues that, in a discrete-time setting, a random walk model is appropriate because stumpage markets are efficient (e.g., Washburn and Binkley [1990]). Our analysis of a monthly series of loblolly pine stumpage prices supports the conjecture that stumpage markets are not characterized by frictionless, continuous trading. Autocorrelation is present in both the monthly series and the quarterly series of opening month prices. However, our analysis shows that the series of quarterly average prices is a non-stationary random walk. This result is likely an artifact of price averaging and not an indication of market efficiency.

The empirical question of whether or not stumpage prices follow a random walk has important implications for the optimal timing of timber harvests. The optimal policies for the autoregressive models for monthly and quarterly intervals involve harvesting when price is above the historical average. In contrast, the optimal policy for the random walk model of quarterly average prices is to cut when the price is low relative to the historical average. Employing such a policy when the price process is autoregressive results in a substantial reduction in expected present value.

The question is raised about the performance of a fixed rotation age, which is the optimal policy for continuous-time models with geometric Brownian motion and no fixed costs (Clarke and Reed [1989]). In comparison with a continuous-time setting, discrete time is a better representation of harvest decisions due to the time required to monitor stumpage prices. Further, price autocorrelation can be explicitly included in discrete-time models. In this setting, a fixed rotation age is inferior to price-dependent harvest rules. A possible advantage of a fixed rotation age is that costs of price monitoring are avoided. Our results show that both monthly and quarterly price-dependent decision rules have higher expected present values than do fixed rotation ages for the likely range of periodic fixed costs in the southeastern United States.

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Appendix A

This appendix contains a heuristic proof of the proposition that averaging a first-order, positive feedback process results in a filtered series that behaves approximately like the original series with a larger autoregressive coefficient. The approximation results from the fact that random shocks decay in a piecewise linear fashion. The resulting process may not be distinguishable from a random walk. The proof is based on the way in which stochastic processes accumulate random shocks. By definition, a random walk is a stochastic process that integrates random shocks over time:

$$\begin{aligned} y(t) &= b_0 + y(t-1) + \varepsilon(t) \\ &= b_0\, t + y(0) + \sum_{j=1}^{t} \varepsilon(j), \end{aligned} \tag{4}$$

where b_0 is drift, t is time, and $\varepsilon(t)$ is a random shock. Each random shock is fully preserved. In contrast, the random shocks in the first-order autoregressive process,

$$\begin{aligned} y(t) &= b_0 + b_1 y(t-1) + \varepsilon(t) \\ &= b_0 + b_1^t y(0) + b_1^{t-1}\varepsilon(1) + \ldots + b_1\varepsilon(t-1) + \varepsilon(t), \end{aligned} \tag{5}$$

decay in an exponential fashion. Also note that a random walk is an autoregressive process of order 1 where $b_1 = 1.0$.

The proof begins by assuming that equation (5) represents the true price process (e.g., the true monthly price series). Consider a process that is obtained by averaging m successive items in an autoregressive series (e.g., let $m = 3$ for quarterly averages). We wish to compare the degree to which random shocks are preserved at $t = \alpha m$, where α is a positive integer. For simplicity, we consider the case where $\alpha = 1$.

Letting $y(0) = \varepsilon(0)$, the equation explaining an observation from the original series at $t = m$ can be written:

$$y(m) = b_1^m \varepsilon(0) + b_1^{m-1}\varepsilon(1) + \ldots + b_1\varepsilon(m-1) + \varepsilon(m). \tag{6}$$

From equation (6), it is seen that the amount of $\varepsilon(0)$ preserved at $t = m$ equals $b_1^m \varepsilon(0)$.

Now consider the expression for the averaged data at the corresponding time point:

(7)

$$\begin{aligned}\bar{y}(m) =& [b_1^m\varepsilon(0) + b_1^{m-1}\varepsilon(1) + \ldots + b_1\varepsilon(m-1) + \varepsilon(m)]/m + \\ & [b_1^{m-1}\varepsilon(0) + b_1^{m-2}\varepsilon(1) + \ldots + b_1\varepsilon(m-2) + \varepsilon(m-1)]/m + \ldots + \\ & [b_1\varepsilon(0) + \varepsilon(1)]/m.\end{aligned}$$

From equation (7), it is seen that the amount of $\varepsilon(0)$ preserved at $t = m$ equals $[(b_1^m + b_1^{m-1} + \ldots + b_1)/m]\varepsilon(0)$.

It remains to show that

$$\varepsilon(0)[b_1^m + b_1^{m-1} + \ldots + b_1]/m > \varepsilon(0)b_1^m. \tag{8}$$

Equation (8) simplifies to

$$b_1^{-1} + b_1^{-2} + \ldots + b_1^{1-m} > m - 1. \tag{9}$$

Note that the left hand side (l.h.s.) of equation (9) contains $m - 1$ terms. If $0 < b_1 < 1$, then each term on the l.h.s. of equation (9) is greater than unity, and the proof is complete. Finally, note that a unit increase in m increases the l.h.s. by b_1^{1-m} while increasing the right hand side by one unit. Consequently, the impact of averaging increases b_1 as m increases for a series exhibiting positive feedback.

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