

# A Multinomial Logit Approach to Estimating Regional Inventories by Product Class

by  
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## Abstract

Current timber inventory projections generally lack information on inventory by product classes. Most models available for inventory projection and linked to supply analyses are limited to projecting aggregate softwood and hardwood. The objective of this research is to develop a methodology to distribute the volume on each FIA survey plot to product classes and simulate the changes in the product distribution over time. A multinomial logit model was developed to estimate sets of product proportion functions to distribute plot volumes by product class for each forest type and size class. The product proportion model is incorporated in the DPSupply system, and is demonstrated using Alabama data.

## INTRODUCTION

Timber inventory projection and supply analysis are important strategic planning tools for forest industry decision makers. However, most recent efforts to project timber inventories have been limited to projecting aggregate softwood and hardwood due to the lack of a mechanism for separating products such as pulpwood and sawtimber from the aggregate data. A procedure to project sawtimber and pulpwood inventories separately is needed to more clearly understand the dynamics of forest inventory and make informed strategic planning decisions.

The overall objective of this research is to develop a methodology to separate products for each potential harvest plot and simulate the structural change in forest inventory over time at the product level. The most recent FIA data for Alabama and Mississippi were used to develop the functions to estimate product proportion for each forest type. Acres available for harvest in each period were determined by a net present value criterion which is calculated in the DPSupply system. Linear programming (LP) was used to determine which acres would be harvested to meet product demands and incorporated the product proportion functions for each forest type. The following sections will describe the development of the product functions for each type and demonstrate the use of these functions in the DPSupply system using Alabama data.

## THE DATA

The data used for this project are Mississippi and Alabama FIA surveys including: MS 1994, MS 1987, AL1990 and AL1 982. These four FIA data sets were pooled for the analysis. Only the data representing timberland acres are included with nonstocked

timberland acres excluded. We considered all live trees. The final data set consisted of 13,740 plots including 1,713 pine plantation plots, 2,739 natural pine plots, 2,758 oakpine plots and 6,530 hardwood plots. For each type and each plot, the percentages of softwood pulpwood, softwood sawtimber, hardwood pulpwood and hardwood sawtimber were calculated as new variables and associated with the other plot level data.

The products definitions used in this study are described by Hansen et al. (1992): (1) **Softwood:** pulpwood -- dbh greater than or equal to 5 inches and less than 9 inches; sawtimber --- dbh greater than or equal to 9 inches; (2) **Hardwood:** pulpwood --- dbh greater than or equal to 5 inches and less than 11 inches; sawtimber --- dbh greater than or equal to 11 inches. The percentages of volume of each product class to total volume on each plot were calculated as separate variables. In addition, we needed an estimate of average dbh for each observation (plot). The observation was first classified according to its stand size class (sawtimber, poletimber and seedling-sapling). Then the average dbh for each observation was calculated based on the rules listed in Table 1. The volume per acre was estimated based on the reported volume of each tree greater than or equal to 5 inches dbh.

## THE MULTINOMIAL LOGIT MODEL

We separated the data into two groups: those observations with average  $dbh \leq 5.0$  inches and those observations with average  $dbh > 5.0$  inches. The final model estimating product distribution has to satisfy the following properties:

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Table 1. Inclusion rules for calculating average dbh for each plot.

Size Class	Pine Plantation	Natural Pine	Oak-Pine	Hardwood
Seedling-sapling	Softwood only & tree dbh <9.0	Softwood only & tree dbh <9.0	Softwood tree dbh <9.0 & Hardwood tree dbh <11.0	Hardwood only & tree dbh <11.0
Poletimber Sawtimber	Softwood only & tree dbh >=5.0	Softwood only & tree dbh >=5.0	Both softwood & hardwood tree dbh >=5.0	Hardwood only & tree dbh >=5.0

(a)  $0 < P_{ij}(dbh, vol) < 1$  for all  $i, j$

(b)  $\sum_i \sum_j P_{ij}(dbh, vol) = 1.0$  for each forest type

Where  $P_{ij}$  — The estimated proportion of the live tree volume in each of the four product classes on the plot,  $i=1,2$  and  $j=1,2$ .

We examined the multinomial logit model and used it to solve this problem. The basic multinomial logit model can be expressed as follows ( Maddala, 1987):

$$P_m = \frac{1}{1 + \sum_{k=1}^{m-1} \exp(B_k'X)}$$

$$P_j = \frac{\exp(B_j'X)}{1 + \sum_{k=1}^{m-1} \exp(B_k'X)} \quad (j = 1, 2, \dots, m-1)$$

Where: X-explanatory variables including dbh and volume; m-categories considered (in our problem, the product classes); P-proportions associated with the categories; B-a vector of parameters.

The multinomial logit model is now being used in a variety of situations in applied econometrics, including occupational choice and transportation choice problems. The only forestry application we are familiar with used the technique to evaluate spruce budworm control efforts (Hughes et al. 1987). For this research, this approach is used to simulate the products composition of stands with a given dbh and volume combination. Four products are classified as softwood pulpwood, softwood sawtimber, hardwood pulpwood and hardwood sawtimber. The proportion of each product in the stand is a function of average dbh and volume per acre by forest type. The parameters of these

proportion functions are estimated by normalizing with respect to softwood pulpwood proportion ( $P_{11}$ ). The maximum likelihood estimates were obtained and the example for pine plantation stands with an average dbh greater than or equal to 5 inches is shown in Table 2.

Table 2. Multinomial logit parameter estimates (Pine plantation, average dbh >=5.0)

Parameters	Products		
	$P_{12}$	$P_{13}$	$P_{14}$
Intercept	-6.1018 (-14.80)	-6.6935 (-9.54)	-7.1908 (-8.92)
dbh	0.6591 (11.47)	0.5114 (544)	0.5274 (4.98)
volume	0.00013 (0.94)	0.000045 (0.18)	-0.000055 (0.19)

(Asymptotic t-statistic5 in parentheses)

Where:  $P_{12}$  = proportion of softwood sawtimber volume;

$P_{13}$  = proportion of hardwood pulpwood volume;

$P_{14}$  = proportion of hardwood sawtimber volume.

The proportions of these four products at the mean vector (dbh=8.498, volume=1383 cf/acre) for pine plantations ( average dbh >=5.0) are:  $P_{11} = 0.528$ ,  $P_{12} = 0.381$ ,  $P_{13} = 0.054$ ,  $P_{14} = 0.038$ . The marginal effects ( $\partial P_{ij} / \partial X_{ij}$ ) are computed at the means of the Xs (dbh and volume) and are listed in Table 3.

Table 3. Marginal Effects Computed at 'the Means

$P_{ij}$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$
dbh	-0.15744	0.13732	0.0114	0.00868
volume	-0.00003	0.00003	-0.0000	-0.0000

For example, at the data set average dbh of 8.498 inches, if average dbh increases one unit, the softwood pulpwood proportion will decrease by 0.157, and the softwood sawtimber proportion will increase by 0.137.

All models contain the same variables, and the models are used in the MANAGE module of DPSupply to calculate net present value for each plot based on the plot's mix of products. They are also used in the HARVEST module of the program to distribute the products and meet individual product level demands,

**THE PRODUCT LEVEL DPSUPPLY SYSTEM**

The prototype DPSupply system was developed several years ago to conduct regional timber inventory projection and supply analysis. While several regional models are available to provide this kind of information, DPSupply is unique in several ways. First, the growth models in DPSupply are developed using **two** FIA surveys and are continuous across the range of potential (projected) dbh and volume combinations. Second, harvest levels in DPSupply are adjusted based on secondary (often severance tax) data, so its more recent harvest information allows for a more accurate picture of the current inventory situation and a more informed forecast of future harvests. Third, harvest in DPSupply is based on net present value and product level demands, that is, harvests are conducted by product class instead of an "oldest first" rule or some other approach which ignores the product requirements of regional harvest activities.

The product functions are used to check the original FIA inventory by product class including **FIA82** and FIA90 for Alabama, FIA87 and FIA94 for Mississippi. The results are shown in Table 4, in which **FIA82\_AL** stands for FIA 1982 Inventory for Alabama, **FIA82E\_AL** stands for estimated 1982 inventory using the product model and volume intervals of **25cf/acre**. Inventory in this table does not include public land and nonstocked plots, and total differences for each State survey are due to rounding errors using the interval approach.

The earlier version of DPSupply used a lookup table to allocate total stand volume to product

classes in the harvesting module. Although this was better than no product recognition in harvesting or inventory projection, the lookup table approach has some drawbacks. One is that it used wide intervals for dbh and volume per acre (dbh used a 1 inch class interval and volume used 500 cf per acre class intervals). Another is that there are many interval combinations that are not represented in the data set and these empty cells in the table have to be interpolated subjectively.

The product model described in the last section does away with these problems and can be applied to a larger region. The model now is incorporated in the DPSupply system. DPSupply traces out the optimal path for harvest through time based on the net present value of the products for each stand. The acres available for harvest are identified and the actual harvests are determined based on the net present value ranking of the stands and the market demand for products using linear programming (LP). The linear programming solver is also **incorporated** in DPSupply.

Results of incorporating the products distribution model in DPSupply are demonstrated below using Alabama inventory data. The harvests are updated to 1996 using severance tax information and assumes that future demands for final products after 1996 are constant. The basic assumptions for projection are as follows:

1. Future demand for forest products are held constant at 1996 levels.
2. Timberland area change basically follows the assumptions made **in** 1993 RPA.

Table 4. **FIA** Estimated Volume and Product Function Estimated Volume **by** Product Classes (MMCF)

Alabama	FIA82_AL	FIA82E_AL	Difference	FIA90_AL	FIA90E_AL	Difference
Total	22048.3	22034.1	-0.06%	22957.7	22943.2	-0.06%
softwood	10885.1	10674.2	-1.94%	10449.3	10524.2	0.72%
<b>Hardwood</b>	<b>11163.2</b>	<b>11359.9</b>	<b>1.76%</b>	<b>12508.5</b>	<b>12419.0</b>	<b>-0.72%</b>
Stwd PW	3201.5	3145.1	-1.76%	2961.7	3013.4	1.74%
Stwd ST	7683.6	7529.2	-2.01%	7487.5	7510.8	0.31%
Hdwd PW	5339.9	5291.3	-0.91%	5256.4	5276.4	0.38%
Hdwd ST	5823.3	6068.6	4.21%	7252.0	7142.6	-1.51%
Mississippi	FIA87_MS	FIA87E_MS	Difference	FIA94_MS	FIA94E_MS	Difference
Total	18854.0	18850.2	-0.02%	18954.9	18852.4	-0.54%
Softwood	7611.3	7647.3	0.47%	7464.7	7446.4	-0.25%
<b>Hardwood</b>	<b>11242.7</b>	<b>11202.9</b>	<b>-0.35%</b>	<b>11490.2</b>	<b>11406.0</b>	<b>-0.73%</b>
Stwd PW	1779.0	1810.4	1.76%	1994.8	2013.2	0.92%
Stwd ST	5832.3	5836.9	0.08%	5469.9	5433.2	-0.67%
Hdwd PW	41270.2	4225.3	2.38%	3995.4	4055.9	1.51%
Hdwd ST	7115.5	6977.6	-1.94%	7494.8	7350.2	-1.93%

(1) areas of natural pine are projected to decrease at an annual rate of **1.5%**;

(2) areas of mixed pine-hardwood are projected to decrease at an annual rate of 0.2%;

(3) areas of pine plantations in the South are projected to increase by 77.7% from 23.03 million acres in 1990 to 40.92 million acres in 2010, or at an annual rate of 2.9%;

(4) Total forest land in the U.S. is expected to decline by about 28 million acres by 2040, mainly in the South and Pacific coast regions. Much of this loss in forest land will be due to conversions associated with roads and urban space utilized by a growing population.

3. The future technology effects accounted for in this simulation include improvements in utilization (a softwood lumber recovery increase of **0.32%/year**), OSB will increase its share of the structural panel market from 26% in 1990 to 51% in 2010, and utilization rates for recovered paper will increase **from** 27% in 1990 to 41% in 2010. These assumptions are also included in recent RPA projections.

Harvest levels are shown in Table 5 and inventory projection results are shown in figures 1-3. In table 5, harvest levels after 1996 are determined based on the assumptions made in the text, and (1), (2), (3) and (4) stand for softwood pulpwood, **softwood** sawtimber, hardwood pulpwood and hardwood sawtimber, respectively.

Using the product level DPSupply system, **timber** inventories in Alabama are projected to decrease for both softwood and hardwood relative to their projected 1996 levels. By 2010, total softwood inventory will decrease 21.5% relative to the 1996 level. Although softwood pulpwood is expected to increase after the year 2000 by 4.1% relative to the 1996 level, **softwood** sawtimber will decrease 31.7% relative to the 1996 level or 38.8% relative to the 1990 level reported by FIA. Total hardwood inventory will increase until 2002 and then decrease gradually. Hardwood pulpwood is projected to decrease by 20.8% relative to the 1996 level.

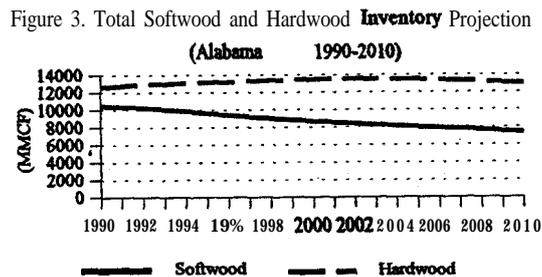
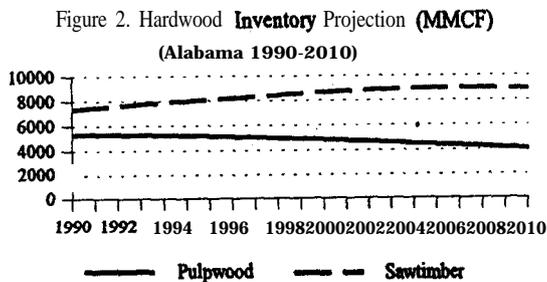
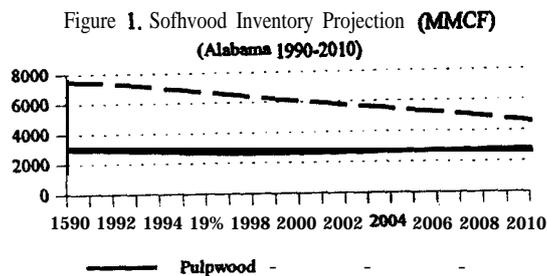
## CONCLUSIONS

The objectives of this study were to **first** develop a method for estimating the product composition of **FIA** plots and then **determine** how to incorporate the method in a timber inventory projection system. Previous methods for projecting inventories

**encountered** difficulties trying to link the **harvest** of products to the aggregate volumes of softwood or hardwood projected into the future. In an earlier version of DPSupply, product distribution of volumes on plots was accomplished using a look up table. This was not completely satisfactory in that the table approach became unwieldy if all observed combinations of dbh and volume were to be represented, and reasonable aggregation still resulted in a table with empty cells that needed to be estimated manually. These problems are avoided with the multinomial **logit** approach since the product distribution functions are continuous across the range of our dbh and volume variables. The multinomial **logit** approach also allows us to more readily generate information on the product distribution of plots in new regions, avoiding the problem of generating new look up tables for each state (or multistate region). Finally, initial, ongoing calibration efforts, with the method indicate that we can accurately reproduce the products distribution of state level surveys by type.

Table 5. Harvest Levels by Product Class in Alabama  
(Unit: MMCF)

Year	Total	Soft	Hard	(1)	(2)	(3)	(4)
1990	1139	719	421	179	540	167	253
1991	1197	742	455	177	565	228	227
1992	1272	816	456	203	613	217	239
1993	1320	830	490	204	626	229	261
1994	1340	861	479	202	660	206	272
1995	1387	886	501	239	647	233	268
1996	1335	851	484	220	631	224	260
1997	1328	846	482	219	627	222	260
1998	1321	841	479	219	623	219	260
1999	1314	837	477	218	619	217	260
2000	1307	832	475	218	615	214	260
2001	1307	831	476	220	611	216	260
2002	1305	830	476	223	607	216	260
2003	1305	829	477	226	603	217	260
2004	1305	827	478	229	599	218	260
2005	1305	826	479	231	595	219	260
2006	1305	825	480	234	591	220	260
2007	1304	824	481	237	587	221	260
2008	1304	822	482	240	583	222	260
2009	1304	821	483	242	579	223	260
2010	1304	820	484	245	575	224	260



### Literature Cited

- Batte, Marvin T., Eugene Jones, and Gary D. Schnitkey. 1990. Computer Use by Ohio Commercial Farmers. *American Journal of Agriculture Economics*, November 1991: 935-945.
- Caffey, Rex H., Richard F. Kazmierczak, Jr. 1993. Factors Influencing Technology Adoption in a Louisiana Aquaculture System. *J. Agr. And Applied Econ.* 26(1), July, 1994:264-274.
- Demaris, Alfred. 1992. Logit Modeling Practical Aoulication. Sage University Paper Series on Quantitative Applications in the Social Sciences, 07-086, Newbury Park, CA, 87p.
- Greene, William. 1990. Econometric Analysis. Macmillan Publishing Company, New York, 783p.

Hansen, Mark H, Thomas Frieswyk, Joseph F. Glover, John F. Kelly. 1992. The eastwide forest inventory data base: Users Manual. United States Department of Agriculture, Forest Service. North Central Forest Experiment Station. General Technical Report NC- 15 1. 48p.

Hughes, Gordon A., Jacqueline L. Robertson and N.E. Savin. 199 1. Multinomial **Logit** Analysis 'of the Effects of Chemical Mixtures. Journal of Economic Entomology, Vol. 84, no. 6. 1957-1968.

Kmenta, Jan. 1986. Elements of Econometrics. Macmillan Publishing Company, New York, 786p.

Madalla, G.S. 1983. Limited-dependent and aualitative variables. in econometrics. Econometric Society Monographs No.3, Cambridge University Press, 401 p.

Pmdyck, Robert S., Daniel L. Rubinfeld. 1981. Econometric Models and Economic Forecasts. MCGRAW-Hill International Editions, Economics Series. 630p.

Wrigley, Neil. 1976. Introduction to the use of loeit models in geography. Geo abstracts Ltd., University of East Anglia, Norwich, NR4 7TJ, 32p.